

## Isospin-mixing corrections for *fp*-shell Fermi transitions

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Isospin-mixing corrections for superallowed Fermi transitions in *fp*-shell nuclei are computed within the framework of the shell model. The study includes three nuclei that are part of the set of nine accurately measured transitions as well as five cases that are expected to be measured in the future at radioactive-beam facilities. We also include some new calculations for  $^{10}\text{C}$ . With the isospin-mixing corrections applied to the nine accurately measured *ft* values, the conserved-vector-current hypothesis and the unitarity condition of the Cabibbo-Kobayashi-Maskawa matrix are tested.

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Superallowed Fermi  $\beta^-$  transitions in nuclei, ( $J^\pi=0^+, T=1 \rightarrow J^\pi=0^+, T=1$ ), provide an excellent laboratory for precise tests of the properties of the electroweak interaction, and have been the subject of intense study for several decades (cf. Refs. [1–5]). According to the conserved-vector-current (CVC) hypothesis, the *ft* values for pure Fermi transitions should be nucleus independent, and given by

$$ft = \frac{K}{G_V^2 |M_F|^2}, \quad (1)$$

where  $K/(\hbar c)^6 = 2\pi^3 \ln 2 \hbar / (m_e c^2)^5 = 8.120\,270(12) \times 10^{-7} \text{ GeV}^{-4} \text{ s}$ ,  $G_V$  is the vector coupling constant for nuclear  $\beta^-$  decay, and  $M_F$  is the Fermi matrix element,  $M_F = \langle \psi_f | T_\pm | \psi_i \rangle$ . By comparing the decay rates for muon and nuclear Fermi  $\beta^-$  decay, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element [6] between *u* and *d* quarks ( $v_{ud}$ ) can be determined and a precise test of the unitarity condition of the CKM matrix under the assumption of the three-generation standard model is possible [5,6].

For tests of the standard model, two nucleus-dependent corrections must be applied to experimental *ft* values. The first is a series of radiative corrections to the statistical rate function *f*, embodied in the factors  $\delta_R$  and  $\Delta_R$ , giving  $f_R = f(1 + \delta_R + \Delta_R)$  [7–12], where  $\delta_R$  is due to standard, electromagnetic (“inner”) radiative corrections (cf. p. 45 in Ref. [7]), while  $\Delta_R$  is what has been referred to as the “outer” radiative correction (cf. p. 47 of Ref. [7]) and includes axial-vector interference terms [9–11]. The second correction is applied to the Fermi matrix element  $M_F$ , and is due to the presence of isospin-nonconserving (INC) forces in nuclei, and is denoted by  $\delta_C$  [2,3,13,14]; namely,  $|M_F|^2 = |M_{F0}|^2 (1 - \delta_C)$ , where  $M_{F0} = [T(T+1) - T_{Z_i} T_{Z_f}]^{1/2}$ .

With the “nucleus-independent”  $\mathcal{F}t$  values defined by

$$\mathcal{F}t = ft(1 + \delta_R + \Delta_R)(1 - \delta_C), \quad (2)$$

the CKM matrix element  $v_{ud}$  is given by [10]

$$|v_{ud}|^2 = \frac{\pi^3 \ln 2}{\mathcal{F}t} \frac{\hbar^7}{G_F^2 m_e^5 c^4} = \frac{2984.38(6) \text{ s}}{\mathcal{F}t}, \quad (3)$$

where the Fermi coupling constant  $G_F$  is obtained from muon  $\beta^-$  decay, and includes radiative corrections. Currently, *ft* values for nine superallowed transitions have been measured with an experimental precision of better than 0.2% [4,15]. Prior to the recent measurement for  $^{10}\text{C}$ , the experimental *ft* values gave some hint of an additional *Z* dependence not presently accounted for. In addition, the unitarity condition for the CKM matrix was not satisfied. This prompted studies to empirically determine the “missing” correction and to satisfy the CVC requirement [16]. Recent results for  $^{10}\text{C}$  [15], however, do not support the conclusion that there may be a “missing” correction, as together all nine  $\mathcal{F}t$  values satisfy the constancy requirement of the CVC hypothesis. The unitarity condition of the CKM matrix, however, is still violated at the level of  $\sim 3\sigma$  [10,11,15], and can only be restored by the application of an across the board correction of approximately 0.3–0.4 %. In the future, a possible *Z* dependence in the  $\mathcal{F}t$  values can be further tested by a remeasurement of  $^{10}\text{C}$  and precise measurements of heavier *fp*-shell Fermi transitions using radioactive beams.

The necessary formalism for computing  $\delta_C$  is given in Refs. [2,14], and conventionally  $\delta_C$  is factored into two components, i.e.,  $\delta_C = \delta_{IM} + \delta_{RO}$  [2]. The correction  $\delta_{IM}$  is due to isospin mixing between different valence shell-model configuration states (e.g., the  $0\hbar\omega 1s0d$  shell). The essential ingredients for  $\delta_{IM}$  are a base isoscalar shell-model Hamiltonian that reproduces the spectra of excited  $J=0$  states and an INC interaction that reproduces experimental mass splittings [14]. The second correction,  $\delta_{RO}$ , is due to the deviation from unity of the radial overlap between the converted proton and the corresponding neutron. This effect corresponds to the influence of states that lie outside the valence shell-model configuration space (e.g.,  $2\hbar\omega$ , one-particle-one-hole configurations). Currently, there are two approaches for evaluating  $\delta_{RO}$  that give roughly the same agreement

TABLE I. List of shell-model configuration spaces and  $\hbar\omega$  used for each nucleus.

Nucleus	Configuration	$\hbar\omega$ (MeV)
$^{46}\text{V}$	full $fp$	10.952
$^{50}\text{Mn}$	$(f_{7/2}, p_{3/2})^{10} + f_{7/2}^{n_7}, f_{5/2}^{n_5}, p_{1/2}^{n_1} (n_5+n_1=1)$	10.550
$^{54}\text{Co}$	$(f_{7/2}, p_{3/2})^{14} + f_{7/2}^{n_7}, p_{3/2}^{n_3}, f_{5/2}^{n_5}, p_{1/2}^{n_1} (n_3+n_5+n_1=2)$	10.486
$^{58}\text{Zn}$	$f_{7/2}^{n_7}, p_{3/2}^{n_3}, f_{5/2}^{n_5}, p_{1/2}^{n_1} (14 \leq n_7 \leq 16)$	10.298
$^{62}\text{Ga}$	$f_{7/2}^{16}, (p_{1/2}, f_{5/2}, p_{1/2})^6$	10.017
$^{66}\text{As}$	$f_{7/2}^{16}, (p_{1/2}, f_{5/2}, p_{1/2})^{10}$	9.681
$^{70}\text{Br}$	$f_{7/2}^{16}, (p_{1/2}, f_{5/2}, p_{1/2})^{14}$	9.424
$^{74}\text{Ga}$	full $fp$	9.203

with the CVC hypothesis, but are in overall disagreement in magnitude. In the first approach [2], the radial wave functions were obtained using a Woods-Saxon (WS) plus Coulomb potential, while in the second [3,14] self-consistent Hartree-Fock (HF) calculations using Skyrme-type interactions (including Coulomb) were performed. The principal feature of the HF procedure is that since the mean field is proportional to the nuclear densities the Coulomb force induces a one-body isovector potential that tends to counter Coulomb repulsion, therefore reducing  $\delta_{\text{RO}}$ . Because of this, the HF values of  $\delta_{\text{RO}}$  are consistently smaller than the WS values by approximately 0.1–0.2 (in %).

In this paper, we reevaluate the isospin-mixing corrections for the  $fp$ -shell transitions  $^{46}\text{V}$ ,  $^{50}\text{Mn}$ , and  $^{54}\text{Co}$  that are included in the set of nine accurately measured transitions using expanded shell-model spaces and improved effective interactions. Comparisons with experimental data on the isospin-forbidden transition to the first excited ( $J=0, T=1$ ) state, which places some constraints on  $\delta_{\text{IM}}$  [17], will also be made. In addition, one application of future radioactive-beam facilities is to extend the data set to the heavier  $fp$ -shell nuclei  $^{58}\text{Zn}$ ,  $^{62}\text{Ga}$ ,  $^{66}\text{As}$ ,  $^{70}\text{Br}$ , and  $^{74}\text{Rb}$  [18]. Such a study may shed light on any possible  $Z$  dependence in the  $\mathcal{F}_t$  values. As such, we present calculations for the important isospin-mixing corrections for these nuclei. We find for these nuclei that both  $\delta_{\text{IM}}$  and  $\delta_{\text{RO}}$  are much larger than in the case of the previous nine transitions. In addition, the difference between the Woods-Saxon and Hartree-Fock calculations for  $\delta_{\text{RO}}$  is more pronounced for these nuclei, and precise measurements of these cases may be able to make a selection between the two approaches.

A calculation of  $\delta_C$  begins with defining the shell-model configuration space and the base isoscalar shell-model Hamiltonian. Naturally, these are not independent choices, as model-space truncations may require renormalizations of the effective interaction. For the nuclei under consideration here, the base configuration space is comprised of the  $0f_{7/2}$ ,  $1p_{3/2}$ ,  $1p_{1/2}$ , and  $0f_{5/2}$  orbitals, or  $fp$  shell. Because of computational restrictions, some model-space truncations must be imposed on all nuclei except  $^{46}\text{V}$  and  $^{74}\text{Rb}$ . The active model space used for each nucleus is listed in Table I. These model-space truncations were found to be adequate except for the cases of  $A=54$  and  $74$  as discussed below. In recent years, progress has been made towards the determination of effective interactions for use in  $fp$ -shell calculations, in particular for the lower part of the shell [19]. In this work, the FPD6 interaction of Ref. [19] was used for  $A \leq 50$ . For  $A=54$  the interaction was taken to be comprised of the two-

body matrix elements of FPD6, while the single-particle energies were renormalized to reproduce the experimental binding energies of  $^{57}\text{Ni}$  assuming a closed  $f_{7/2}$  core (FPD6\*). In the upper part of the  $fp$  shell, the interaction is less well determined, and for  $58 \leq A \leq 74$  we compare the results obtained using FPD6\* and the FPVH interaction of Ref. [20]. The calculations presented here were performed using a Unix version of the shell-model code OXBASH [21] on Silicon Graphics computers at Oak Ridge National Laboratory.

Another popular interaction used recently, but not here for the reasons outlined below, is a modified version of the original Kuo-Brown interaction referred to as KB3 [22]. Although this interaction gives very nearly the same results as FPD6 and FPD6\* in the lower  $fp$  shell, it begins to diverge drastically from either FPD6\* or FPVH for  $A \geq 60$ . The reason for this is that in the upper part of the shell monopole terms in KB3 tend to push the  $0f_{5/2}$  orbit up, creating a large gap between the  $p$  orbitals and the  $0f_{5/2}$  orbit. In fact, for the single-hole nucleus  $A=79$ , KB3 predicts the ground state to be  $J^\pi=5/2^-$  with excitation energies for the  $1/2^-$  and  $3/2^-$  hole states of 3.753 MeV and 7.010 MeV, respectively. This is in strong disagreement with spherical Hartree-Fock calculations, where, for example, the Skyrme  $M^*$  force [24] predicts the ground state to be  $J^\pi=1/2^-$ , with excitation energies for the  $5/2^-$  and  $3/2^-$  hole states of 0.591 MeV and 1.460 MeV, respectively. Both FPD6\* and FPVH are in excellent agreement with the HF results.

To evaluate the configuration-mixing contribution  $\delta_{\text{IM}}$  we use an INC interaction derived in the same manner as in Ref. [23]. An important ingredient of the INC interaction is the mass scaling of the Coulomb two-body strength and single-particle energies as governed by the oscillator parameter  $\hbar\omega$  [cf. Eq. (3.5) of Ref. [23]]. Since there are important deviations from the usual smooth formulas for  $\hbar\omega$  around  $A \sim 53$ – $59$ , and we want a uniform parametrization across the  $fp$  shell, we have chosen  $\hbar\omega$  so as to reproduce the rms point proton radii obtained from a spherical Hartree-Fock calculation using the Skyrme  $M^*$  force. The values of  $\hbar\omega$  used here are listed in Table I. Using these values of  $\hbar\omega$ , the parameters of the INC interaction of Ref. [23] were redetermined. In addition, the single-particle energies of the  $0f_{5/2}$  and  $1p_{1/2}$  orbits were not well determined by the data set in Ref. [23], and were chosen to reproduce the Coulomb splittings for the  $J^\pi=5/2^-$  and  $1/2^-$ ,  $A=57$ ,  $T=1/2$  multiplets [25] assuming a closed  $^{56}\text{Ni}$  core. The parameters of the INC interaction used are  $\epsilon(0f_{7/2})=7.487$  MeV,  $\epsilon(1p_{3/2})=7.312$

TABLE II. List of isospin-mixing corrections  $\delta_{IM}$  and  $\delta_{RO}$  (in %), theoretical and experimental excitation energies for the first  $J=0, T=1$  excited state (in MeV), and theoretical and experimental values of  $\delta_{IM}^I$ . Values of  $\delta_{IM}$  obtained by setting the theoretical excitations equal to experiment are indicated by the additional subscript *s*. Values of  $\delta_{RO}$  for Hartree-Fock and Woods-Saxon wave functions are denoted by the superscripts HF and WS, respectively. The results obtained for  $A \geq 58$  are shown for the FPD6\* interaction.

$A$	$E_{x,th}^I$	$E_{x,expt}^I$	$\delta_{IM}^I$	$\delta_{IM,s}^I$	$\delta_{IM,expt}^I$	$\delta_{IM}$	$\delta_{IM,s}$	$\delta_{RO}^{HF}$	$\delta_{RO}^{WS}$
$^{46}\text{V}$	4.295	2.611	0.020	0.054	0.053(5) <sup>a</sup>	0.040	0.094	0.286	0.36(6) <sup>b</sup>
$^{50}\text{Mn}$	3.620	3.69	0.014	0.015	<0.016 <sup>a</sup>	0.026	0.017	0.325	0.40(9) <sup>b</sup>
$^{54}\text{Co}$	6.423	2.561	0.0004	0.003	0.035(5) <sup>a</sup>	0.003	0.006	0.397	0.56(6) <sup>b</sup>
$^{58}\text{Zn}$	2.850	2.943	0.196	0.183		0.227	0.214	0.974	1.677
$^{62}\text{Ga}$	1.876	2.33	0.261	0.169		0.471	0.379	0.885	1.217
$^{66}\text{As}$	0.848		0.066			0.499		0.911	1.236
$^{70}\text{Br}$	1.083		0.089			0.313		0.801	1.377
$^{74}\text{Rb}$	2.258		0.069			0.223		0.831	1.716

<sup>a</sup>From Ref. [17].

<sup>b</sup>From Ref. [2].

MeV,  $\epsilon(0f_{5/2}) = 7.582$  MeV,  $\epsilon(1p_{1/2}) = 7.240$  MeV,  $S_C = 1.006$ ,  $S_0^{(1)} = 0.0$ , and  $S_0^{(2)} = -4.2 \times 10^{-2}$ .

Shown in Tables II (FPD6\* for  $A \geq 58$ ) and III (FPVH for  $A \geq 58$ ) are the results of shell-model calculations for  $\delta_{IM}$  for the *fp*-shell nuclei under consideration. In addition, the theoretical and experimental values for the excitation energy of the first excited  $J^\pi = 0^+, T=1$  state are shown. Generally, for  $A < 58$  one finds that  $\delta_{IM}$  is of the order of 0.02–0.10 %, while for the heavier nuclei it can be as large as 0.4%. One reason for the increase in  $\delta_{IM}$  for  $A \geq 62$  is that the excitation energy of the lowest  $J=0, T=0$  state is steadily decreasing in these nuclei, eventually becoming equal to or less than that for the  $J=0, T=1$  state. The effect of  $T=0$  mixing in the  $T_z=0$  parent is to remove Fermi strength from the transition, therefore increasing  $\delta_{IM}$ . The second reason for the enhancement in  $\delta_{IM}$  is that the excitation energy of the first excited  $J=0, T=1$  state is lower in these nuclei than for  $A \leq 54$ . The contribution to  $\delta_{IM}$  due to mixing with this state is given by

$$\delta_{IM}^I = [\alpha(0) - \alpha(-1)]^2, \quad (4)$$

where  $\alpha(T_z)$  is the amplitude for mixing the first excited state into the ground state for the nucleus with third component of isospin  $T_z = (Z-N)/2$  ( $Z$  and  $N$  denoting the number of protons and neutrons, respectively). In perturbation theory, the mixing amplitude  $\alpha$  is determined by the ratio of the matrix element of the INC interaction and the energy difference between the states, i.e.,

$$\alpha = \langle \psi_1 | V_{\text{INC}} | \psi_0 \rangle / \Delta E_{01}. \quad (5)$$

Therefore a dependence in  $\delta_{IM}$  on the isoscalar interaction and shell-model configuration space is manifested in the reproduction of the energy spectrum of  $J=0$  states. Improved values for  $\delta_{IM}^I$  and  $\delta_{IM}$  maybe obtained by scaling  $\delta_{IM}^I$  by the square of the ratio of the theoretical and experimental excitation energies  $(\Delta E_{01}^{\text{th}} / \Delta E_{01}^{\text{expt}})^2$ . The results are tabulated in Tables II and III with the additional subscript *s*. In addition, for  $^{46}\text{V}$  the contribution due to the second excited state,  $\delta_{IM}^2 = 0.012\%$ , was also scaled by the ratio  $(5.84/3.57)^2$  to account for the difference between the experimental and

theoretical excitation energies for this state as well. As is pointed out in Ref. [17], the experimentally measured Fermi matrix element for the isospin-forbidden transition from the ground state of the parent to the first excited  $J=0, T=1$  state in the daughter can be related to  $\delta_{IM}^I$  [26]. The experimental and theoretical values are compared in Table II, where overall good agreement is achieved except for  $A = 54$ .

Two nuclei in this study deserve special mention in regard to model-space truncations. The first is  $A = 74$ . Towards the upper end of the *fp* shell, it is apparent that deformation effects are beginning to become important as can be seen by the steady decrease with nucleon number  $A$  in the excitation energy of the lowest  $J^\pi = 2^+$  states in even-even  $N=Z$  nuclei [27,28] as shown in Table IV. Also shown in Table IV is a comparison between the experimental excitation energies and those obtained from a shell-model calculation using the FPD6\* and FPVH interactions. A clear change is observed between  $A = 72$  and  $76$ , and for this reason a proper calculation for  $A = 74$  should probably include the  $0g_{9/2}$  orbit. At present such a calculation is not feasible, and we express caution regarding the results for  $A = 74$  and the hope that more thorough calculations can be performed in the near future. The second case is  $A = 54$ , where, to first order, the ground-state wave function is comprised of two  $f_{7/2}$  holes. Excited  $J=0$  states, which are important for  $\delta_{IM}$ , have at least two particles excited outside the  $0f_{7/2}$  orbit [i.e., a two-particle-four-hole ( $2p-4h$ ) configuration relative to the  $^{56}\text{Ni}$  closed shell]. The effect of including these configurations, however, is to decrease the binding energy of the ground state relative to the  $2p-4h$  states, leading to an artificially large excitation energy for the excited states. In prin-

TABLE III. Same as Table II for  $A \geq 58$  using the FPVH interaction.

$A$	$E_{x,th}^I$	$E_{x,expt}^I$	$\delta_{IM}^I$	$\delta_{IM,s}^I$	$\delta_{IM}$	$\delta_{IM,s}$	$\delta_{RO}^{HF}$	$\delta_{RO}^{WS}$
$^{58}\text{Zn}$	2.850	2.943	0.224	0.258	0.231	0.265	0.997	1.762
$^{62}\text{Ga}$	1.460	2.33	0.201	0.079	0.408	0.286	1.029	1.409
$^{66}\text{As}$	1.250		0.019		0.388		1.243	1.577
$^{70}\text{Br}$	1.545		0.017		0.330		1.082	1.596
$^{74}\text{Rb}$	2.988		0.090		0.237		0.670	1.409

TABLE IV. Comparison between theoretical and experimental excitation energies (in MeV) of the first  $J^\pi=2^+$  state in even-even  $N=Z$  nuclei.

<i>A</i>	FPVH	FPD6*	Expt.
$^{60}\text{Zn}$	1.134	0.825	1.004 <sup>a</sup>
$^{64}\text{Ge}$	0.914	0.700	0.902 <sup>b</sup>
$^{68}\text{Se}$	0.939	0.600	0.854 <sup>b</sup>
$^{72}\text{Kr}$	0.976	0.707	0.709 <sup>b</sup>
$^{76}\text{Sr}$	0.892	0.752	0.261 <sup>b</sup>
$^{80}\text{Zr}$			0.289 <sup>b</sup>

<sup>a</sup>From Ref. [27].

<sup>b</sup>From Ref. [28].

ciple, if computational limitations permitted, the inclusion of  $4p$ - $6h$  states would decrease this gap. A calculation utilizing no restrictions with the  $0f_{7/2}$  and  $1p_{3/2}$  orbits is feasible, and the gap between the ground state and excited states is reduced considerably. The effects of isospin mixing in this space, however, are quite small, and are in disagreement the experimental results obtained in Ref. [17]. In addition, when excitations involving two particles into the  $0f_{5/2}$  orbit are included, the gap worsens, indicating that  $4p$ - $6h$  excitations to the  $0f_{5/2}$  orbit are important for describing the energy of the first excited state. An alternative approach is that of Ref. [13] where the isoscalar interaction was renormalized in the  $2p$ - $4h$  space so that the excitation spectrum had the correct energies. In that work,  $\delta_{\text{IM}}^1$  and  $\delta_{\text{IM}}$  were found to be 0.037(8)% and 0.045(5)%, respectively, and are in good agreement with the experimental value for  $\delta_{\text{IM}}^1$  of 0.035(5). Given the computational limitations and the experimental data, probably the best value of  $\delta_{\text{IM}}$  for  $^{54}\text{Co}$  when testing CVC and the unitarity of the CKM matrix is 0.04(1)%.

The radial overlap correction  $\delta_{\text{RO}}$  was evaluated using the procedures outlined in Refs. [2,14]. Shown in Tables II (FPD6\*) and III (FPVH) are the results for  $\delta_{\text{RO}}$  using Hartree-Fock (HF) and Woods-Saxon (WS) single-particle wave functions. The HF results were computed using the Skyrme  $M^*$  force [24], which generally gives better overall agreement with many experimental observables than do other Skyrme forces, in particular some isovector quantities

such as the centroid energies for giant-dipole and giant-isovector-monopole resonances [29]. Therefore we have chosen to present all the results with Skyrme  $M^*$ . However, we believe the dependence on the parameters of the Skyrme interaction should be further investigated [30]. The WS values for  $A \geq 58$  were computed using the Woods-Saxon parameters given in Ref. [31].

An interesting feature of  $\delta_{\text{RO}}$  is that it is much larger for the  $A \geq 58$  cases. This is primarily due to (1) the larger difference between the proton and neutron separation energies  $\sim 10$  MeV; (2) the last proton being rather weakly bound  $\sim 2.5$  MeV, as opposed to 5–6 MeV for  $A \leq 54$ ; and (3)  $\delta_{\text{RO}}$  being dominated by the  $1p_{3/2}$  orbit, which has a lower centrifugal barrier than in the case for  $A \leq 54$ , which is dominated by the  $0f_{7/2}$  orbit. In addition, unlike the  $0f$  orbitals, the  $1p_{3/2}$  orbit has a node, which is a feature that also tends to increase the mismatch integral [2]. Finally, it is apparent from Tables II and III that the difference between the HF and WS evaluations of  $\delta_{\text{RO}}$  is considerably larger for the heavier nuclei, ranging from 0.3–0.7 %, as opposed to 0.02–0.2 % for the  $A \leq 54$  cases (cf. Ref. [3]). As such, CVC tests including accurate measurements of the  $ft$  values for the heavier  $fp$ -shell cases may lead to a differentiation between the two approaches.

To complete the survey of isospin-mixing corrections for Fermi transitions, the values of  $\delta_{\text{IM}}$  and  $\delta_{\text{RO}}^{\text{HF}}$  (and the sum  $\delta_C$ ) for the nine accurately measured nuclei are listed in Table V. The  $\delta_{\text{RO}}^{\text{HF}}$  values were obtained using the Skyrme  $M^*$  force. The values presented for  $^{10}\text{C}$  were evaluated using the full  $0p_{3/2}, 0p_{1/2}$  shell-model space and the CKPOT isoscalar interaction [32] and the INC interaction of Ref. [3].

Aside from the systematic difference between the HF and WS estimates of  $\delta_{\text{RO}}$  the theoretical uncertainty in  $\delta_C$  for  $A \leq 54$  is of the order 0.09% in most cases [3]. This arises from the addition in quadrature of 0.05% for  $\delta_{\text{IM}}$ , 0.06% for  $\delta_{\text{RO}}$ , and 0.05% as a conservative estimate for the spectator mismatch, which as discussed in Refs. [3,33] is expected to be negligible. For  $A \geq 58$  there are some differences between the results obtained with the FPD6\* and FPVH interactions. For the most part, the  $\delta_{\text{IM}}$  values are in overall agreement with differences of the order of 0.05 %. For  $\delta_{\text{RO}}$  the mean

TABLE V. List of isospin-mixing corrections  $\delta_{\text{IM}}$ ,  $\delta_{\text{RO}}$ , and  $\delta_C$  (in %), and  $f_R t$  and  $\mathcal{F}t$  (in seconds) for the accurately measured cases.

<i>A</i>	$\delta_{\text{IM}}$	$\delta_{\text{RO}}^{\text{HF}}$	$\delta_C$	$f_R t^{\text{c,d}}$	$\mathcal{F}t^{\text{d}}$
$^{10}\text{C}$	0.04	0.11	0.15(9)	$3154.4 \pm 5.1 \pm 2.4$	$3149.7 \pm 5.8 \pm 2.4$
$^{14}\text{O}$	0.01 <sup>a</sup>	0.14	0.15(9)	$3151.1 \pm 1.8 \pm 2.4$	$3146.4 \pm 3.4 \pm 2.4$
$^{26m}\text{Al}$	0.01 <sup>a</sup>	0.29	0.30(9)	$3157.8 \pm 1.7 \pm 2.4$	$3148.3 \pm 3.3 \pm 2.4$
$^{34}\text{Cl}$	0.06 <sup>a</sup>	0.51	0.57(9)	$3167.0 \pm 1.9 \pm 2.4$	$3148.9 \pm 3.4 \pm 2.4$
$^{38m}\text{K}$	0.11 <sup>a</sup>	0.48	0.59(9)	$3166.5 \pm 2.6 \pm 2.4$	$3147.8 \pm 3.8 \pm 2.4$
$^{42}\text{Sc}$	0.11 <sup>a</sup>	0.31	0.42(9)	$3168.1 \pm 1.4 \pm 2.4$	$3154.8 \pm 3.2 \pm 2.4$
$^{46}\text{V}$	0.09	0.29	0.38(9)	$3165.5 \pm 1.8 \pm 2.4$	$3153.5 \pm 3.4 \pm 2.4$
$^{50}\text{Mn}$	0.02	0.33	0.35(9)	$3164.2 \pm 1.6 \pm 2.4$	$3153.1 \pm 3.3 \pm 2.4$
$^{54}\text{Co}$	0.04	0.40	0.44(9) <sup>b</sup>	$3166.4 \pm 1.1 \pm 2.4$	$3152.5 \pm 3.1 \pm 2.4$

<sup>a</sup>From Ref. [3].

<sup>b</sup>Using  $\delta_{\text{IM}}=0.04(1)$  as discussed in the text.

<sup>c</sup>From the new Chalk River data set [35].

<sup>d</sup>The systematic uncertainty of 2.4 s is due to the systematic uncertainty of 0.08% in  $\Delta_R$  [10].

difference between the two interactions is 0.124%, but can be as large 0.33%. These differences are primarily attributed to differences in the excitation energies of the  $T=3/2$  states in the  $A-1$  parent. For more precise studies in the future, it will be necessary to improve upon the base shell-model isoscalar interaction. Nonetheless, both interactions predict large differences between the HF and WS approaches to  $\delta_{\text{RO}}$ .

A test of the CVC hypothesis is performed by applying  $\delta_C$  to the  $f_R t$  values, which are also listed in Table V. Here,  $f_R t$  was computed by applying the radiative corrections listed in column 1 of Table III in Ref. [10] and the average of the  $(\alpha/\pi)C_{NS}$  corrections listed in Refs. [10,11] to the  $ft$  values of the new Chalk River compilation [34]. Applying  $\delta_C$  to  $f_R t$  (note that the  $\mathcal{F}t$  are also listed in Table V) and taking the error-weighted average, we find  $\mathcal{F}_{av,t}=3150.8 \pm 1.2 \pm 2.5$  s with  $\chi^2/\nu=0.78$ . Using Eq. (3) and  $v_{us}=0.2199(17)$  [10] and  $v_{ub}<0.0075$  (90% confidence level) [35], the unitarity condition of the CKM matrix is found to be  $0.9956 \pm (0.0008)_{\text{stat}} \pm (0.0007)_{\text{sys}}$ . Thus, from the constancy of the  $\mathcal{F}t$  values, we conclude that the CVC hypothesis is satisfied, but that the unitarity condition of the CKM matrix is violated at the level of  $(3-4)\sigma$ , and can only be achieved with an additional negative correction of 0.3–0.4 % applied uniformly to each nucleus. It is important to note that a correction of this magnitude lies well outside the range of acceptable uncertainties in the nuclear corrections.

In summary, the isospin-mixing corrections for Fermi transitions in *fp*-shell nuclei were evaluated. The evaluation

also included transitions involving heavier nuclei that are expected to be measured in the future radioactive-beam facilities. It was found that the isospin-mixing corrections were considerably larger for the  $A \geq 58$  cases. In addition, the difference between the Hartree-Fock and Woods-Saxon method of evaluating  $\delta_{\text{RO}}$  was much larger for these nuclei. As such, accurate measurements of the  $ft$  values for these nuclei might lead to a discrimination between the two methods. In regard to the accurately measured transitions, it was found that the newer evaluations give better agreement with experiment for the configuration-mixing term  $\delta_{\text{IM}}$ , with the noted exception of  $^{54}\text{Co}$ , which poses a significant computational challenge. Lastly, it is found that the corrected  $\mathcal{F}t$  values are in excellent agreement with the CVC hypothesis, but that the unitarity condition of the CKM matrix is violated at the level of  $(3-4)\sigma$ .

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