# Neutron radii and the neutron equation of state in relativistic models

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The root-mean-square radii for neutrons in nuclei and their relationship to the neutron equation of state are investigated in the relativistic Hartree model. A correlation between the neutron skin in heavy nuclei and the derivative of the neutron equation of state is found which provides a linear continuation of results obtained from nonrelativistic Hartree-Fock models. The relativistic models tend to give larger neutron radii and an associated stiffer neutron equation of state compared with the nonrelativistic models.

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It has recently been proposed to perform an experiment at Jefferson Laboratory [1,2] to measure the neutron radius in $^{208}$Pb via the parity violation effect in electron scattering more precisely than it has been determined previously from hadronic probes. The parity violation in this experiment comes from the $Z^0$ exchange between the electron and the nucleus which is dominated by interaction with neutrons. The sensitivity of the neutron radius to the parameters of the nonrelativistic Skyrme Hamiltonian for nuclear Hartree-Fock calculations and its rather simple linear relationship to the derivative of the neutron equation of state (EOS) was recently pointed out [3]. Since this result was based on nonrelativistic models, it is important to study to what extent similar conclusions are obtained from other models. In this Brief Report we investigate the results obtained from relativistic Hartree models. Although there are calculations for the neutron EOS based upon the known two-nucleon potential [4–6], there is a large uncertainty from the three-nucleon potentials, and any constraints coming from the properties of nuclei such as the neutron radii are extremely important.

Since the proton rms radii are accurately known from electron scattering experiments, it is useful to discuss the neutron radii in terms of the neutron "skin" defined by $S=\sqrt{\langle r_n^2\rangle}-\sqrt{\langle r_p^2\rangle}$. In [3] it was shown that the main quantity of importance for the neutron skin was the derivative of the neutron EOS for a value of the neutron density near that which exists in stable nuclei, chosen in [3] to be $\rho_0=0.10$ neutrons/fm$^3$. This quantity is not well determined from other nuclear properties. More physically the derivative may be expressed in terms of the pressure $P=\rho^2 d(E/A)/d\rho|_{\rho_0}$, but we will continue to use the derivative at $\rho_0=0.10$ neutrons/fm$^2$ which is numerically just a factor of 100 larger than the pressure. In the next few paragraphs we review the relativistic formulations, and then we finish with a comparison to the Skyrme model results. Some of the relativistic models we discuss have recently been used [7] to make the connection between the neutron skin and the properties of neutron stars. A more general review of the relationship between the neutron EOS and neutron star properties is given in [8].

The relativistic description of nuclear systems uses a field theoretical approach (quantum hydrodynamics) where the interaction of nucleons is described by an exchange of mesons. Investigation of this model began in the 1950s [9] and was developed in the pioneering work of Walecka [10]. From a given Lagrangian density containing nucleons and scalar and vector mesons as degrees of freedom the corresponding field equations are obtained, which can be solved in various approximations. Usually, a mean-field Hartree approximation is employed where the meson fields are treated as classical fields and negative-energy states of the nucleons are neglected (the no-sea approximation). Walecka was able to describe saturation properties of nuclear matter by adjusting the parameters (masses and coupling constants) of the theory assuming a linear coupling of nucleons and mesons. In the Dirac equation for the nucleons large scalar and vector self-energies appear which naturally lead in a nonrelativistic approximation to the well-known central and spin-orbit interactions.

Various extensions of the linear Walecka model have been proposed in order to obtain a more quantitative description of nuclear matter and atomic nuclei. Boguta and Bodmer [11] introduced nonlinear cubic and quartic self-interactions of the scalar $\sigma$ meson, which maintain the renormalizability of the theory, to describe an effective medium dependence of the interaction. This approach has become a standard choice in relativistic mean-field calculations due to its success in a quantitative description of many nuclear phenomena, like ground state properties of spherical and deformed nuclei, collective nuclear excitations, etc. It has also been used for exploring more extreme conditions, e.g., neutron stars and heavy-ion collisions.

The first serious attempt to develop a parametrization of this nonlinear model was made by Reinhard *et al.* [12]. They fitted the parameters to reproduce properties, i.e., binding energies, diffraction radii, and surface thickness in this case, of a selection of mostly doubly magic nuclei. This set (called NL1) was able to describe many properties of stable nuclei. For exploring exotic nuclei, an improvement of the NL1 parametrization was required, which can be related to its overprediction of the symmetry energy for nuclear matter. Including more exotic nuclei in the fitting procedure to binding

 

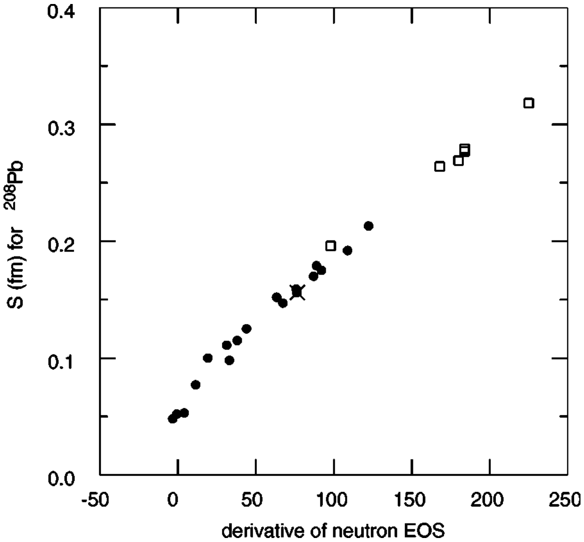


FIG. 1. The derivative of the neutron EOS at $\rho_0 = 0.10$ neutrons/fm$^3$ (in units of MeV fm$^3$/neutron) vs the $S$ value in $^{208}$Pb for 18 Skyrme parameter sets (filled circles) and for the six relativistic models (squares). The cross is the SKX Skyrme Hamiltonian [18].

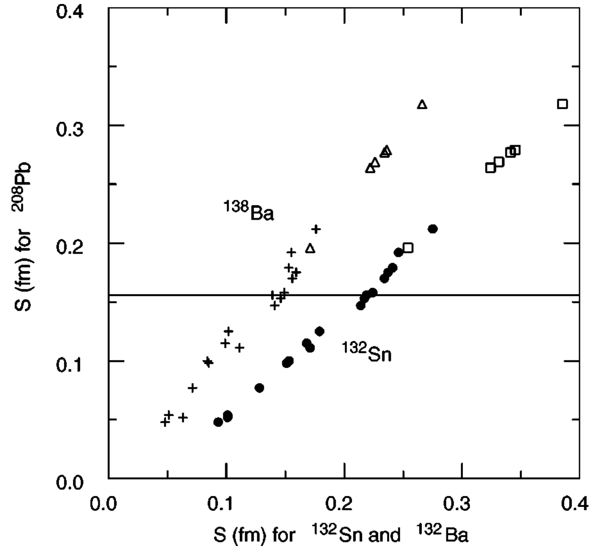


FIG. 2. The $S$ value for $^{208}$Pb vs the $S$ values for $^{132}$Sn (filled circles and squares) and $^{138}$Ba (pluses and triangles) for 18 Skyrme parameter sets (filled circles and pluses) and six relativistic models (squares and triangles). The horizontal line is the SKX value for $^{208}$Pb.

energies and charge radii, the parametrizations NL-SH [13] and NL3 [14] were obtained. They give a good description of many, even more unstable, nuclei. These nonlinear models lead to a wide range of values for the nuclear matter incompressibility; $K = 211$ MeV for NL1, $K = 270$ MeV for NL3, and $K = 356$ MeV for NL-SH.

Motivated by results of Dirac-Brueckner calculations of nuclear matter, Sugahara and Toki [15] introduced quartic self-interactions of the $\omega$ meson in the Lagrangian density to describe the observed flattening of the vector self-energy at high densities; it does not rise linearly as predicted by the standard nonlinear models. Their parameter set TM1 was obtained from a fit to nuclear binding energies and charge radii yield and gives $K = 281$ MeV, which is similar to the Dirac-Brueckner results.

In order to include medium effects in the description also a density dependence of the coupling constants in the original Walecka Lagrangian can be assumed without adding nonlinear self-interactions of the mesons. This approach is again influenced by Dirac-Brueckner calculations, where an effective density dependence of the self-energies can be extracted. In earlier models only a parametric dependence was introduced, but for a thermodynamically consistent theory so-called ''rearrangement'' contributions in the self-energies have to be considered. A fit to binding energies of nuclei and nuclear matter properties in this full theory leads to a successful description of nuclei in the parametrization VDD [16]. Here, the couplings of both isoscalar $(\sigma, \omega)$ and isovector mesons $(\rho)$ to the nucleons are assumed to be dependent on the vector density. The results give softer equations of state for symmetric ($K = 240$ MeV) and asymmetric nuclear matter.

The introduction of additional couplings of the mesons to the (derivative of the) nucleon field instead of meson self-interactions or density dependent coupling constants was investigated in [17]. From the various possibilities a model DC1 was extracted which leads to an effective density dependence of the meson masses. Here, the parameters were deduced from a fit to binding energies and charge distributions of nuclei. The predictions of this model are very similar to those of the nonlinear parametrization NL3.

To draw our conclusions we calculated the neutron EOS and the neutron skins for $^{132}$Sn, $^{138}$Ba, and $^{208}$Pb with these six relativistic models (NL1, NL3, NL-SH, TM1, DC1, and VDD). The figures from [3] based upon 18 Skyrme (nonrelativistic) parameter sets are extended to include these six relativistic models. In Fig. 1 we show the neutron skin in $^{208}$Pb vs the derivative of the neutron EOS. The relativistic points (squares) form a linear continuation of the nonrelativistic points (filled circles). The cross is the SKX Skyrme Hartree-Fock value which is based partly upon a fit to the Friedman-Pandharipande neutron EOS as discussed in [3,18]. The smallest value for $S$ is obtained with VDD and this value overlaps with the upper end of the Skyrme model points. The highest value for $S$ is obtained for NL1. The $S$ values for the other four models (NL3, NL-SH, TM1, and DC1) cluster in between.

In Fig. 2 we show the neutron skin in $^{208}$Pb vs the neutron skins in $^{138}$Ba and $^{132}$Sn (the horizontal line is the SKX Skyrme value for $^{208}$Pb). Again we find that the relativistic and nonrelativistic models form a unified linear set of points. The shift in skin thickness between $^{138}$Ba and $^{132}$Sn is due to the change in the Fermi energies for protons and neutrons [3].

The implication of these comparisons is that an accurate measurement of the neutron skin in one nucleus such as

$^{208}$Pb will provide a new constraint on both relativistic and nonrelativistic models. This constraint will enable more accurate calculations for the neutron radii in other nuclei as well as for the neutron EOS. Accurate models for the neutron rms radii are important for the interpretation of the ''weak-charge'' measurements in atoms which provide a test of the standard model of weak interactions [19]. Knowledge of the neutron EOS is of course crucial for understanding neutron stars [8].

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