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Parity inversion in the $N=7$ isotones and the pairing blocking effect

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The parity inversion anomaly in the spectra of $N=7$ isotones is discussed based on contemporary shell-model wave functions It is shown that the proton–neutron monopole interaction in the mean field has a large effect on the energy gap between the $\frac{1}{2}^-$ and $\frac{1}{2}^+$ states in ^{11}Be, but not enough to explain the inverted spectrum We point out that the effects of quadrupole core excitation and pairing blocking are equally important to make the parity inversion in ^{11}Be It is predicted that ^{9}He has also a parity inversion in the ground state due to the pairing blocking

The essential mechanism which causes the parity inversion in ^{11}Be has been a long-standing open question [1] the $J^\pi=\frac{1}{2}^+$ state is the ground state of ^{11}Be, while the naive shell-model would predict a $J^\pi=\frac{1}{2}^-$ state as the ground state of this odd–even 1p-shell nucleus Although many large-basis shell-model calculations [2–6] have been carried out, they do not all reproduce the parity inversion For those calculations that do reproduce the inversion [7–9,5], it is important to go beyond just the numerical results and to understand the qualitative physical mechanisms responsible In this letter we explore this issue from the weak coupling picture of the core and valence particle The validity of this model is discussed in comparison with realistic shell model wave functions We also study how these mechanisms affect other odd–even $N=7$ isotones, in particular ^{9}He The shell-model calculations are performed by using the recent Warburton–Brown effective interaction [7] This new interaction was obtained from a least squares fit of single-particle energies, two-body matrix elements and two-body interaction potential strengths to 51 1p shell data and 165 1p–1d2s (cross shell) data on binding energies and excitation energies in nuclei with $A=10$–22 The RMS deviation between the theoretical and experimental energies was about 330 keV This interaction represents a significant improvement over the Millener–Kurath interaction [3] in many ways, but most importantly for this work, in its ability to accommodate a realistic N and Z dependence of the single-particle energies over a wide range of nuclei

In the early 1960s, Talmi and Unna first noticed the parity inversion problem in ^{11}Be [1] and suggested that the proton–neutron monopole interaction in the mean field was the main cause We have performed Hartree–Fock (HF) calculations for the $N=7$ isotones to study the effect of the proton–neutron interaction in the mean field spectra The HF results in fig 1 are obtained by using the Skyrme force SGII [10], and the particles are assumed to occupy the lowest single-particle orbits in the HF potential in order One observes large changes in the single-particle energies of the 1p- and 2s1d-shell orbits when one decreases the proton numbers This is due to a shallower neutron potential for smaller $N=7$ nuclei, and the neutron–proton interaction is responsible for this change Notice also the inversion of the $2s_{1/2}$-state and $1d_{5/2}$-orbits which is caused by a smaller kinetic energy for the loosely-bound $2s_{1/2}$ than that of $1d_{5/2}$

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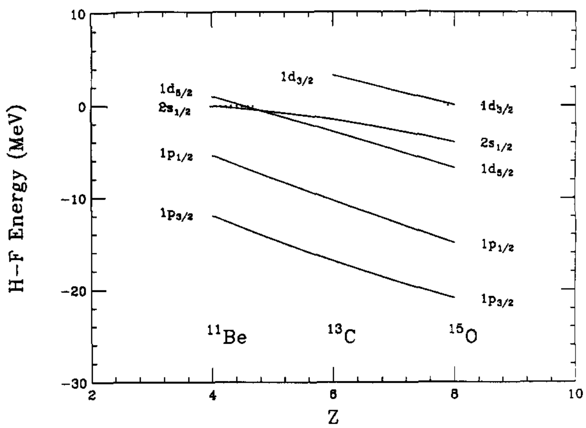

Fig 1 HF single particle energies for the $N=7$ isotones calculated with the Skyrme interaction SGII [10]

However, the energy gap between the $2s_{1/2}$- and $1p_{1/2}$-state is still larger than 5 MeV and the proton–neutron monopole interaction is thus not enough to explain the parity inversion

We note that this 5 MeV is the energy difference of the $2s_{1/2}$ single-particle state and the $1p_{1/2}$ single-hole state of "^{12}Be", where by "^{12}Be" we mean the semi-closed 1p-shell configuration for this nucleus The corrections which remain to be considered are just the 1p-shell proton pairing for the $1p_{1/2}$ state But for the $2s_{1/2}$ state we need to consider in addition the effect of removing two neutron holes which leads to contributions from (1) the quadrupole collectivity of ^{10}Be, (2) the 1p-shell neutron pairing, and (3) the change in monopole interaction It turns out with the Warburton–Brown interaction that (3) can be ignored We will now discuss the contributions from (1) and (2)

A large deformation can also give an inverted spectrum of the positive- and negative-parity states the positive-parity state with the asymptotic quantum number $[Nn_3\Lambda\Omega]=[220\,\frac{1}{2}]$ comes lower in energy than the negative parity state $[101\,\frac{1}{2}]$ near the quadrupole deformation $\beta_2\simeq 0\ 7$ in the deformed Nilsson diagram (see ref [11] p 221) These states trace back to $1d_{5/2}$- and $1p_{1/2}$-states, respectively, in the spherical limit Ragnarson et al [12] performed cranking shell-model calculations with these Nilsson wave functions In their calculations with different cranking Hamiltonians, the $\frac{5}{2}^+$ state came lower in energy than or at most about the same energy as the $\frac{1}{2}^+$ state This is due to the fact that the deformed Nilsson wave function with the asymptotic quantum number $[Nn_3\Lambda\Omega]=[220\,\frac{1}{2}]$ at $\beta_2\simeq 0\ 7$ is 80% $1d_{5/2}$, and the $2s_{1/2}$-state is a small component Since the $\frac{5}{2}^+$-state must be an excited state with $E_x\geqslant 1\ 7$ MeV in the experiment, a large deformation is unlikely in the ground state of ^{11}Be It has also been pointed out that a large deformation cannot explain the experimental data for the total reaction cross sections of ^{11}Be both at high and medium energy heavy ion experiments [13] This is due to the fact that the dominant $1d_{5/2}$-wave function in the Nilsson $[220\,\frac{1}{2}]$ state does not have a large halo due to the centrifugal barrier

Although a large deformation in the intrinsic frame is unlikely, the 2^+ core excitation coupled with the $1d_{5/2}$-state is an important component in the lowest $\frac{1}{2}^+$ shell-model state in ^{11}Be We will now estimate the effect of the mixing of the core excitation The $\frac{1}{2}^+$ ground state of ^{11}Be can be expressed approximately as [14]

$$|\tfrac{1}{2}^+\rangle=\alpha_s\,|2s_{1/2}\times 0^+\rangle+\alpha_d\,|1d_{5/2}\times 2^+\rangle\,, \qquad (1)$$

where 0^+ and 2^+ are the ground state and the first excited state at $E_x=3\ 37$ MeV in ^{10}Be and α_d will be calculated in perturbation theory The single-particle energy of the $1d_{5/2}$-state is 1–2 MeV higher than that of the $2s_{1/2}$-state in the HF calculations, and thus the total energy denominator for α_d is about 5 MeV The coupling matrix element $\langle 2s_{1/2}\times 0^+|V_{\rm coup}|1d_{5/2}\times 2^+\rangle$ is proportional to the transition amplitude of the core excitation in the particle–vibration coupling model (see ref [11] p 417) Using the experimental transition strength from the ground state to the 2^+ state in ^{10}Be which is several times larger than the single-particle unit, the coupling matrix element is estimated to be 2 7 MeV Thus $\alpha_d^2\sim 0\ 20$ and the energy of the ground state is lowered by about 1 5 MeV This estimate is consistent with the full shell-model calculation as well as the fact that the ^{10}Be to ^{11}Be $2s_{1/2}$ spectroscopic factor is large We note, however, that the most direct test of the α_d^2 component will come from a measurement of the ^{11}Be to ^{10}Be 2^+ $1d_{5/2}$ spectroscopic factor using a radioactive beam

Finally, we will discuss the effect of the pairing correlations The specific issue we address is the Pauli blocking effect due to the presence of the last neutron in $N=7$ isotones Suppose the dominant configuration of the ground state is $2s_{1/2}$ and the first excited state is $1p_{1/2}$ which is the case in ^{11}Be There are four available pairing configurations in the 1p-shell orbits for the ground state in ^{10}Be as is shown in fig 2 The configurations (a) and (b) are available for both the positive- and the negative-parity states On the other hand, the configurations (c) and (d) contribute to the positive-parity state, but not to the negative-parity state because of the Pauli blocking effect due to the presence of one additional neutron in the $1p_{1/2}$-orbit The difference makes a substantial effect on the off-diagonal pairing matrix element as we will now show We note that even though the diagrams on fig 2 are based upon a ^{10}Be core, we only use this core with respect to an evaluation of the pairing and quadrupole contributions, and, as mentioned above, the single-particle energy themselves are with respect to a ^{12}Be core

Let us now study the pairing correlations quantitatively by using more realistic shell model wave functions We expand the even–even core part of the shell-model wave function as

$$|i\rangle = \sum_{\alpha==a,b,c,d} C(i,\alpha)|\alpha\rangle\,, \tag{2}$$

where $C(i,\alpha)$ is the shell-model amplitude for each

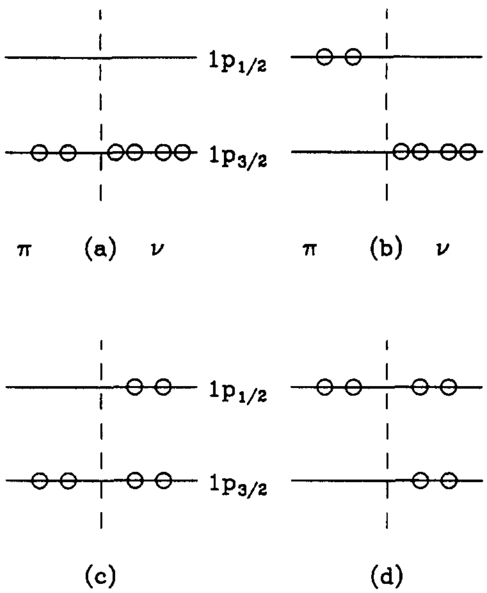

Fig 2 Pairing configurations for the ^{10}Be core All configurations contribute to the pairing correlation for the $\frac{1}{2}^+$ state, while only two configurations (a) and (b) contribute for the $\frac{1}{2}^-$ state

configuration The wave functions are calculated with a full $0\hbar\omega$-configuration space for negative-parity states and a full $1\hbar\omega$-configuration space for positive-parity states by using the same effective interaction [7] The pairing matrix element for the $\frac{1}{2}^-$ state is

$$\begin{aligned}\langle \tfrac{1}{2}^- | V_{\rm pair} | \tfrac{1}{2}^- \rangle &= C(\tfrac{1}{2}^-,a)^2\langle a|V_{\rm pair}|a\rangle + C(\tfrac{1}{2}^-,b)^2\langle b|V_{pair}|b\rangle \\ &+2\times C(\tfrac{1}{2}^-,a)C(\tfrac{1}{2}^-,b)\langle a|V_{\rm pair}|b\rangle \end{aligned} \tag{3}$$

There is only one cross term in eq (3) The matrix element for the positive-parity state has more terms

$$\begin{aligned}\langle \tfrac{1}{2}^+ | V_{\rm pair} | \tfrac{1}{2}^+ \rangle &= C(\tfrac{1}{2}^+,a)^2\langle a|V_{\rm pair}|a\rangle + C(\tfrac{1}{2}^+,b)^2\langle b|V_{\rm pair}|b\rangle \\ &+C(\tfrac{1}{2}^+,c)^2\langle c|V_{\rm pair}|c\rangle + C(\tfrac{1}{2}^+,d)^2\langle d|V_{\rm pair}|d\rangle \\ &+2\times C(\tfrac{1}{2}^+,a)C(\tfrac{1}{2}^+,b)\langle a|V_{\rm pair}|b\rangle \\ &+2\times C(\tfrac{1}{2}^+,a)C(\tfrac{1}{2}^+,c)\langle a|V_{\rm pair}|c\rangle \\ &+2\times C(\tfrac{1}{2}^+,b)C(\tfrac{1}{2}^+,d)\langle b|V_{\rm pair}|d\rangle \\ &+2\times C(\tfrac{1}{2}^+,c)C(\tfrac{1}{2}^+,d)\langle c|V_{\rm pair}|d\rangle \end{aligned} \tag{4}$$

The two-body pairing matrix elements for 1p-shell nuclei are

$$\langle (p_{3/2})^2|V_{\rm pair}|(p_{3/2})^2\rangle = -3\ 85\ \text{MeV},$$
$$\langle (p_{3/2})^2|V_{\rm pair}|(p_{1/2})^2\rangle = -3\ 84\ \text{MeV},$$
$$\langle (p_{1/2})^2|V_{\rm pair}|(p_{1/2})^2\rangle = -1\ 22\ \text{MeV}$$

for the present calculations [7] The pairing energy difference between the two states is found to be large

$$\begin{aligned}\Delta E_{\rm pair} &= \langle \tfrac{1}{2}^- | V_{\rm pair} | \tfrac{1}{2}^- \rangle - \langle \tfrac{1}{2}^+ | V_{\rm pair} | \tfrac{1}{2}^+ \rangle \\ &= 2\ 19\ \text{MeV}\end{aligned} \tag{5}$$

The 5 MeV energy difference between the $2s_{1/2}$ and the $1p_{1/2}$ states in the mean field, as shown in fig 1, is thus effectively reduced to about zero by a combination of the core excitation and the Pauli blocking effects The blocking effect exists also for other $N=7$ nuclei In ^{9}He, the value $\Delta E_{\rm pair}$ is 3 28 MeV which could result in another inverted spectrum in this nucleus The value of $\Delta E_{\rm pair}$ in ^{13}C is 2 42 MeV which is not enough to make the parity inversion because of the larger single-particle energy gap, and a somewhat smaller effect of the 2^+ core excitation compared to ^{11}Be

In summary, we have studied the possible mechanisms for the parity inversion spectrum of ^{11}Be We have pointed out that the core excitation to the first 2^+-state and the pairing blocking effect are both important to produce the parity inversion The proton–neutron monopole interaction in the mean field is significant, but not enough to make the inverted spectra as in shown in fig 1 Our study suggests a possible parity inversion also in ^{9}He The combined effects of 2^+ core excitation and Pauli blocking are important for reducing the excitation energy of "core excited" states of nuclei in a wide region of the mass table [15,16] However, it is only when these are combined with the reduced single-particle spacing, which occurs for loosely bound states in nuclei far from stability, that the peculiar feature of "parity inversion" and "islands of inversion" occurs

The feature we have discussed here for the $N=7$ isotones has interesting implications for other nuclei in this mass region near the neutron-drip For example, the Warburton–Brown interaction [7] predicts that the configurations with two neutrons excited from the 1p to the 1d2s shell are degenerate with the 1p shell configuration for ^{12}Be and ^{11}Li because of the same physical reasons described in this letter It will be important in future theoretical investigations to explore the consequences of the mixing between these 1p and 1d2s configurations [17] and the relationship with the cluster and three-body continuum models which have been used for ^{11}Li [18]

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