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# Isospin symmetry breaking and the nuclear shell model

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**Abstract**

The effect of isospin symmetry breaking on level statistics has been examined with the nuclear shell model. The eigenvalues and electromagnetic transitions were calculated with the program OXBASH for the nuclide $^{26}$Al for conserved isospin and for broken isospin. The long-range correlations of the eigenvalues, as measured by $\Delta_3$, show good agreement between the experimental results and the calculations. However, there are discrepancies between data and calculations for the short-range correlations of the eigenvalues.


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The analysis by Wigner [1] of the spacing distribution of neutron resonances is usually considered the first application of random matrix theory (RMT). The status of RMT some 20 years later was summarized in the classic review by Brody et al. [2]. Following the proposed connection between RMT and chaos [3] and major theoretical developments—both in the 1980s—there was an explosion of applications of RMT in many fields. The most recent review of RMT by Guhr, Müller–Groeling, and Weidenmüller [4] has over 800 references!

Although nuclear physics is usually considered the birth place of RMT, there are surprisingly few tests of RMT in nuclei. The reason is the stringent requirements placed on the data by the standard measures employed: the spectra must be complete (few or no missing levels) and pure (few or no misassignments). In practice it is extremely difficult and time consuming to obtain the required purity and completeness, and thus there is a scarcity of careful tests of RMT in nuclei. Analysis [5–7] of neutron and proton resonance data yielded excellent agreement with the Gaussian orthogonal ensemble (GOE) version of RMT. Analysis of low-lying states throughout the

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periodic table [8] suggested that the spectra of low-lying states in light nuclei appear to be chaotic (agree with the Wigner distribution), while similar data in heavier nuclei appear to be regular (agree with the Poisson distribution). A study of higher spin states in the rare earth region [9] found a Poisson distribution. The level statistics of an ensemble of scissors-mode states in heavy deformed nuclei also agree with the Poisson distribution [10].

Our interest has focused on the effect of symmetry breaking on approximate symmetries. Dyson [11] predicted that the effect on the level statistics of breaking a discrete symmetry would be amplified in a dense spectrum. Some 20 years later Pandey [12] drew similar conclusions. The first direct observation of this effect was for the approximate symmetry isospin in the nuclide $^{26}$Al [13,14]. Guhr and Weidenmüller [15] showed that these data were consistent with independent estimates of the isospin symmetry breaking in $^{26}$Al. Hussein and Pato [16] obtained similar results. We recently determined the complete spectroscopy of another nuclide—$^{30}$P [17]; the level statistics agree very well with our earlier measurements. In addition measurements in analog systems—acoustic resonances in quartz blocks [18] and electromagnetic resonances in microwave cavities [19]—have demonstrated that the effect of symmetry breaking on the level statistics is very well described by RMT.

We then turned our attention to the study of electromagnetic transitions. At the time there was no explicit RMT prediction for the effect of symmetry breaking on the distribution of $\gamma$-ray transition strengths, although heuristic arguments suggested that the distribution would always remain Porter–Thomas [20]. Our experiments in $^{26}$Al and $^{30}$P showed significant deviations from the PT distribution [17,21]. The first RMT theoretical studies of this effect [22,23] predicted that the effect of symmetry breaking changes the experimental distributions (from the PT distribution). A recent analog measurement of two coupled superconducting microwave cavities determined the distribution of the product of partial widths [24]. The experimental data deviate from the $K_0$ distribution (the product of two Porter–Thomas distributions). These new results thus agree qualitatively with theory and our experimental data on $^{26}$Al and $^{30}$P.

Experimentally there are differences between the distributions for different transition types (e.g., E1 and M1, or dipole and quadrupole). This raises the general question of how far one can proceed in describing such data with the generic predictions of RMT (i.e., without any dynamics). This motivated us to consider shell model calculations in order to examine the extent to which symmetry breaking effects on level statistics and on transitions could be described within the nuclear shell model. The shell model also offers the advantage that concerns about completeness and purity are minimal. In the present Letter we consider only the level statistics.

Shell-model calculations were performed for the nuclei $^{22}$Na, $^{26}$Al, and $^{34}$Cl using the code OXBASH [25]. For each nuclide, the calculations were first performed with an isospin-conserving (IC) Hamiltonian and then repeated with an isospin-nonconserving (INC) Hamiltonian. The INC Hamiltonian is that of Ormand and Brown [26]. In addition, for $^{26}$Al two INC calculations were performed, one using single-particle energies based on the $A = 39$ system (hereafter labeled $\mathrm{INC}_A$) and one using single-particle energies based on the $A = 17$ system ($\mathrm{INC}_B$).

In terms of the input, isospin mixing comes from the $j$-dependence of the isovector single-particle energies and the $J$-dependence of the isovector and isotensor two-body matrix elements. If these isovector and isotensor matrix elements are constant (all the same for each kind) there is a constant Coulomb displacement energy and no isospin mixing. The INC interaction is based upon the Coulomb contribution calculated in an oscillator basis plus an INC strong interaction with strengths obtained from the experimental Coulomb displacement energies. With $\mathrm{INC}_A$ the isospin mixing is dominated by the $J$-dependence of the two-body INC matrix elements. The $j$-dependence of the single-particle energies in $\mathrm{INC}_B$ is much larger than that of $\mathrm{INC}_A$ due to the 400 keV Thomas–Ehrman shift of the $s_{1/2}$ orbit in $A = 17$ relative to the $d_{5/2}$ orbit. This results in isospin mixing matrix elements for $^{26}$Al which are about four times larger with $\mathrm{INC}_B$ compared to $\mathrm{INC}_A$.

In each calculation, the lowest $\approx 30$ positive-parity levels for each total angular momentum $J$ in the range 0—5 were determined. Our analysis included states up to the energy $E_{\max}$ at which any value of $J$ had its last level; therefore, we have completeness for $0 \leqslant E_x \leqslant E_{\max}$ and $0 \leqslant J \leqslant 5$. This provided sufficient statistics for study and provided an energy range comparable to

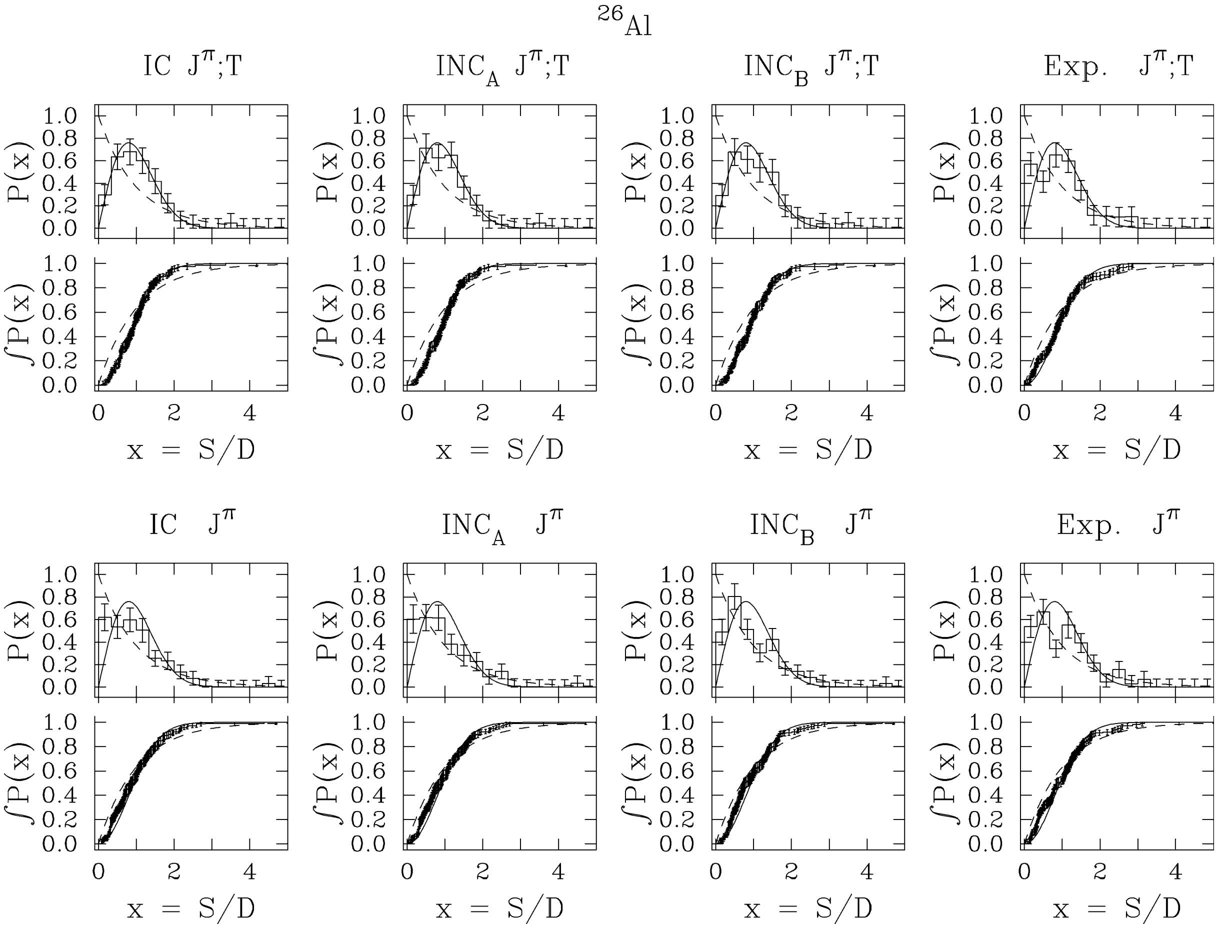


Fig. 1. Nearest-neighbor spacing distributions for shell-model eigenvalues in $^{26}$Al. The labels on each graph indicate which quantum numbers identify the sequences and which Hamiltonian was used in the calculation. Solid curves show GOE distributions, while dashed curves show the corresponding Poisson distributions.

the experimental data. (The experimental data extend to $E_{\mathrm{max}} = 8.066$ MeV in $^{26}$Al and 8.014 MeV in $^{30}$P, but the experimental data include negative parity states as well.)

We focus here on the nuclide $^{26}$Al, for which we have both experimental data and theoretical analyses. For this nuclide, the value of $E_{\mathrm{max}}$ was $\approx 9$ MeV; the total number of levels was 132 for the IC and $\mathrm{INC}_A$ Hamiltonians and 133 for the $\mathrm{INC}_B$ Hamiltonian. We will utilize two common eigenvalue statistics, the nearest-neighbor spacing distribution (NNSD) first described by Wigner [1] and the $\Delta_3$ statistic introduced by Dyson and Mehta [27]. The NNSD emphasizes short-range correlations, whereas $\Delta_3$ reflects long-range correlations ("spectral rigidity") in the eigenvalue spectrum. In Fig. 1 the results from the various shell model calculations as well as the experimental results are shown for the NNSD. The solid (dotted) line is the Wigner (Poisson) distribution, which corresponds to chaotic (integrable) behavior [4]. Both the distribution function and its integral are shown. The procedure for generating spacing distributions, including how we account for the energy dependence of the mean level spacing, has been described in detail by Shriner and Mitchell [28]. As shown in the upper left-hand corner of Fig. 1, the agreement with the Wigner distribution for the IC calculation appears quite good. One can obtain a quantitative measure of the distribution by fitting the data to an empirical interpolation formula due to Brody [29]. The data are

Table 1
Values of $\omega$ and $\mu$ for the $^{26}$Al shell-model calculations and for the experimental data for $^{26}$Al and $^{30}$P with $T$ included ($J^\pi; T$) and with $T$ omitted ($J^\pi$)

| Nuclide | IC/INC/Exp | $\omega$ | | $\mu$ | |
|---|---|---|---|---|---|
| | | $J^\pi; T$ | $J^\pi$ | $J^\pi; T$ | $J^\pi$ |
| $^{26}$Al | IC | $0.89 \pm 0.13$ | $0.49 \pm 0.10$ | $0.99 \pm 0.02$ | $0.81 \pm 0.02$ |
| $^{26}$Al | $\text{INC}_A$ | $0.87 \pm 0.13$ | $0.46 \pm 0.10$ | $0.98 \pm 0.02$ | $0.79 \pm 0.02$ |
| $^{26}$Al | $\text{INC}_B$ | $0.77 \pm 0.12$ | $0.48 \pm 0.10$ | $0.92 \pm 0.03$ | $0.80 \pm 0.02$ |
| $^{26}$Al | Exp | $0.51 \pm 0.11$ | $0.47 \pm 0.10$ | $0.93 \pm 0.04$ | $0.85 \pm 0.02$ |
| $^{30}$P | Exp | $0.47 \pm 0.15$ | $0.39 \pm 0.12$ | $0.93 \pm 0.04$ | $0.84 \pm 0.06$ |

then characterized by a parameter $\omega$, where $\omega = 1$ for a Wigner distribution and $\omega = 0$ for a Poisson distribution. This particular distribution is characterized by $\omega = 0.89 \pm 0.13$ (we have used a maximum-likelihood method to determine $\omega$). Next we ignore the isospin symmetry and repeat the analysis. As the lower left part of Fig. 1 shows, the agreement with the Wigner distribution is destroyed; this distribution is characterized by $\omega = 0.49 \pm 0.10$. This behavior is what one expects: when a conserved symmetry is ignored, the statistical distribution dramatically reflects this omission.

Next we repeat the calculation with isospin broken according to the prescription of Ormand and Brown [26]. States are now classified according to their spin, parity, and dominant isospin. The dominant isospin is determined by calculating the overlaps of each INC wavefunction with the IC wavefunctions for the same spin. The state is then assigned $T = 0$ or $T = 1$ according to which isospin component is larger. The majority of states remain dominated by a single isospin; this is not surprising since the average level spacings at the highest energies for states of a single $J$ are $\approx 20$ times the rms interaction matrix element for $\text{INC}_A$ and $\approx 4$ times the rms matrix element for $\text{INC}_B$.

As the center portions of Fig. 1 show, the results look similar to those when isospin was conserved. This is borne out by the numerical results. When $T$ is treated as a good quantum number, values of $\omega = 0.87 \pm 0.13$ and $\omega = 0.77 \pm 0.12$ are obtained for $\text{INC}_A$ and $\text{INC}_B$, respectively. When $T$ is ignored the corresponding values of $\omega$ are $0.46 \pm 0.10$ and $0.48 \pm 0.10$. The fact that $\omega$ changes so drastically when $T$ is ignored suggests that $T$ is still a very good symmetry. The quantitative results are summarized in Table 1.

Now consider the results for the experimental spacing distribution in $^{26}$Al, shown at the right of Fig. 1 and also listed in Table 1. Naturally one has only the INC case, since nature has already performed the symmetry breaking. In this case the value of $\omega$ is unchanged within experimental error whether we keep track of $T$ or ignore $T$—$0.51 \pm 0.11$ and $0.47 \pm 0.10$. Contrasting this result—very little change in $\omega$ whether isospin is considered or ignored—with that when isospin is conserved—a significant change between the two cases—indicates that the small symmetry-breaking effect has had a large impact on the statistical distribution. In fact, this is what Dyson [11] predicted—although the symmetry is only slightly broken, the effect on the level statistics is large. Detailed theoretical analyses of these data [15, 16] agree with this interpretation. Experimental data for $^{30}$P, the only other system for which data of this nature are available, show similar behavior. The differences between the $\text{INC}_B$ calculation and the experimental results can be seen in more detail in Fig. 2. The major difference in the two distributions is at small values of $x$; the "level repulsion" is much stronger in the calculation than in the data.

We then repeated the entire analysis considering the Dyson–Mehta $\Delta_3$ statistic [27]. The results are shown in Fig. 3 and listed in Table 1. For each of the three calculations, there is a noticeable shift in the values away from the GOE result when $T$ is ignored. We use the interpolation formula of Seligman and Verbaarschot [30] to quantify these results. The expected value of the interpolation parameter $\mu$ is 1 for a GOE spectrum. For the IC calculations, the parameter $\mu$ changes from 0.99 to 0.81 when $T$ is ignored. For the INC cases, the values of $\mu$ change from 0.98 and 0.92 to 0.79 and 0.80 for $\text{INC}_A$ and $\text{INC}_B$, respectively. The values of $\mu$ for the IC and $\text{INC}_A$ calculations are essentially the same. The experimental results for the interpolation parameter

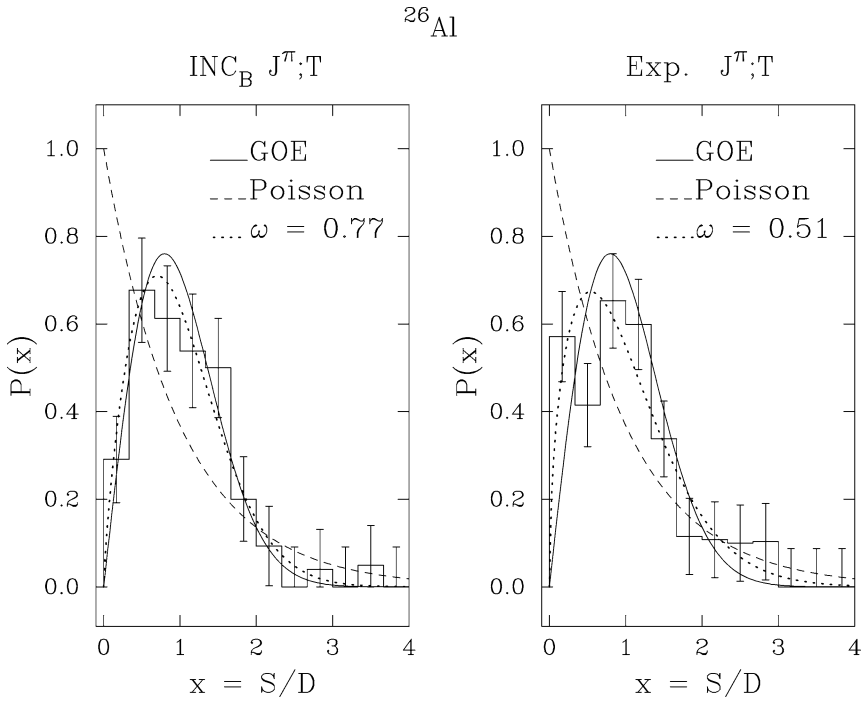


Fig. 2. Probability density functions for nearest-neighbor spacing distributions for the $\mathrm{INC}_B$ calculation and the experimental data in $^{26}$Al. Solid lines show the GOE distribution, dashed lines show the Poisson distribution, and dotted lines show the Brody distribution [29] for the best-fit value of the parameter $\omega$.

$\mu$ are also listed in Table 1. The $\mathrm{INC}_B$ calculation is in reasonable agreement with the experimental behavior.

A way to quantify this result is to examine the root-mean square value of the isospin mixing matrix elements. Guhr and Weidenmüller [15] obtained a value of $\approx 20$ keV in $^{26}$Al by examining the behavior of $\Delta_3$ for the experimental data. The overlap in the $\mathrm{INC}_A$ calculation between $2^+$ states with pure $T$ and those $2^+$ states with mixed $T$ was calculated in order to determine the INC matrix element, and a value of $\approx 6$ keV was obtained. In contrast, the $\mathrm{INC}_B$ calculations give a Coulomb matrix element of 24 keV which is consistent with the value of $\approx 20$ keV that Guhr and Weindermüller obtained from the random matrix analysis of $\Delta_3$.

It remains a puzzle why the spacing distributions for the shell model and the experiment differ as much as they do when $\Delta_3$ for the shell model and experiment appear to agree. One possible explanation could be that levels in $^{26}$Al are missing, or spins and/or parities have been misassigned in the data set; those effects are known [28] to shift spacing distributions in a fashion similar to what has been observed in $^{26}$Al. However, the fact that very similar spacing distributions are also observed in independent data for $^{30}$P (see Table 1) seems to suggest that this is not a likely explanation. Such an explanation would also seem inconsistent with the nearly one-to-one correspondence between experimental positive-parity states and shell-model states [31] that suggests the experimental data are of the necessary high quality required for the statistical analyses. It should be emphasized that the spin, parity, and isospin assignments in $^{26}$Al represent the combined results of many different experiments and extensive analyses (see especially Ref. [31]) and are of the highest quality.

A second explanation could be that for some reason the shell model does not reproduce the short-range correlations. This too does not seem likely, because the shell model with isospin conserved does produce good agreement with the Wigner distribution (see Fig. 1 and Table 1). A third possible explanation is that the discrepancy at small $x$ in Fig. 2 indicates that the rms

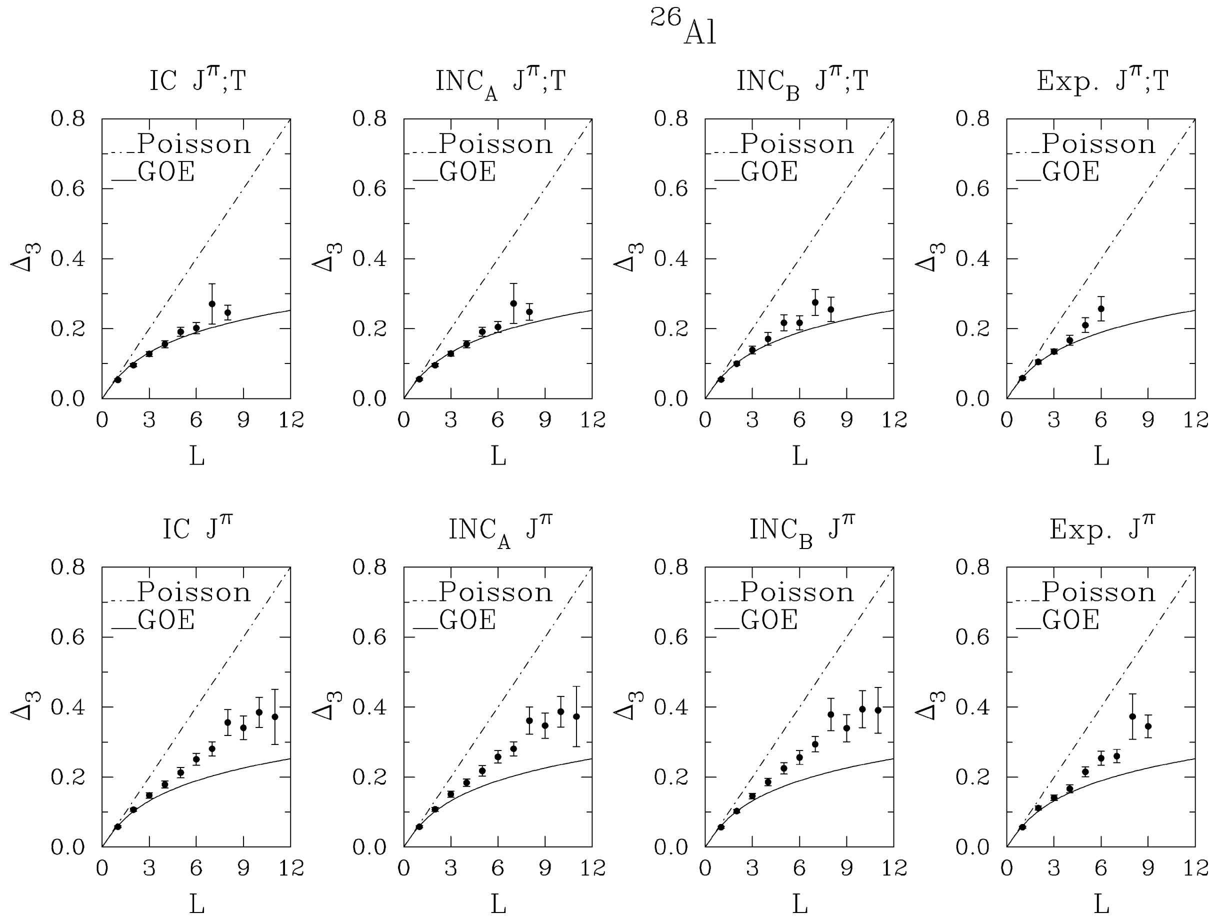


Fig. 3. $\Delta_3$ for shell-model eigenvalues in $^{26}$Al. The labels on each graph indicate which quantum numbers identify the sequences and which Hamiltonian was employed in the calculation.

INC matrix element may be even larger than 24 keV. As discussed above, the isospin mixing in $INC_B$ is determined primarily by the Thomas–Ehrman shift of the $s_{1/2}$ orbit. In this context the data would indicate that the effective Thomas–Ehrman shift for the $s_{1/2}$ component of the excited states in $^{26}$Al is as large or even larger than the 400 keV observed in $A = 17$. The extraction of the Coulomb matrix element for these data by Guhr and Weidenmüller [15] utilized only $\Delta_3$, and an extraction of the Coulomb matrix element directly from the spacing distribution has not been performed. We note that there is evidence [28] that $\Delta_3$ values for small sample sizes are much more variable than spacing distributions; this at least allows the possibility that the spacing distribution might be consistent with a larger matrix element.

We have also analyzed electromagnetic transitions, which were calculated for the same three nuclei and measured experimentally in $^{26}$Al. A variety of effects are observed that depend on the transition type (electric or magnetic; dipole or quadrupole) and on the isoscalar or isovector nature of the transition. These results will be presented elsewhere [32].

In conclusion, we have studied eigenvalue statistics for shell-model calculations for $^{26}$Al and compared them with results for experimental data. Three different shell-model Hamiltonians were employed, one of which conserved isospin and two of which did not. The isospin-conserving Hamiltonian and an isospin-nonconserving calculation utilizing single-particle energies based on the $A = 39$ system produced essentially identical nearest-neighbor spacing distributions;

this suggested that the amount of symmetry breaking in this INC calculation was too small, and a determination of the Coulomb matrix element in this system seemed to confirm this. In contrast, an isospin-nonconserving calculation utilizing single-particle energies based on the $A = 17$ system had a Coulomb matrix element in good agreement with the value extracted from the experimental values of the $\Delta_3$ statistic. For the spacing distribution the $\mathrm{INC}_B$ calculation is different from that for the isospin-conserving case, but does not agree with the experimental distribution. The origin of this discrepancy is unclear. In spite of this remaining question, the present results show that the shell-model can be useful in studying statistical properties when isospin is a broken symmetry and illustrate the sensitivity of level statistics as a tool in studies of symmetry breaking.

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