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Gamow-Teller strength as a function of excitation energy

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Abstract

We study the dependence of the total Gamow-Teller strength in an $N=Z$ nucleus (^{24}Mg) as a function of excitation energy within a complete $0\hbar\omega$ shell-model space (the $0d1s$ shell). We find an essentially monotonic increase in strength with increasing excitation energy. We are able to relate this behavior to a decrease in spatial symmetry (increase in $SU(4)$ symmetry) as a function of excitation energy. © 1997 Elsevier Science B.V.

Nuclear shell-model calculations appear to be one of the most promising candidates for probing the structure of complicated wave functions in quantum many-body systems with strong interactions. The reliability and the predictive power of the shell model with realistic residual interactions are well established [1,2]. Numerous calculations of nuclear energy spectra and other observables invariably show good agreement with the growing body of data. Going beyond the lowest energy region we can rely on the shell-model predictions in the analysis of highly excited states whose individual properties are inaccessible by current experimental tools of nuclear spectroscopy. On this road, important conclusions about statistical features of complicated states and their relations to quantum chaos, thermalization and phase transitions can be reached [3,4].

In this letter we study Gamow-Teller (GT) transitions between highly excited states in the sd-shell. Since the pioneering work by Wigner 60 years ago [5], spin-isospin symmetry and the properties of nuclear interaction related to the GT strength have been studied in detail [6]. From the considerable work carried out for low lying states (see for example [7]), it is apparent that full $0\hbar\omega$ shell-model calculations (presently possible up to $A\sim 50$ [8]) provide an excellent description of measured GT matrix elements apart from an overall quenching of the free GT operator ($g_A=1.26$) corresponding to $g_A^{\rm eff}\sim 1$. This quenching is attributed both to excitation to configurations outside of the $0\hbar\omega$ shell-model space and to delta-isobar excitations [1]. However, the study of GT transitions from excited states remains virtually non-existent. Such calculations can give us important information about the behavior of a simple excitation mode in a realistic environment of inco-

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herent nuclear interactions (the evolution of the isospin-invariant pairing as a function of excitation energy was discussed in [3]).

The GT operator is defined as a sum over the nucleons of the $\sigma\tau$ operator:

$$O_{\mu}^{(\pm)} = \sum_{\text{nucleons}} \sigma_{\mu} t_{\pm} = \sum_{k,k'} \left(\sigma_{\mu} t_{\pm}\right)_{k,k'} a_k^{\dagger} a_{k'} \qquad (1)$$

where the last term is given in second quantized form, k represents a complete set of single-particle quantum numbers including the isospin projection, and $\pm$ corresponds to the $\beta^{\mp}$ direction for the transition. The operator $t_{\pm}$ converts a neutron into a proton and vice versa ($t_{\pm} = \mp 1/\sqrt{2}\,\tau_{\pm}$ where $\tau_{\pm}$ are the spherical components of $\tau = 2t$). The GT strength is obtained by calculating the matrix element squared for the $\langle f | O^{\pm} | i \rangle$ transition between the many-body eigenstates $|i\rangle$ and $|f\rangle$. We study the strength function summed over all final states $|f\rangle$ to give the *total* GT strength, B_{GT}, of the initial state $|i\rangle$,

$$B_{\text{GT}}^{(\pm)}(i) = \sum_f B_{\text{GT}}^{(\pm)}(i \to f) = \sum_f \sum_{\mu} |\langle f | O_{\mu}^{(\pm)} | i \rangle|^2$$
$$= \sum_{\mu} \langle i | O_{\mu}^{(\pm)\dagger} O_{\mu}^{(\pm)} | i \rangle \qquad (2)$$

where the closure summation over $|f\rangle$ was used. According to the GT sum rule which follows from the definition (2), the difference in total strength $B_{\text{GT}}^{(+)}(i) - B_{\text{GT}}^{(-)}(i) = 3(N_i - Z_i)$ is directly related to the number of protons and neutrons in the initial nucleus. For the $T=0$ states of an $N=Z$ nucleus, which we use as an example in our study, it suffices to consider one of these sums, say $B_{\text{GT}}^{(-)} \equiv B_{\text{GT}}$.

We compute the GT strengths for a system of $n=8$ valence nucleons in the *sd*-shell which can be seen as a subset of states in the ^{24}Mg nucleus. Using a realistic hamiltonian, we find the eigenfunctions $|J^{\pi}T\rangle$ with the exact quantum numbers of angular momentum and isospin and use them to calculate the amplitudes for the GT transitions. The Wildenthal hamiltonian [1,9] defines the single-particle energies and the interaction between the valence particles by fitting more than 400 binding and excitation energies for the *sd*-shell nuclei. The total GT strength (2) is shown in Fig. 1(a) as a function of excitation energy for all 325 individual $J^{\pi}T = 0^{+}0$ states in ^{24}Mg found in the *sd* shell model. We see a clear monotonic increase in the total B_{GT} strengths with excitation energy.

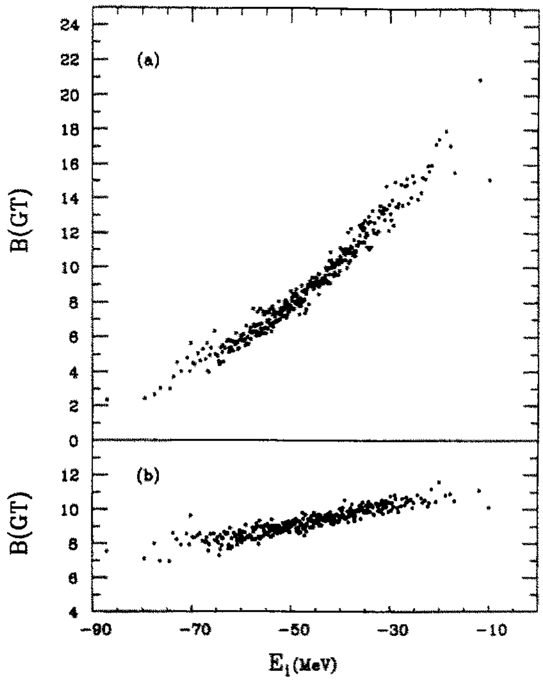

Fig. 1. (a) The total Gamow-Teller strength from the 0^{+} states of ^{24}Mg plotted as a function of the absolute energy of the 0^{+} state. (b) The total strength obtained from the incoherent sum over the partial contributions shown in Fig. 2.

We now look for an understanding of the above result, obtained by explicit summation over shell-model final states, in terms of either statistical ideas or in the limit of well-defined symmetries for the shell-model states. The structure of the shell-model eigenstates is determined largely by competition between the central and spin-orbit components of the effective interaction. The spin-orbit interaction enters mainly at the one-body level by splitting the energy of the single-particle states with the same orbital momentum ($d_{5/2}$ and $d_{3/2}$ in our case). The central interaction is largely $SU(4)$ conserving, as can be demonstrated by turning off the other components of the interaction, and the appropriate basis is the Wigner supermultiplet scheme characterized by a Young tableaux $[f]$, quantum numbers $(\lambda\mu)$ of the $SU(3)$ group, orbital L, spin S and total angular momentum J, isospin T and additional quantum numbers for multiple occurrences of L or TS. The [2222] symmetry ([44] spatial symmetry) ground state has $B_{GT}=0$. On the other hand, the spin-orbit interaction alone leads to pure jj coupling and a $d^8_{5/2}$ ground state with seniority zero has $B_{GT}=112/15$ (6/7 of this from $d_{5/2}\to d_{3/2}$ transitions).

First we consider a statistical treatment within the framework of the jj coupling scheme. The matrix element $\langle f|O|i\rangle$ of any one-body operator between complicated many-body states is the coherent sum of the products of single-particle matrix elements, $(l,j\|\sigma_\mu t_-\|l,j')$ in our case, and the one-body transition densities $\langle f\|a^\dagger_{lj}a_{lj'}\|i\rangle$. (the GT operator does not change the orbital momentum l). The non-zero values for the sd-case correspond to the following (lj,lj') transitions: $(d_{5/2},d_{5/2})$, $(d_{5/2},d_{3/2})$, $(d_{3/2},d_{3/2})$, $(d_{3/2},d_{5/2})$, and $(s_{1/2},s_{1/2})$. Assuming that the very complicated eigenstates have nearly random phases for its components [3,4], we can substitute the one-body transition densities by their mean values for the heated Fermi-liquid, $\sqrt{n_{lj'}(1-p_{lj})}$, in terms of the mean neutron, n, and proton, p, occupation probabilities to obtain (for the $B^{(-)}_{GT}$ direction)

$$B_{GT}=\sum_{l,j,j'} n_{lj'}(1-p_{lj})6(2j+1)(2j'+1)\times\begin{Bmatrix} j & 1 & j' \\ 1/2 & l & 1/2\end{Bmatrix}^2 . \quad (3)$$

This approximation neglects all coherent effects and leads to a very weak energy dependence of the strength. In particular, the lowest unperturbed state with 8 particles in the $d_{5/2}$ level would have $n_{d5/2}=p_{d5/2}=2/3$ and $B_{GT}=8.3$ while the highest unperturbed state with the filled $d_{3/2}$ orbit would have $B_{GT}=9.6$. The middle of the shell-model spectrum corresponds to the equipopulation of the orbitals (''infinite temperature'', [3,10]). Then (3) simplifies to the universal result

$$B_{GT}=3N\left(1-\frac{Z}{\Omega_p}\right) \quad (4)$$

where the Pauli blocking factor is just the average proton population of available Ω_p orbitals; Eq. (4) gives 8 in the case under study. Precisely the same result (4) follows from the statistical spectroscopy [11] as an average for all states allowed for given N and Z in a truncated shell-model space, regardless of their exact quantum numbers. This number is close to what we see in the middle of the spectrum.

The statistical consideration neglects the interference effects between 5 partial transitions contributing to the total GT strength. This interference, destructive at low energies and constructive at higher energies, is responsible for the regular behavior seen in Fig. 1(a). The partial contributions to the total GT strength for all 5 groups of transitions can be seen in Fig. 2. The transitions with $(j=j')$ have relatively flat distributions while those that involve two different orbitals $(j\neq j')$, have strengths that slightly decrease and increase with excitation energy. In agreement with (3), this reflects an average population of $d_{5/2}$ and $d_{3/2}$ orbitals according to their single-particle energy levels in the sd-shell with spin-orbit coupling. The sum of the partial strengths is shown in Fig. 1(b). As a direct result of the absence of interference, the sum is close to the single-particle estimate and the energy behavior is much flatter than that of the original, unrestricted strength.

We now turn to the limit of pure $SU(4)$ symmetry which prevails, to a good approximation, if the one-body spin-orbit splitting between the $d_{5/2}$ and $d_{3/2}$

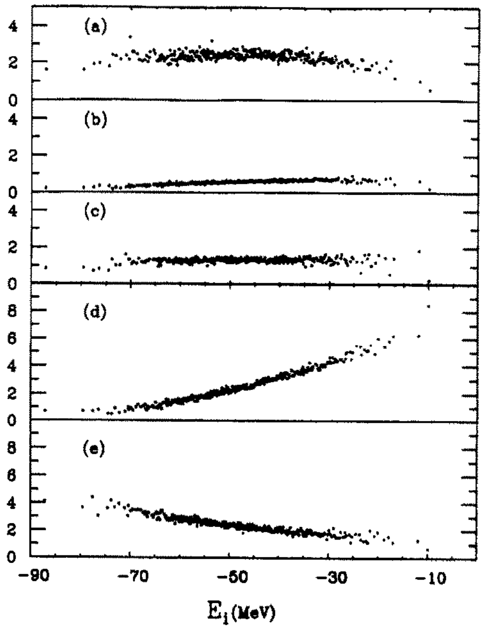

Fig. 2. Partial contributions to the total GT strength for (lj',lj) = (a) $(d_{5/2},d_{5/2})$, (b) $(d_{3/2},d_{3/2})$, (c) $(s_{1/2},s_{1/2})$, (d) $(d_{3/2},d_{5/2})$, and (e) $(d_{5/2},d_{3/2})$.

orbits is set to zero. Since the components of the Gamow-Teller operator are among the generators of the $SU(4)$ group, their matrix elements may be calculated using the Racah algebra for the $SU(4)$ group developed by Hecht and Pang [12], see also [13]. However, for $T=0$ initial states the closure expression for B_{GT} is particularly simple

$$B_{GT} = \tfrac{1}{3}\langle i \| \tfrac{1}{2} \sum_{ab} \sigma_a \cdot \sigma_b \tau_a \cdot \tau_b \| i \rangle \tag{5}$$

where we have used the fact that $B_{GT}^{(+)}$, $B_{GT}^{(-)}$, and the correspondingly defined $B_{GT}^{(0)}$, are all equal, and have taken one third of their sum. The expression (5) can be rewritten in a number of equivalent forms involving the space-exchange operator $P_{ab} = -P_{ab}^{\sigma} P_{ab}^{\tau}$, T^2, S^2, the number of particles in the shell n, the Casimir operator $C[SU(4)]$ for $SU(4)$, the projection operators P_{ab}^{S} and P_{ab}^{T} for a given value of spin and isospin of a pair or the number of pairs $N_{TS} = \sum_{a<b} P_{ab}^{T} P_{ab}^{S}$ with given T and S in the initial wave function. For example,

$$B_{GT} = \tfrac{1}{3}\langle i \| \frac{9n}{2} + \sum_{a<b} \sigma_a \cdot \sigma_b \tau_a \cdot \tau_b \| i \rangle \tag{6}$$

$$= \tfrac{2}{3}\langle i \| C[SU(4)] - S^2 - T^2 \| i \rangle \tag{7}$$

$$= \tfrac{2}{3}\langle i \| 4n - \frac{n^2}{4} - 2\sum_{a<b} P_{ab} - S^2 - T^2 \| i \rangle \tag{8}$$

$$= \langle i \| \frac{3n}{2} - N_{01} - N_{10} + 3N_{00} + \tfrac{1}{3}N_{11} \| i \rangle \tag{9}$$

Thus, for pure supermultiplet states with the total spin S and zero isospin, the sum rule is

$$B_{GT} = \tfrac{2}{3}\{C[SU(4)] - S(S+1)\} \tag{10}$$

where the eigenvalues are $C[SU(4)] = F(F+4) + F'(F'+2) + (F'')^2$ in terms of $F = (f_1 + f_2 - f_3 - f_4)/2$, $F' = (f_1 - f_2 + f_3 - f_4)/2$, and $F'' = (f_1 - f_2 - f_3 + f_4)/2$ for the Young tableaux $[f]$. The strong space-exchange component in the two-body effective interaction acts to put the states in order of decreasing spatial symmetry before the spin-orbit force disrupts the symmetry. Since $C[SU(4)]$ increases with decreasing spatial symmetry, this explains the resulting increase in the total GT strength apparent in Fig. 1(a). For the states $J=T=0$ in ^{24}Mg the minimum strength is zero for [2222] $SU(4)$ symmetry ([44] spatial symmetry), and the maximum is $B(\mathrm{GT}) = 76/3$ for $[f]=[62]$ ([221111] spatial symmetry)and $S=2$. The symmetry argument does not depend on the J values, and thus even though we have only carried out the microscopic calculations for $J=0$ we expect the results for the higher J values to follow the same magnitude and increasing trend as a function of increasing energy.

The GT sum-rule for the shell-model initial states of ^{24}Mg will be a linear combination of the values in Eq. (10) weighted by the intensities for each $[f]$ and S in the wave function. The smooth increase in the GT strength observed in Fig. 1a comes about by the mixing of the $SU(4)$ configurations by the $SU(4)$ nonconserving parts of the residual interaction. In a sense, the total GT strength is a measure of the $SU(4)$ symmetry breaking and of the strength of the spin-orbit interaction. Vogel and Ormand [14] have studied the symmetry breaking in the ground-states of *sd*-shell nuclei and find 51.2% [44] spatial symmetry and 36.8% [431] symmetry in the ground state of ^{24}Mg. The [431] symmetry contributes 1.47 to the full shell-model B_{GT} of 2.3.

We also note that the two-body part of the Wildenthal hamiltonian [1] is dominated by the the large attractive $S=0, T=1$ and $S=1, T=0$ spatially symmetric matrix elements. Indeed, it is the spin-orbit single-particle splitting which is primarily responsible for breaking the $SU(4)$ symmetry. If we repeat the calculations for ^{24}Mg with the spin-orbit splitting set to zero, the extreme range of the calculated B_{GT} strength becomes very close to the $SU(4)$ limits of zero and 76/3. The role of the major $SU(4)$ conserving components of the Wildenthal interaction in accounting for the essential features of Fig. 1(a) can be made more explicit. The coefficient of the monopole space-exchange component $\sum_{ab} P_{ab}$ of the interaction is -1.675 MeV. The contribution of this interaction to the diagonal energy separation of the highest [44] and lowest [221111] symmetry states is $22 \times 1.675 = 36.8$ MeV. Likewise the $Q.Q$ component of the interaction can be shown to contribute roughly 18 MeV to the energy difference between the configurations with (8 4) and (2 1) $SU(3)$ symmetry associated with the above spatial symmetries. Thus 55 MeV of the ~ 77 MeV between the lowest and highest 0^+ states in Fig. 1 is accounted for.

In order to carry out a set of numerically tractable calculations, we have concentrated here on the results for 0^+ states in ^{24}Mg. However, all of the sum rules we discuss are independent of the total angular momentum J and hence the results for all $T = 0$ states of ^{24}Mg should follow the same trend. (This should be confirmed numerically.) In addition, our conclusions are qualitatively applicable to all *sd*-shell nuclei as well as the *fp*-shell and *sdg*-shell nuclei. The systematics of the Gamow-Teller strength for the ground states of all even-even nuclei with $N = Z$ [7] show a large reduction in the Gamow-Teller strength compared to the single-particle estimates. Vogel and Ormand [14] have shown that the ground states of most nuclei in the sd-shell are dominated by the lowest two $SU(4)$ symmetries, the first of which gives the smallest possible GT strength (zero for ^{24}Mg), and the second of which gives the next smallest GT strength (as discussed explicitly above for ^{24}Mg). Once the reduction of the GT strength in the ground state is established and its connects to the $SU(4)$ symmetry is understood, the effect of increasing GT strength as a function of excitation energy is inevitable.

The $0\hbar\omega$ spectrum corresponds to the actual physical spectrum only for the first 10–20 MeV, and after that physical level density becomes dominated by multi-particle multi-hole configurations outside of the $0\hbar\omega$ model space. Thus, although our specific $0\hbar\omega$ calculations are no longer valid above 10–20 MeV excitation energy, the general arguments about the change in spatial symmetry as a function of excitation energy are still valid.

For heavy nuclei, one usually starts with zeroth-order wave functions in the simplest *jj*-coupling basis [e.g. $(0d_{5/2})^8$ for ^{24}Mg], and then adds on a core-polarization or RPA correction to the GT strength [15]. For the *sd*-shell such perturbative calculations for the ground-state strength [15] typically go about half way from the extreme *jj*-coupling limit (8.3 for ^{24}Mg) toward the full $0\hbar\omega$ result (2.3 for ^{24}Mg). The perturbation theory correction for the highest energy states can also be considered, but at the top of the energy spectrum, the energy denominator for the states which are mixed in changes sign and the result is an enhancement from the *jj* zeroth-order value. Although, we have suggested that the *jj*-coupling plus RPA calculation may be adequate for nuclei above the *sd*-shell [15], we note that the GT strengths for the ground states in nuclei near ^{100}Sn are closer to the $SU(4)$ coupling limit (zero) than to the *jj*-coupling limit [16].

The entire picture of the GT strength is determined by the main features of the residual interaction related to the spatial and spin-isospin symmetry. This picture is seen clearly against the background of the incoherent collision-like interactions. The coexistence of regular nuclear motion with chaotic single-particle dynamics was discussed in a different context in [17]. The exotic $N \approx Z$ nuclei toward ^{100}Sn can supply additional information concerning the role of symmetry in nuclear stability. The results of shell-model calculations for GT transitions from excited states are related to the rates of certain astrophysical reactions, especially for the late stages of supernova [18]. Since excitation of the parent states of only up to about 10 MeV is important, the effect will not be large, but the general trend of increasing GT strength as a function of excitation energy should not be ignored. However, in addition to the total strength studied in this paper, the shape of the GT strength distribution is also important and this aspect remains to be investigated.

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