Toward a Consistent Description of Parity Nonconservation in $A = 18-21$ Nuclei

Mihai Horoi[1,2] and B. Alex Brown[1]

[1]*National Superconducting Cyclotron Laboratory, East Lansing, Michigan 48824*
[2]*Institute of Atomic Physics, Bucharest, Romania*
(Received 10 August 1994)

The current theoretical analyses of the parity nonconserving (PNC) experiments in light nuclei fail to provide a consistent description of the experimental data for ^{18}F, ^{19}F, and ^{21}Ne. We perform a "piecewise" analysis of the PNC matrix elements in the $0p_{1/2}$-$0d_{5/2}$-$1s_{1/2}$ model space and show that by using a proper overall normalization of the isoscalar and isovector parts we can consistently describe all three PNC experimental data. New calculations are also presented for the s-p-sd-pf model space which support our analysis.

The investigations of low energy parity nonconservation (PNC) phenomena in light nuclei have as a goal to provide more reliable results for the hadron-meson weak coupling constants. These couplings are of importance for our understanding of the quark behavior inside the nucleons under the influence of the fundamental interactions. These investigations necessitate both very delicate experiments and very reliable nuclear structure calculations of the matrix elements for a correct extraction of the weak nucleon-meson coupling constants.

Most of the results on the experimental and theoretical PNC studies in light nuclei have been presented in a review paper [1]. From the proposed cases during the last 25 years in this range of nuclei, four cases have been selected as reliable enough for experimental and theoretical analysis. They involve parity mixed doublets (PMD) [1] in ^{14}N, ^{18}F, ^{19}F, and ^{21}Ne. Two others cases involving PMD's in ^{16}O [2] and ^{20}F [3] have been proposed recently. From the four mentioned cases, only the ^{19}F has been measured with a result larger then the experimental error. All other cases have been measured with errors larger (^{18}F and ^{21}Ne) or near the result (^{14}N). However, the upper limits obtained for ^{18}F and ^{21}Ne are so small that they impose severe constraints on the different contributions to the PNC matrix elements. These results, compared to current theoretical calculations, have shown a discrepancy, which has not yet been solved (see also Fig. 11 from Ref. [4]). Namely, if one interprets the small limit of the (experimentally) extracted PNC matrix element (<0.029 eV) for ^{21}Ne as a destructive interference between the isoscalar and the isovector contribution [1], then it is difficult to understand why the isovector contribution in ^{18}F is so small (<0.09 eV) and the constructive isoscalar + isovector contribution in ^{19}F is relatively large (0.40 ± 0.10).

In the last few years, efforts have been made to improve the shell-model calculations with special emphasis on the description of the weak observables [5,6]. Recently, two new interactions have been developed by Warburton and Brown [7], which were designed to accurately describe the energies of states in nuclei with $A = 10-22$. The interaction was designed for pure oscillator excitations [e.g., $0\hbar\omega$ $(0p)^n$ configurations, $1\hbar\omega$ $(0p)^{n-1}(0d1s)^1$ configurations, etc.], but methods have been developed to use them when mixed $n\hbar\omega$ excitations contribute [6,8]. Recently, we performed a $(0 + 1 + 2 + 3 + 4)\hbar\omega$ calculation in ^{14}N [8] and we obtained a PNC matrix element with a magnitude consistent with the experimental result [9,10]. These results encouraged us to look to the effect of the higher $n\hbar\omega$ excitations for the ^{18}F, ^{19}F, and ^{21}Ne cases. The previous theoretical analysis of these cases is based on the calculations of Haxton [1]: $(0 + 1 + 2)\hbar\omega$ for ^{18}F (this result was shown to be comparable with the matrix element extracted almost model independent from the first forbidden beta decay of ^{18}Ne [11]) and $(0 + 1)\hbar\omega$ calculations for ^{19}F and ^{21}Ne.

At present, it is impossible to perform $(0 + 1 + 2 + 3 + 4)\hbar\omega$ calculations for all of these nuclei in the full p-sd model space or any extension of it. However, the small Zucker-Buck-McGrory (ZBM) model space [12] contains the most important PNC transition $(0p_{1/2} \rightarrow 1s_{1/2})$ and, moreover, includes up to $4\hbar\omega$ excitations. The ZBM model space assumes a ^{12}C core and $A - 12$ valence nucleons moving in the $0p_{1/2}$, $0d_{5/2}$, and $1s_{1/2}$ single particle orbitals. We thus reanalyze the results obtained in this model space with respect to the contribution of the higher $n\hbar\omega$ excitations and compare, when possible, with the results from the larger s-p-sd-fp model spaces. Table I presents the probabilities of the various $n\hbar\omega$ contributions to the wave functions of the parity mixed doublets in $A = 18-21$ nuclei. The calculations have been performed in the ZBM model space with the F-psd interaction [13]. One can see that in all cases $2\hbar\omega$, $3\hbar\omega$, and $4\hbar\omega$ contributions are significant. Similar results are obtained with all the available interactions in this model space [14].

It is interesting to see how the effect of these higher $n\hbar\omega$ amplitudes is reflected in the magnitude of the PNC matrix element. Table II presents the $n\hbar\omega \rightarrow (n + 1)\hbar\omega$ decomposition of the PNC matrix elements.

TABLE I. Probabilities for the $n\hbar\omega$ configurations obtained for the parity mixed doublets in $A = 18-21$ nuclei. F-psd indicates the ZBM model space and F-psd interaction; WBT indicates the $(0 + 1 + 2 + 3)\hbar\omega$ model space and WBT interaction [7]. 0, $2\hbar\omega$, and $4\hbar\omega$ apply for the positive parity states; $1\hbar\omega$ and $3\hbar\omega$ apply to the negative parity ones.

Nucleus	Model	$J^\pi T$	$0\hbar\omega$	$1\hbar\omega$	$2\hbar\omega$	$3\hbar\omega$	$4\hbar\omega$
	F-psd	0^+1	0.443		0.424		0.133
^{18}F	F-psd	0^-0		0.739		0.261	
	WBT	0^+1	0.554		0.446		
	WBT	0^-0		0.552		0.448	
^{19}F	F-psd	$\frac{1}{2}\pm\frac{1}{2}$	0.618	0.701	0.333	0.299	0.049
^{20}F	F-psd	$1^\pm1$	0.637	0.800	0.326	0.200	0.036
^{21}Ne	F-psd	$\frac{1}{2}\pm\frac{1}{2}$	0.463	0.697	0.422	0.303	0.115

In all our calculations we have used the Desplanques-Donoghue-Holstein (DDH) best values [15] for the weak coupling constants. One can observe the alternation of sign for different contributions which has been explained in Ref. [1] in a simple Nilsson quadrupole plus pairing scheme. The message of this behavior is that one has to take into account an appropriate number of $n\hbar\omega$ in order to "smooth out" this cancellation behavior. For instance, taking only $(0 + 1)\hbar\omega$ could give very distorted results due to the missing, and opposite in sign, $1 \rightarrow 2\hbar\omega$ contribution. This cancellation effect is particularly strong for the ^{18}F and ^{21}Ne nuclei. For both those nuclei the $2 \rightarrow 3\hbar\omega$ and $3 \rightarrow 4\hbar\omega$ contributions are also important. Moreover, in the ^{21}Ne case the most important contribution is $1 \rightarrow 2\hbar\omega$, so that the analysis based on $(0 + 1)\hbar\omega$ [1] turns out to be inappropriate for ^{21}Ne.

One cannot fully verify this conclusion in a larger model space. However, one can perform some tests for ^{18}F. A calculation for the parity mixed doublet in this nucleus has been carried out using the Warburton-Brown interaction in the first four major shells including up to $3\hbar\omega$ excitations. In all these calculations we see the following: (a) In the four major shell calculation the spuriousity due to the center-of-mass (c.m.) motion has been removed with the method described in Ref. [16]. Spuriousity was not removed from the ZBM wave functions. However, the ZBM model space has only a small c.m. spuriousity [17], and when a moderately large c.m. Hamiltonian is added to the ZBM Hamiltonian [18] the PNC matrix elements are changed by relatively little [19]. (b) The effect of the short-range correlations has been taken into account as described in Ref. [20]. (c) The effect of the Saxon-Woods tail of the single particle wave functions has been checked and found to be small. Tables I and II include the results for ^{18}F of the $n\hbar\omega$ decomposition of the wave functions and PNC matrix element calculated in the ZBM model space and in the four major shell model space calculated with the WBT interaction [7]. The various contributions are relatively similar. This gives us some confidence that the small model space calculations contain the most important physics necessary for the analysis of the PNC matrix elements in light nuclei.

Table III presents the isoscalar-isovector decomposition of the PNC matrix elements for these nuclei. The results are stable with respect to various interactions used. The isoscalar contribution in the ^{21}Ne case is small. This fact and the smallness of the pion weak coupling constant (as deduced from the ^{18}F experiment [1,4]) can explain the smallness of the PNC matrix element for ^{21}Ne. A similar conclusion with respect to the ^{21}Ne PNC matrix element

TABLE II. Partial contributions to the PNC matrix elements in $A = 18-21$ nuclei. Units are eV. F-psd and Z-psd interactions are taken from Ref. [13]. The DDH best values have been used as weak coupling constants.

Nucleus	Interaction	ΔT	$0\hbar\omega - 1\hbar\omega$	$2\hbar\omega - 1\hbar\omega$	$2\hbar\omega - 3\hbar\omega$	$4\hbar\omega - 3\hbar\omega$
	F-psd	1	1.045	-0.815	0.549	-0.187
^{18}F	Z-psd	1	1.119	-0.778	0.462	-0.148
	WBT	1	0.921	-0.502	0.533	?
		0	0.566	-0.097	0.227	-0.073
^{19}F	F-psd	1	0.744	-0.212	0.221	-0.032
		0	0.633	-0.134	0.184	-0.055
	Z-psd	1	0.858	-0.164	0.187	-0.023
		0	0.473	-0.026	0.099	-0.018
^{20}F	F-psd	1	0.806	-0.261	0.195	0.018
		0	0.446	-0.086	0.037	-0.007
	Z-psd	1	0.762	-0.222	0.108	0.002
		0	0.290	-0.370	0.139	-0.172
^{21}Ne	F-psd	1	-0.164	0.558	-0.095	0.257
		0	0.404	-0.348	0.092	-0.076
	Z-psd	1	-0.246	0.555	-0.053	0.103

TABLE III. Isoscalar (IS) and isovector (IV) contributions to the PNC matrix elements in $A = 18-21$ nuclei.

Nucleus	Interaction	IS	IV	Total
^{18}F	F-psd	-	0.592	0.592
	Z-psd	-	0.734	0.734
^{19}F	F-psd	0.627	0.722	1.349
	Z-psd	0.629	0.858	1.487
^{20}F	F-psd	0.518	0.757	1.262
	Z-psd	0.389	0.650	1.032
^{21}Ne	F-psd	-0.113	0.556	0.442
	Z-psd	0.071	0.359	0.430

FIG. 1. Analysis of the PNC result for ^{18}F, ^{19}F, and ^{21}Ne using Eq. (1) and ZBM F-psd calculations. Solid lines represent the limits imposed by the experimental errors. The dotted line is the experimental result for ^{19}F. The shadowed region indicates the consistency of the experimental result with the present analysis.

has been recently presented in Ref. [21]. A different conclusion has been presented by Brandenburg et al. [22], who claim that the isoscalar contribution is stable and the isovector contribution changes sign when F-psd or Z-psd interactions are used. We have consistently calculated the relative phases of the PNC matrix elements (and wave functions) for both interactions and find out that the sign of the isovector contribution is stable while the sign of the isoscalar part changes; the relative phase of Brandenburg et al. is incorrect.

One can go a step further in the analysis of all $A = 18-21$ results using a graphic picture, similar to Fig. 11 from Ref. [4]. One can write the PNC matrix element for all these cases in terms of the isoscalar (IS) and isovector (IV) contributions calculated with some "standard" weak coupling constants (DDH best values in our case) and some weighting factors, $\alpha_{IS(IV)}$ and $\beta_{IS(IV)}$

$$\langle V_{PNC} \rangle = \alpha_{IS}\beta_{IS} \langle V_{PNC}^{DDH}(IS) \rangle_{ZBM}$$
$$+ \alpha_{IV}\beta_{IV} \langle V_{PNC}^{DDH}(IV) \rangle_{ZBM} . \quad (1)$$

We note that the IS matrix element is dominated by the ρ exchange term proportional to h_ρ^0 and the IV matrix element is dominated by the π exchange term proportional to f_π [1,20]. The β factors take into account the renormalization effects due to the orbitals missing in the ZBM model space. Since the $0p_{3/2} \leftrightarrow 0d_{3/2}$ and $0d1s \leftrightarrow 0f1p$ transitions are small in amplitude, their effect should be perturbative and the β_{IS} and β_{IV} factors may be about the same for all three nuclei (^{18}F, ^{19}F, and ^{21}Ne). The results presented in Fig. 1 are based on the F-psd interaction [13]. A value of $\beta_{IV} = 0.59$ can be obtained from the comparison with the ^{18}Ne $\rightarrow$ ^{18}F first forbidden beta decay result [4]. A value of $\beta_{IS} = 0.48$ was estimated from a comparison with a recent $(0 + 1 + 2 + 3 + 4)\hbar\omega$ calculation [8] in ^{14}N. The α factors represent the ratio of the actual weak coupling constants to the DDH best values [15]. An $(\alpha_{IS}, \alpha_{IV})$ plot, similar to that in Fig. 11 from Ref. [4], is presented in Fig. 1; it shows an overlapping region for the ^{18}F, ^{19}F, and ^{21}Ne data. The $(\alpha_{IS}, \alpha_{IV})$ values in the overlapping region are in the range $(0.6-1.2, 0.07-0.26)$.
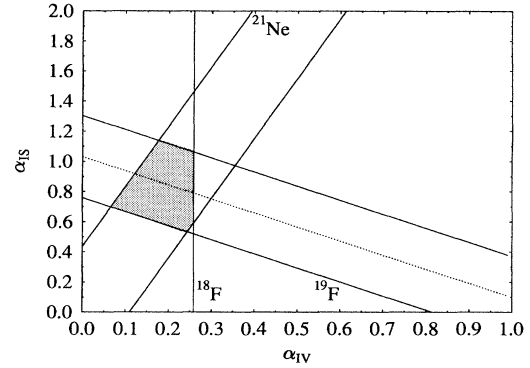
The F-psd and Z-psd interactions have been constructed [13] to describe the spectra, the electromagnetic transition probabilities, and the spectroscopic factors in nuclei up to $A = 20$. We checked the spectra of $A = 21$ and 22 nuclei and obtained very good agreement with the experiment. The electromagnetic transition probabilities analyzed in Table II of Ref. [23] have been calculated and shown to be in reasonable agreement with the results of the model of Ref. [23], and with the experimental values, when available (taking into account an estimate from the $0p_{3/2}$, $0d_{3/2}$, and $0f1p$ contributions). We have also examined other observables related to the $\frac{1}{2}^\pm$ states in ^{21}Ne to test the ZBM wave functions. The spectroscopic factors connecting the $\frac{1}{2}^\pm$ states with the ground states of the ^{20}Ne and ^{22}Ne have been calculated and are in very good agreement with the available experimental results from Table 21.12 of Ref. [24]. These comparisons give some confidence in the reliability of the ZBM wave functions.

In conclusion, we have theoretically analyzed the PNC experiments in $A = 18-21$ nuclei using the small ZBM model space with emphasis on the higher $n\hbar\omega$ contributions. We concluded that it is essential to estimate the effect of $2\hbar\omega$, $3\hbar\omega$, and $4\hbar\omega$ contributions. The present analysis suggests that the usual interpretation of the smallness of the ^{21}Ne matrix element due to a cancellation between the IS and IV contribution, which was obtained in a $(0 + 1)\hbar\omega$ calculation, has to be modified due to the strong $1 \rightarrow 2\hbar\omega$ contribution. We conclude that the correct mechanism is based on a very small IS part of the PNC matrix element, due to the nuclear structure involved, and the smallness of the IV part, due to the small pion weak coupling constant. Our analysis shows that a consistent understanding of the PNC experiments in $A = 18-21$ nuclei is possible if one includes

the appropriate number of $n\hbar\omega$ excitations in the nuclear structure calculations.

[1] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 501 (1985).

[2] N. Kniest, M. Horoi, O. Dumitrescu, and G. Clausnitzer, Phys. Rev. C **44**, 491 (1991).

[3] M. Horoi and G. Clausnitzer, Phys. Rev. C **48**, R522 (1993).

[4] S. A. Page et al., Phys. Rev. C **35**, 1119 (1987).

[5] W. C. Haxton and C. Johnson, Phys. Rev. Lett. **65**, 1325 (1990).

[6] E. K. Warburton, B. A. Brown, and D. J. Millener, Phys. Lett. B **293**, 7 (1992).

[7] E. K. Warburton and B. A. Brown, Phys. Rev. C **46**, 923 (1992).

[8] M. Horoi, G. Clausnitzer, B. A. Brown, and E. K. Warburton, Phys. Rev. C **50**, 775 (1994).

[9] V. J. Zeps et al., A.I.P. Conf. Proc. **176**, 1098 (1989).

[10] V. J. Zeps, E. G. Adelberger, A. García, C. A. Gossett, H. E. Swanson, W. Haeberli, P. A. Quin, and J. Sromicki (to be published).

[11] W. C. Haxton, Phys. Rev. Lett. **46**, 698 (1981).

[12] A. P. Zucker, B. Buck, and J. B. McGrory, Phys. Rev. Lett. **21**, 39 (1968).

[13] J. B. McGrory and B. H. Wildenthal, Phys. Rev. C **7**, 974 (1973); B. S. Reehal and B. H. Wildenthal, Part. Nuclei **6**, 137 (1973).

[14] B. A. Brown et al., MSUNSCL Report No. 524, 1988.

[15] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Ann. Phys. (N.Y.) **124**, 449 (1980).

[16] D. H. Gloeckner and R. D. Lawson, Phys. Lett. **53B**, 313 (1974).

[17] J. B. McGrory and B. H. Wildenthal, Phys. Lett. **60B**, 5 (1975); R. R. Whitehead, A. Watt, B. J. Cole, and I. Morrison, Adv. Nucl. Phys. **9**, 123 (1977).

[18] We multiplied the c.m. Hamiltonian by a factor of 20 as suggested in P. R. Rath, A. Faessler, H. Muther, and A. Watt, J. Phys. G **16**, 245 (1990), and checked that the quality of the spectrum and the overall strength of the PNC matrix elements are maintained.

[19] With the c.m. Hamiltonian added as described in [18] and F-psd interaction used, the following PNC matrix elements (in units of eV) are obtained for ^{18}F, ^{19}F, and ^{21}Ne, respectively (see also Table III): (IS) 0.0, 0.517, and 0.084; (IV) 0.616, 0.725, and 0.470.

[20] B. A. Brown, W. A. Richter, and S. Godwin, Phys. Rev. Lett. **45**, 1681 (1980).

[21] B. Desplanques and O. Dumitrescu, Nucl. Phys. **A565**, 818 (1993).

[22] R. A. Brandenburg et al., Phys. Rev. Lett. **41**, 618 (1978).

[23] D. J. Millener, E. K. Warburton, K. A. Snoover, R. von Lintig, and P. G. Ikossi, Phys. Rev. C **18**, 1878 (1978).

[24] P. M. Endt, Nucl. Phys. **A521**, 1 (1990). For comparison with the spectroscopic factors in Table 21.12 we give the following values obtained with the F-psd interaction: S_n^- are 1.0, 0.22, and 2.79 for the $\frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{5}{2}^+$ states, respectively; $(2J + 1)S_n^+$ are 0.46, 1.18, and 3.47 for the $\frac{1}{2}^-$, $\frac{1}{2}^+$, and $\frac{5}{2}^+$ states, respectively.