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Statistical correlations in nuclear many-body states

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Abstract

Statistical correlations of nuclear many-body states are explored in the $0d1s$ shell model as a function of the strength of residual interactions. Model independent predictions for correlation functions and distributions, developed from parametric random-matrix theory, are found to describe the observed nuclear behavior for excited states. In particular, we find that correlations generally decrease as a power law.

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Already at a few MeV of excitation energy, the exponentially increasing level density makes the detailed analysis of all individual nuclear many-body states impractical. In this regime, it is reasonable to separate the average global behavior of nuclear properties as a function of excitation energy (or temperature) and the remaining exact integrals of motion, from the local correlations and fluctuations. Such considerations were the original impetus for the application of random matrix techniques to the nuclear structure problem of the neutron resonances [1,2]. Since then, random matrix predictions have been put to the test in both the experimental arena and in a variety of nuclear models [3]. Statistical enhancement of the weak interaction effects in compound nuclear states is one of the extensively studied examples. Predictions for level spacing statistics, distributions of electromagnetic decay strengths, distributions of widths of resonances, etc., have been explored in great detail experimentally in nuclear properties across the periodic table, and theoretically in a full scope of nuclear models. These studies have validated the use of random matrix techniques as one of the few methods that can be used to address the complex many-body behavior of excited nuclei. The complexity of the nuclear many-body problem results in features being close to the optimal statistical limits provided by random matrix theory. The detail analysis reveals also the systematic deviations of nuclear many-body dynamics from this "stochastic" limit of complete mixing [4].

We consider here the extension of these types of ideas to correlation functions of adiabatically evolving nuclei. Such correlations result, for example, from a general parametric variation of the nuclear many-body Hamiltonian, which might be achieved through a self-consistent constrained Hartree-Fock calculation for a

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nucleus undergoing fission. It can be analyzed as well in terms of a parametric variation in the framework of the nuclear shell model. In either case, the Hamiltonian is smoothly varied along some path, either self-consistently, or forcefully. As a consequence, there is a continuous modification of the mean field resulting in redistributions of nucleons among the changing single-particle orbitals [5], and an ensuing decorrelation of the adiabatic many-body states. If we assume that the ground state of the nucleus is approximately a Slater determinant, the decorrelation of this state, denoted by $\langle \Psi_1(\lambda)|\Psi_1(0)\rangle$ where λ is some parameter, is given by the Onishi formula [6]. If λ corresponds to quadrupole distortions (β_2) of the nuclear surface, the correlation length is approximately $\Delta\beta_2 \sim 5/A$ [7]. For the excited states, due to the high level density, it is not meaningful to consider the extension of this formula to individual states. Rather, the equivalent statistical correlations in a particular energy window are of practical interest. What we develop and test here are the statistical extensions of the Onishi formula to excited many-body states.

As the energy levels evolve as a function of changing parameters, the pattern of multiple avoided crossings visualize the process of mixing and correlation. The deep analogy of level dynamics [8–11] with the collisional dynamics of the gas particles was recognized long ago by Dyson [12] in his Brownian motion description of random hamiltonians. In the random matrix limit, statistical distributions of crossing parameters [13–16,4] as well as correlation functions of various spectral characteristics [17–19] can be obtained analytically and numerically.

Our studies are motivated by recent results where parametric extensions of random matrix theory were used to develop universal or model independent predictions for correlation functions that depend of wavefunctions and energies [20,21]. These functions have been studied, in collective nuclear excitations in the framework of the interacting boson model, where good agreement was found [22]. While this model describes collective excitations, it is natural to expect that the results might be different in many-fermion dynamics where correlations might be suppressed by factors on the order of $1/A$. We will see below that by varying the strength of residual two-body interactions, the nuclear many-body wavefunctions statistically correlate and decorrelate in agreement with predictions from parametric random matrix theory. This will provide not only a statistical extension of the Onishi formula to excited many-body states, but a more general class of predictions for more general correlations as well as distributions of nuclear matrix elements.

The nuclear shell model provides an excellent framework to explore the properties of finite, strongly interacting many-fermion systems. The first step, for our purposes, is to generate a parametric dependence of the Hamiltonian. While this is not a unique procedure, results from the analysis of collective excitations suggest that the choice of the parameter, nor the number of parameters, are unimportant: general single- or multi-parameter deformations of the Hamiltonian imply the same statistical results, providing that the Hamiltonian is sufficiently chaotic for all values of the parameter(s) [22] and the proper scaling is performed. So the approach we choose is to modify directly the strength of the residual interactions, generating a Hamiltonian of the form $H(\lambda) = H_{\rm diag} + \lambda H_{\rm res}$, where $H_{\rm diag}$ is the diagonal (single-particle plus two-body interaction) contribution, and $H_{\rm res}$ represents the off-diagonal residual two-body interactions. Again, both the linear dependence of $H(\lambda)$ on the new parameter λ, or the choice of only one parameter, will not be important.

We will consider the full $0d1s$ shell-model basis, which provides a realistic description of low-lying levels in nuclei in the mass region A = 18–38 [23]. The entire spectra for the $0d1s$ shell-model states have recently been investigated from the point of view of level statistics, information entropy, complexity and thermalization [4,24,25]. While the physical strength is $\lambda = 1$, we will consider the variation of this strength and the resulting effects on the many-body states. The evolution of spectra, the structure of complicated eigenstates and the spreading widths of shell-model configurations are studied as a function of λ in [4,26].

For practical reasons, we investigate the spectra and wave functions for the 325 states of the $(0d1s)^8$ $J^\pi T = 0^+0$ basis. Larger $0d1s$ shell-model basis configurations such as $(0d1s)^{12}$ were analyzed in [4]. However, 325 is about the optimal number of states for the present computation which involves $\sim 10^3$ separate diagonalizations, and we are convinced that the found results are typical of all $0d1s$ shell spectra. The results are obtained with the Wildenthal (USD) two-body interaction [23] computed with the OXBASH shell-model code [27]. For the range of $0 \leq \lambda \leq 1$, with $\lambda = 1$ corresponding to the empirical strength in this shell, the adiabatic energies $E_n(\lambda)$ were computed. For the analysis of energy related statistics and correlations, the unfolded spectrum was obtained by first computing the staircase cumulative level number, $N(E,\lambda) = \sum_n \theta[E - E_n(\lambda)]$, approximating it by the smooth function $\bar{N}(E,\lambda)$ and then extracting the unfolded energy spectrum according to $\widetilde{E_n}(\lambda) = \bar{N}[E_n(\lambda),\lambda]$. The function $\bar{N}(E,\lambda)$ was approximated by a sixth order polynomial in E and a second order polynomial in λ, allowing a smooth extrapolation over the range $0 \leq \lambda \leq 1$. In Fig. 1a we display the total spectrum of 325 $J^\pi T = 0^+0$ states for 8 valence particles in the $0d1s$ major shell. The unfolded version of this same spectrum is shown on a number scale in Fig. 1b, and a magnified fragment of the same spectrum containing about 40 levels in the middle of the spectrum in Fig. 1c. The avoided crossings can be seen to become less frequent with increasing λ.

The statistical predictions can be made using parametric random matrix theory. This is a parametric generalization of the usual constant random matrix theory used in the past to study nuclear fluctuations. By this we mean a random matrix, in this case Gaussian Orthogonal Ensemble (GOE) hamiltonian $H(x)$ which depends on a parameter x, but has GOE fluctuations at every x. This would be consistent with the observed behavior of the shell-model states at various values of λ [4,24]. One of the more convenient formulations is in terms of the stochastic integral [20,21]

$$H(x) = \int f(x-y) V_{ij}(y)\, dy,$$
$$\overline{V_{ij}(x) V_{kl}(y)} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2\delta(x-y). \qquad (1)$$

In this case, $\overline{H(x)H(y)} = F(x-y) = \int f(x-y-z) f(z)\, dz$. The fact that in this model we have translational invariance in parameter space is not important, since we will apply the results locally over a certain parameter region, rather than globally [20].

There are two parameters that enter into correlation functions that we compute from random matrix theory: the dimension N of the matrix, and the leading order behavior of the autocorrelation function $F(z) \cong 1 - c_\alpha |z|^\alpha + \cdots$. However, both c_α and N can be removed by an appropriate rescaling of the parameter. The scaling is computed by observing that the leading order behavior from perturbation theory for the energy levels (such as Fig. 1b) looks on short distance scales like a diffusion process, with

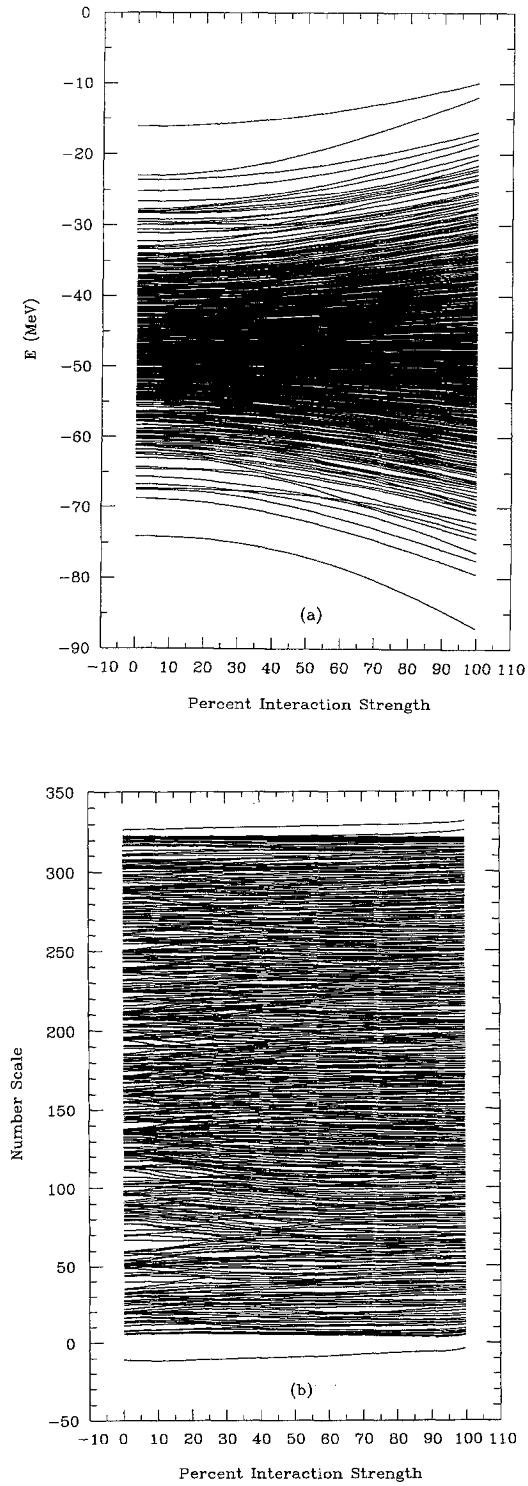
$$\overline{[E_n(z) - E_n(0)]^2} = \overline{(\Delta E_n)^2} \approx D_\alpha z^\alpha,$$
$$D_\alpha \equiv \frac{4 c_\alpha N}{\pi^2}, \qquad (2)$$

where the diffusion constant D_α contains all model specific dependencies. Hence by rescaling the parameter as $\tilde{z} = (D_\alpha)^{1/\alpha} z$, all results computed in random matrix theory will be dimensionless, model independent functions: all model dependence on the parametric realization of our random matrix Hamiltonian is now gone. It is worth noting that for any smooth functional dependence of $H(x)$ on the parameter x, be it linear or otherwise, we will have $\alpha = 2$, simply because the convolution which defines the autocorrelation function $F(z)$ will always have a quadratic short distance behavior [21]. By using this rescaling, we now compute correlation functions and obtain what are known as *universal predictions*.

For many nuclear phenomena, such as the description of fission, diffusion constants in large amplitude nuclear shape evolution [28], or the statistical decay of high spin states [29,30], it is important to understand the overlap matrix elements of nuclear many-body configurations, for instance the basic overlaps $\langle \Psi_1 | \Psi_2 \rangle$. Because we are considering the eigenstates, the unfolding of the energy levels plays no role. The statistical extension of the Onishi result for Slater determinants is obtained by studying the adiabatic survival probabilities, averaged over a number of neighboring states. We would like to consider the equivalent formulas for diagonal and off-diagonal correlations in the statistical regime. Parametric random matrix theory predicts that for diagonal matrix elements

$$\overline{|\langle \Psi_n(\tilde{z}) | \Psi_n(0) \rangle|^2} = \frac{1}{1 + c\tilde{z}^2}, \qquad (3)$$

where c is a constant. This is compared to the computed shell-model survival probabilities in Fig. 2a. The "data points" were extracted from a region of about 50 levels in the middle of the spectrum over a range of λ values of 0.2 to 0.4. A total of about 200 correlations functions were averaged. In general the agreement is quite good. The correlation length is typically on the scale of one level crossing ("mean free path"



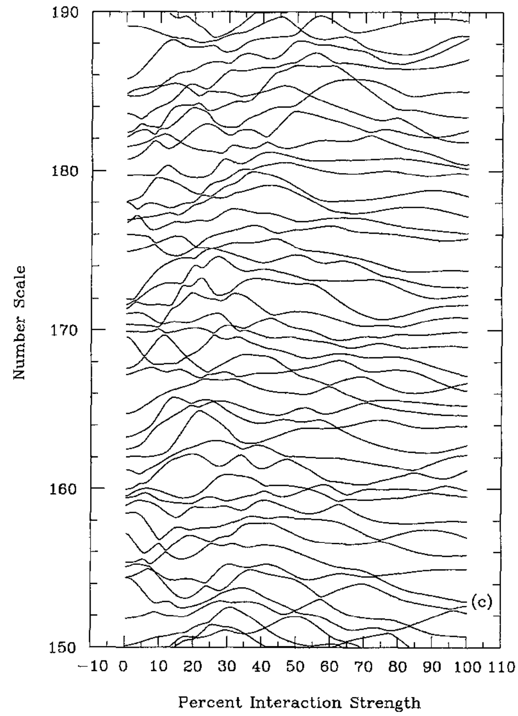


Fig. 1. Parametric spectrum of 8 fermion states in the $0d1s$ major shell. (a) All 325 $J^{\pi} = 0^{+}$ $T = 0$ states in the $(0d1s)^{8}$ shell-model space are shown, as a function of the residual interaction strength λ given as a percent of its normal physical strength of 100 percent. (b) Then same spectrum, now unfolded. (c) An expanded portion of the unfolded spectrum.

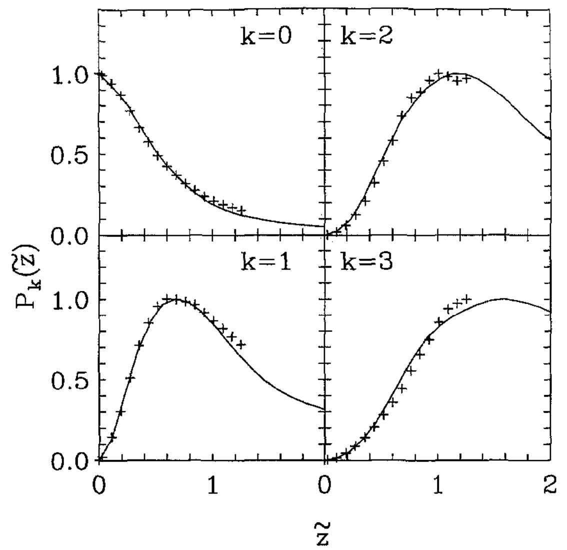


Fig. 2. Statistical correlation (+) of fermionic many-body states in the *sd* shell. The solid line corresponds to the universal predictions. $k = 0$ is the diagonal decorrelation function of Eq. (3), and $k = 1, 2, 3$ are the off-diagonal correlation functions of Eq. (5).

of Brownian "particles"). Although this might seem quite rapid, one should contrast this to known results from many-body theory. Using perturbation theory, to leading order, we then know that the (regular) ground state correlations of many-body (bosonic) vacua behave as:

$$\langle \Psi_1(x) | \Psi_1(x') \rangle = \exp(-A|x - x'|^2) \tag{4}$$

where A is a coefficient which is related to the matrix elements of the Hamiltonian. (Compare this to the exact result for the overlap of two harmonic oscillator states with the equilibrium points $x = 0$ and $x = \xi$, where $\langle n; 0 | n; \xi \rangle = (\rho^{1/2}) P_n(\rho)$ where P_n is the Legendre polynomial and $\rho = \exp(-\xi^2)$. The form of non-chaotic correlators is clearly model dependent.) The distribution (4) has finite moments. On the other hand, the statistical correlations found here show a lorentzian decorrelation for excited states, which results in power law decay at large distances, rather than exponential decay. As a consequence, all moments are infinite. For off-diagonal matrix elements, the approximate behavior is given by [20,22]:

$$\overline{|\langle \Psi_n(\tilde{z}) | \Psi_m(0) \rangle|^2} = C \frac{\tilde{z}^2}{k(k - 3/4) + \tilde{z}^4}, \tag{5}$$

where $k = |m - n| > 0$, and C is the overall normalization given by orthonormality of the wavefunctions, which is also power law at large distances. These are compared to results from the shell model for $k = 1, 2, 3$ in Figs. 2b–d.

Finally we consider here the curvature properties of the shell-model levels in Fig. 1. The rigidity of the spectrum results in the fact that the most probable curvatures are small. Contrary to expectations, the probability of finding a large curvature $k \gg 1$ is smaller in the chaotic case than for regular dynamics. In terms of $\tilde{z}$, the distribution of curvatures is [14,15]

$$P(k) = \frac{c}{(1 + k^2)^{3/2}}, \quad k = \frac{1}{\pi} \frac{d^2 \tilde{E}_n}{d\tilde{z}^2}. \tag{6}$$

The limiting behavior at large k in (6), $P_c(k) \propto k^{-3}$, directly follows from the power law $P(s) \sim s^2$ for the nearest level repulsion in the Gaussian ensembles, since the asymptotics of large k presumably corresponds to a pairwise interaction of two levels when the influence of remote levels is negligible. Then the curvature is determined by the small energy denominator and the distribution of the level spacings s is translated into the curvature distribution, $P_c(k \gg 1) \rightsquigarrow P(s = 1/k)/k^2$. The fractional power level repulsion $\sim s$, would give the asymptotics $P_c(k) \sim 1/k^3$ and the decrease of $P_c(k)$ is rather slow so that the second moment of the distribution diverges logarithmically for the GOE. We display in Fig. 3 the empirical curvature distribution cumulated over various values of λ for 50 unfolded levels in the middle of the spectrum and then binned as a function of k. The data for the intervals $\Delta\lambda = 0.2$ are grouped together. When the single-particle Hamiltonian $H_{\rm diag}$ dominates, as in the regime $\lambda < 0.2$, there is an excess of small and a deficit of larger curvatures. The small k behavior is determined mostly by a presence of gaps in the irregular Poisson-like chain of levels in a weakly interacting system. A pure random sequence of levels with a weak random interaction would generate a singularity $\sim 1/\sqrt{k}$ at small k. This contribution is still present at $\lambda = 0.0$–0.2. However the range $\lambda = 0.2$–0.4 gives a good agreement with the GOE result (6). This is the same value of strength which manifests the onset of chaos in the nearest level spacing distribution [4].

We have found that the statistical correlations of excited many-body fermionic states, computed in the

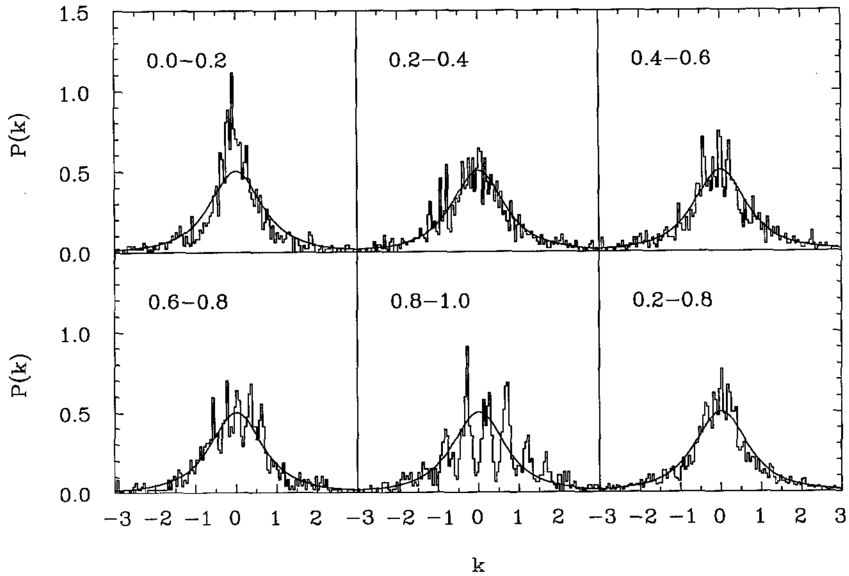


Fig. 3. Curvature distributions of the spectrum in Fig. 1b (histogram). The six panels show the results selected for various ranges of λ. The solid line corresponds to the random matrix prediction, Eq. (6).

shell model, are well described by parametric random matrix theory. In analogy to ground state correlations, the typical statistical correlation length for excited states is seen to be on the scale of the "mean free path", or equivalently, one level crossing. While this distance is shorter for excited states, the statistical correlations are power law and hence decay much more gradually. The resulting functions have infinite moments. This is the case for both adiabatic survival probabilities and transitional overlaps. This provides a statistical extension of the Onishi formula for excited states when the Hamiltonian is sufficiently complex in the parameter range of interest.

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