High-energy reaction cross sections of light nuclei

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(Received 26 September 1988)

The high-energy reaction cross sections of Li and Be isotopes are calculated using a simplified Glauber model and densities constrained by the empirical binding energies. We find excellent agreement with experiment, reproducing the large increase for the most neutron-rich nuclei.

The reaction cross sections of unstable nuclei can now be measured, using exotic isotope beams produced by projectile fragmentation in high-energy heavy-ion collisions.[1,2] An important motivation for these measurements is to determine the size of very neutron-rich nuclei. A simple analysis of the cross sections was made by Tanihata *et al.*,[1] who inferred the nuclear radius from the formula

$$\sigma_I = \pi[R_I(p)+R_I(t)]^2 \ . \tag{1}$$

Here $R_I(p)$ and $R_I(t)$ are the projectile and the target radius, respectively. These authors found that the radii of ^{11}Li, ^{12}Be, and ^{14}Be were much larger than expected from the standard formula $R=1.2A^{1/3}$ fm, where A is the mass number. They also checked Eq. (1) with Glauber calculations using phenomenological densities.

Theoretical densities from Hartree-Fock theory were applied to Glauber model calculations by Sato and Okuhara.[3] Their calculated results show reasonable agreements with experimental data on He, Li, and the lighter Be isotopes. The cross section of ^{11}Li is underpredicted, which has led to suggestions that effects beyond Hartree-Fock theory cause the larger cross sections for neutron-rich nuclei. For example, recently Hansen and Jonson[4] revived the dineutron model of Migdal,[5] assuming that a dineutron orbits around a ^{9}Li core. They also investigated the possibility of Coulomb excitation contributing to the reaction cross section. However, this is surely negligible under the conditions of the experiment. According to Ref. 6, the Coulomb excitation of the giant dipole resonance in a light nucleus by heavy target is about 40 mb. One gains a factor of the logarithm of the relative excitation energies when exciting a low-lying state, but this is more than offset by the dependence on the target charge which is quadratic. Coulomb excitation by a carbon target should be completely negligible. However, it could be tested by varying the target Z.

In this paper we will present a refined calculation of the cross sections extending to the extreme neutron-rich Be isotopes. The two ingredients of the study are the reaction cross section model and the densities. We use a simplified Glauber model to calculate the reaction cross section, which is discussed below. Our densities are obtained from mean-field theory constrained to fit the separation energy of the last neutron. This constraint turns out to be an important difference with respect to Ref. 3.

We want to calculate the probability that two nuclei interact, neglecting all transverse motion while the nuclei pass each other. We consider the interactions to take place between independent tubes in the projectile and target. These tubes are oriented along the beam axis, as shown in Fig. 1. The probability of interaction for i and j tubes can be written by $P_{ij}n_in_j$ where n_i (n_j) is the number of nucleons in the tube i (j) and P_{ij} is the absorption probability associated with the nucleon-nucleon interaction. Then the probability of transmission for tubes i,j is given by

$$1-P_{ij}n_in_j \approx \exp(-P_{ij}n_in_j) \ .$$

We further assume that all the tubes contribute independently to the absorption of the projectile. Then the total transmission probability can be expressed as the product

$$\prod_{i,j} \exp(-P_{ij}n_in_j) = \exp\left[-\sum_{i,j} P_{ij}n_in_j\right] \ .$$

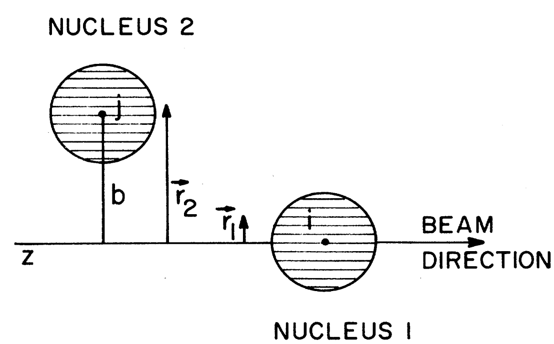

FIG. 1. Diagrammatic representation of the nucleus-nucleus collision. Each nucleus is divided into tubes denoted by i and j. The value b is the impact parameter.

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Taking the limit of small tubes, the above sums are replaced by integrals

$$\exp\left[-\int d^2r_1\int d^2r_2P(\mathbf{r}_1,\mathbf{r}_2)\rho_z'(\mathbf{r}_1)\rho_z^2(|\mathbf{r}_2-\mathbf{b}|)\right],$$

where ρ_z is defined by $\rho_z(\mathbf{r})=\int dz\,\rho(r^2+z^2)^{1/2}$. For a finite-range interaction the nucleon-nucleon absorption probability is $P(\mathbf{r}_1,\mathbf{r}_2)=f(|\mathbf{r}_1-\mathbf{r}_2|)\sigma_{nn}$ where the range function f is normalized by $\int fd^2r=1$. The nucleus-nucleus reaction cross section is then given by

$$\sigma_R=2\pi\int bdb\left[1-\exp\left[-\sigma_{nn}\int d^2r_1\int d^2r_2f(|\mathbf{r}_1-\mathbf{r}_2|)\rho_z'(\mathbf{r}_1)\rho_z^2(|\mathbf{r}_2-\mathbf{b}|)\right]\right].\quad(2)$$

In the calculation below, we will use both a zero-range interaction, with a delta function, and a finite-range interaction of the form $f(r)=\exp(-r^2/r_0^2)/\pi r_0^2$. For the zero-range interaction Eq. (2) becomes

$$\sigma_R=2\pi\int bdb\left[1-\exp\left[-\sigma_{nn}\int d^2r_1\rho_z'(\mathbf{r}_1)\rho_z^2(|\mathbf{r}_1-\mathbf{b}|)\right]\right],\quad(3)$$

which was first applied to heavy-ion reaction cross sections in Ref. 7. The formulas (2) and (3) are approximations to the Glauber model, but they are entirely adequate for our present purpose. (The usual treatment of the Glauber model[3] expands in powers of the nucleon-nucleon scattering amplitude, requiring higher order calculation to achieve equivalent accuracy.)

We used an effective cross section $\sigma_{nn}=40$ mb and an interaction range $r_0=1$ fm in the following calculations. This interaction range is consistent with the absorptive t matrix of Ref. 8.

We first calculated the densities by Hartree-Fock theory, which we later modified as described below. The Hartree-Fock calculations were carried out in a similar way to earlier studies,[9] assuming spherical symmetry and occupation numbers determined by single-particle energies. The interaction was the Skyrme parametrization SGII.[10] The resulting densities do not reproduce the sharp rise in the cross section of ^{11}Li, as was also found in the Hartree-Fock calculations of Ref. 3. Also, the ground state of ^{11}Be is incorrectly predicted to be $\frac{1}{2}^-$ rather than $\frac{1}{2}^+$.

The Hartree-Fock theory is evidently unable to reproduce the separation energies to better than an MeV or so, which can have a marked effect on the density distribution near the drip line. We therefore decided to recompute the densities using a potential that reproduces the empirical separation energies of the neutrons. This is done by renormalizing the SGII single-particle potential by a factor depending on orbital. As a practical matter, only the nuclei of mass 11 and higher are affected by this procedure. The p-orbital potential in the neutron-filled shells of ^{11}Li and ^{12}Be is adjusted to fit the one-neutron separation energy. This requires a renormalization factor in the range 0.82–0.86.

There is a valence pair in ^{14}Be in the s-d shell, and here we adjusted the sd-orbit energies to fit the one half of the two-neutron separation energy of ^{12}Be. The nucleus ^{11}Be has a single nucleon in the s-d shell, so here we fit the one-neutron separation energy. In these cases, the normalization factor is close to 1.

The neutron(s) in the s-d shell will have partial occupation probabilities in the different orbits, which affects the density distribution. The calculations of Ref. 11 give probabilities for the ^{11}Be neutron as 0.26, 0.03, and 0.71 for the $d_{5/2}$, $d_{3/2}$, and $s_{1/2}$ orbits. For ^{14}Be we assumed the same probability per valence neutron. The densities are then computed from the single-particle wave functions of the renormalized potential, weighted with the appropriate occupation probabilities.

The predicted rms point nuclear matter radii of Be and Li isotopes are listed in Table I. The calculated charge radii are also shown in Table I together with available experimental data. The calculated reaction cross sections for Li and Be isotopes on ^{12}C target are shown in Figs. 2 and 3. Two lines correspond to results with (without) the effect of the finite-range interaction. In the case of Li isotopes, the cross section varies smoothly with mass until ^{9}Li. This agrees with the results of Ref. 3. There is a sudden change in the radii between ^{9}Li and ^{11}Li. Our calculation reproduces this change fairly well, while that of Ref. 3 did not. The essential difference stems from the constraint for the separation energy of the last neutron orbit in ^{11}Li which is 1 MeV. The loosely bound neutrons in the last orbit increase the rms radius and also the reaction cross section considerably.

We can see in Figs. 2 and 3 that the results with the finite-range interaction are systematically 10% larger than those of the zero-range one. This is easily understood in the following way. As we can see in Eq. (2), the finite-range factor $f(r)$ plays the same role in Eq. (2) as the finite-size factor does in the analogous equation for the nuclear density. In both cases, the surface part of the density is increased while the inner part is decreased. Since the strong absorption radius is in the outer surface, this change increases the reaction cross section.

The reaction cross sections of Be isotopes on ^{12}C target are shown in Fig. 3. As in the case of Li isotopes, the cross section varies smoothly with mass up to $A=10$. Above $A=10$ we can see large variations of both the predicted and experimental cross sections. The ground state of ^{11}Be has spin $J^\pi=\frac{1}{2}^+$ and is assigned a configuration with one neutron in the $0s1d$ shell, while the first excited state has the spin $J^\pi=\frac{1}{2}^-$ which suggests the unpaired

TABLE I. rms radii and separation energies for Li and Be isotopes. The matter radius is calculated with the point-density operator taking into account the center-of-mass effect, while the charge radius is obtained from the proton density folded the finite nucleon size of 0.65 fm. The charge radii data are from Ref. 12, while the data for separation energies are taken from Ref. 13.

A	J^π	S_n (MeV)	S_{2n} (MeV)	Matter radius (fm) Theory	Charge radius (fm) Theory	Charge radius (fm) Experiment
^{6}Li	1^+	−5.7	−27.2	2.034	2.118	2.57(10)
^{7}Li	$\frac{3}{2}^-$	−7.2	−12.9	2.072	2.139	2.41(10)
^{8}Li	2^+	−2.0	−9.3	2.176	2.157	
^{9}Li	$\frac{3}{2}^-$	−4.1	−6.1	2.224	2.132	
^{11}Li	$\frac{3}{2}^-$	−1.0(3)	−0.19(11)	2.846	2.249	
^{7}Be	$\frac{3}{2}^-$	−10.7		2.086	2.330	
^{8}Be	0^+	−18.9	−29.6	2.152	2.320	
^{9}Be	$\frac{3}{2}^-$	−1.7	−20.6	2.182	2.284	2.519(12)
^{10}Be	0^+	−6.8	−8.5	2.253	2.295	
^{11}Be	$\frac{1}{2}^+$	−0.5	−7.3	2.772	2.372	
	$\frac{1}{2}^-$	−0.18	−7.1	2.722	2.360	
^{12}Be	0^+	−3.2	−3.7	2.572	2.397	
^{14}Be	0^+		−1.3(1)	3.607	2.426	
^{12}C	0^+			2.309	2.467	2.471(6)

neutron stays in the p shell. We calculated both configurations constraining the single-particle energy to reproduce the empirical neutron separation energy. The calculated cross sections, shown in Fig. 3, are nearly the same and agree well with the measured value. More surprisingly, our predicted results reproduce the small decrease of the ^{12}Be cross section and large increase of the ^{14}Be cross section as a function of mass. This trend stems entirely from the effective separation energy of each nucleus which is 0.5 MeV for ^{11}Be, 3.2 MeV for

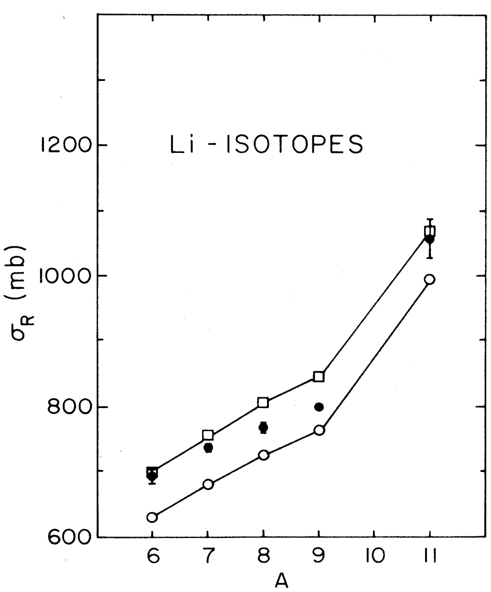

FIG. 2. The reaction cross section for Li isotopes on ^{12}C target. A solid line connecting boxes (circles) shows the theoretical cross sections obtained by Eq. (2) [Eq. (3)] with the finite-range interaction (zero-range interaction). The nucleon-nucleon cross section parameter σ_{nn} is taken as 40 mb, while the finite-range size r_0 of the interaction in Eq. (2) is taken as 1 fm. The data are taken from Ref. 1.

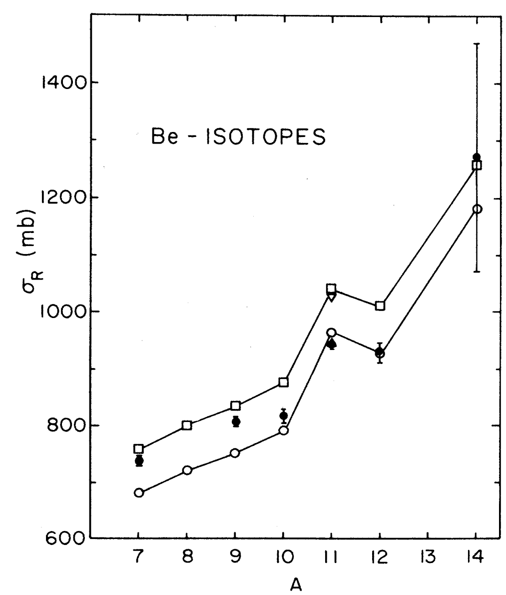

FIG. 3. The reaction cross section for Be isotopes on ^{12}C target. Two solid lines connecting boxes and circles show the theoretical cross sections obtained by Eqs. (2) and (3), respectively. The triangles for ^{11}Be show the results for the first excited state. The data are taken from Ref. 1. For details, see the caption to Fig. 2.

^{12}Be, and 0.6 MeV for ^{14}Be.

In Table I we notice that the calculated rms charge radii of ^{6}Li, ^{7}Li, and ^{9}Be are somewhat smaller than the available experimental data. The calculated charge radii are increased when one uses the Sk V Skyrme interaction[14] and become close to the experimental data. For example, the charge radius obtained from Sk V is 2.42 fm, which is close to the empirical one of 2.519 fm. This change of the density distribution increases the reaction cross section for the point interaction (the finite-range interaction) to $\sigma_R = 785$ mb (864 mb) from the case of SGII force $\sigma_R = 750$ mb (833 mb). However, the difference of the two calculations is only a few percent and does not make any significant contribution to our final conclusion.

We took the value $\sigma_{nn} = 40$ mb as the effective nucleon-nucleon cross section for all calculations. This value reproduces the reaction cross sections of light nuclei to within a few percent accuracy. To see the sensitivity to this parameter, we decreased σ_{nn} by 20% and found that the heavy-ion cross sections are reduced by 7%. A reduction of the range of the interaction by 20% reduces σ_R by 2.5%. Medium effects such as Pauli blocking could require the interaction to be modified on these scales. However, we have not attempted such refinements since our main goal is to understand the systematics going to the neutron-rich nuclei.

In summary, we see that reaction cross sections of light nuclei can be calculated quite well with a simple reaction model and densities from Hartree-Fock theory. The main ingredient of the calculations is the density which is obtained from the Hartree-Fock single-particle wave functions combined with the shell model occupation probabilities. The essential difference in our calculation to previous ones is the constraint that the separation energy of the last occupied orbit reproduce the experimental value. We found that both the rms radius and the reaction cross section are very sensitive to the separation energy. Surprisingly, our calculated results reproduce the large increases of the cross section of unstable nuclei ^{11}Li, ^{12}Be, and ^{14}Be fairly well.

One of the authors (H.S.) acknowledges the hospitality of the Michigan State University Cyclotron Laboratory. The research was supported in part by the National Science Foundation under Grant No. PHY-87-14432.

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