

Coulomb energies in  $^{17}\text{Ne}$  and the ground state mass of  $^{18}\text{Na}$ 

H. T. Fortune

*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*

R. Sherr

*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

B. A. Brown

*Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*

(Received 17 January 2006; published 30 June 2006)

For negative-parity mirror states in  $^{17}\text{N}$  and  $^{17}\text{Ne}$ , we computed Coulomb energies in three different models. Results, along with spectroscopic factors for  $^{18}\text{Na}$  (ground state) to various states in  $^{17}\text{Ne}$ , are used to calculate the  $^{18}\text{Na}$  mass excess.

DOI: [10.1103/PhysRevC.73.064310](https://doi.org/10.1103/PhysRevC.73.064310)

PACS number(s): 21.10.Sf, 21.10.Dr, 27.20.+n

The low-lying negative-parity states of  $^{17}\text{N}$  [1] are reasonably well described as two  $sd$ -shell neutrons coupled to the  $^{15}\text{N}$  ground state (gs). Two different sets of wave functions within this space were used to analyze results from the  $^{15}\text{N}(t, p)$  reaction [2]. Both sets worked well, and comparison between experiment and calculations allowed several suggestions of  $J^\pi$  and a dominant configuration. One minor exception was the first  $3/2^-$  state, whose energy and strength in  $^{18}\text{O}(d, ^3\text{He})$  [3] indicate about 10% configuration with a  $p3/2$  hole (rather than  $p1/2$ ).

The states in  $^{17}\text{Ne}$  should be mirrors of those in  $^{17}\text{N}$ , but the different values of  $N, Z$ —together with significant occupancy of the  $2s1/2$  orbital—can produce appreciable differences in energy splittings in the two nuclei. We have computed Coulomb energies in  $^{17}\text{Ne}$  using one set of wave functions (labeled LSF) from Ref. [2] and a set from a recent shell-model (sm) calculation [4]. The LSF wave functions result from a pure two-nucleon calculation, using two-body matrix elements (Constrained II) from Ref. [5] and “local” (i.e.,  $^{16}\text{N}$ ) single-particle energies.

The shell-model calculation uses the  $p$ - $sd$  model space in which the wave functions for  $A = 17$  have two particles in the  $sd$  shell and one hole in the  $p$  shell ( $p1/2$  or  $p3/2$ ). We use the WBP Hamiltonian that was obtained from the USD interaction for the  $sd$ -shell matrix elements and from fitted potential-model interactions for the  $p$ -shell and cross-shell  $p$ - $sd$  matrix elements. This Hamiltonian and the data considered for its determination are discussed in Ref. [4].

Our assumption is that the admixture coefficients in the wave functions are equal in  $^{17}\text{N}$  and  $^{17}\text{Ne}$  and that only the shape of the radial wave function is different for mirror levels. Such an approach worked well for Coulomb energies in  $^{18}\text{Ne}$  [6].

Our wave functions are computed in a Woods-Saxon potential ( $r_0 = 1.25, a = 0.65$  fm), plus angular momentum and Coulomb terms (uniform sphere) as appropriate. For the LSF wave functions (Table I), we couple  $s^2, d^2$ , and  $ds$  to the gs of  $^{15}\text{N}$  or  $^{15}\text{O}$ . For the newer sm set, we use  $sp$  spectroscopic factors (Table II) computed with the interaction of Ref. [4]

TABLE I. Wave function amplitudes for  $^{17}\text{N}$  levels coupled to  $^{15}\text{N}$  (gs) (LSF from Ref. [2]).

$2J^-$	$n$	$\Delta J^a$	Wave function amplitude <sup>b</sup>		
			$d^2$	$s^2$	$ds$
1	1	0	0.845	0.536	—
1	2	0	0.536	−0.845	—
3	1	2	0.787	—	0.572
3	2	2	−0.599	—	0.798
5	1	2	0.787	—	0.572
5	2	2	−0.599	—	0.798
5	3	3	0	—	1.0
7	1	4	0.989	—	—
7	2	3	0	—	1.0
9	1	4	0.989	—	—

<sup>a</sup>Angular momentum for the pair of  $sd$ -shell nucleons.<sup>b</sup>Small components have been omitted.

for  $A = 16, T = 1$  to  $A = 17, T = 3/2$ . Results are listed in Table III. Whenever the states are known in  $^{17}\text{Ne}$  [1,7] both calculations do reasonably well, but the two predictions do differ somewhat. The newest experimental value of the  $^{17}\text{Ne}$  mass excess is 16461(27) keV [8], supplanting the previous value of 16480(50) keV [9]. If the experimental state is not known, we still list the two computed energies, because we wish to use these to approximate the  $^{18}\text{Na}(\text{gs})$  mass. We note that, relative to the central experimental value, the LSF wave functions get the  $^{17}\text{Ne}(\text{gs})$  too low by 6 keV and the sm too high by 26 keV, but both are within the present experimental uncertainty. Throughout this work, all energies were computed to the nearest keV but rounded to 10 keV for most of the quoted results. For the five known levels, the average of the absolute value of measured minus calculated energy is 60 keV for LSF and 40 keV for sm. For the other states, the sm energies average about 30 keV lower than the LSF ones. In both calculations, one of the largest discrepancies is for the first  $7/2^-$  state, whose wave function is very simple in both sets of wave functions.

TABLE II. Single-nucleon spectroscopic factors for  $^{16}\text{N}$  to  $^{17}\text{N}$  from a shell-model calculation.

$2J^-$		Spectroscopic factors <sup>a</sup> to core states			
$n$		0 <sup>-</sup>	1 <sup>-</sup>	2 <sup>-</sup>	3 <sup>-</sup>
1	1	0.119	0.00, 0.360	0.589	0.784
1	2	0.366	0.005, 1.113	0.204	0.269
3	1	0.034	0.397, 0.0002	0.648, 0.391	0.331
3	2	0.018	0.466, 0.00008	0.471, 0.488	0.275
5	1	0.209	0.362	0.207, 0.009	0.591, 0.469
5	2	0.322	0.105	0.286, 0.027	0.744, 0.385
5	3	0.370	0.527	0.035, 0.921	0.060, 0.00003
7	1	—	0.0006	1.531	0.288, 0.0006
7	2	—	0.955	0.003	0.034, 0.945
9	1	—	—	0.116	1.664

<sup>a</sup> $S$ 's for  $d5/2$  and  $d3/2$  have been added. Whenever two  $l$  values are allowed, the  $S$ 's are listed as  $l = 2, l = 0$ .

The present procedure for computing Coulomb energies was used previously [10] to estimate the amount of  $s^2$  in the  $^{17}\text{Ne}(\text{gs})$  as 22% using the earlier experimental mass excess. With the newer value, the method of [10] gives  $24 \pm 3\%$  for the  $s^2$  occupancy in the gs. The wave function in Table I has 28%  $s^2$ , the sm slightly less. All these are significantly lower than estimates of others [11], who prefer a preponderance of  $s^2$ .

For computation of the  $^{18}\text{Na}(\text{gs})$  mass excess, we use  $A = 17, T = 3/2$  to  $A = 18, T = 2$   $sp$  spectroscopic factors (Table IV) from the recent shell-model calculation. If the energy of the  $^{17}\text{Ne}$  core state is known, we use it. Otherwise we perform calculations for both sets of  $^{17}\text{Ne}$  computed energies and for a set called wc (for weak coupling) that we obtained by assuming the same energy shift in  $^{17}\text{Ne}$   $[(sd)^2(1p)^{-1}]$  as for the corresponding state in  $^{18}\text{Ne}$   $[(sd)^2]$ . These wc energies are listed in Table III for states not known experimentally. The three results for the proton separation energy are 1402, 1387, and 1382 keV for wc, LSF, and sm, respectively. Note the difference in these values is considerably less than

TABLE III. Experimental and computed energies in  $^{17}\text{Ne}$ .

$2J^-$	$n$	Ex ( $^{17}\text{N}$ ) (MeV) <sup>a</sup>	Energy <sup>b</sup> (MeV) in $^{17}\text{Ne}$			
			Exp. <sup>c</sup>	wc	LSF	sm
1	1	0	16.461(27)	—	16.455	16.487
1	2	3.66	—	2.85	2.75	2.67
3	1	1.37	1.29	—	1.36	1.24
3	2	3.20	—	2.90	2.83	2.77
5	1	1.91	1.76	—	1.84	1.75
5	2	(3.91)	—	3.61	3.42	3.53
5	3	(4.42)	—	3.56	3.50	3.45
7	1	3.13	3.00	—	3.15	3.08
7	2	(4.81)	—	3.95	3.73	3.68
9	1	3.63	3.55	—	3.57	3.57

<sup>a</sup>Reference [1].

<sup>b</sup>Mass excess for gs, excitation energy otherwise.

<sup>c</sup>References. [7,8].

TABLE IV. Spectroscopic factors<sup>a</sup> from a shell-model calculation for lowest 1<sup>-</sup> and 2<sup>-</sup> states of  $^{18}\text{Na}$  to core states in  $^{17}\text{Ne}$ .

$2J^-$	$n$	$S(1^-)$	$S(2^-)$
1	1	0.0118, 0.0054	0.700
1	2	0.00009, 0.0017	0.0002
3	1	0.672, 0.195	0.209, 0.033
3	2	0.054, 0.153	0.135, 0.087
3	3	0.016, 0.030	0.072, 0.00004
5	1	0.891	0.110, 0.001
5	2	0.085	0.074, 0.002
5	3	0.184	0.022, 0.194
7	1	0.425	0.992
7	2	0.020	0.0003
9	1	—	0.135

<sup>a</sup>When two  $l$  values are allowed, they are listed as  $l = 2, l = 0$ .  $S$ 's for  $d5/2$  and  $d3/2$  have been added together.

the experimental uncertainty in the gs mass excess of  $^{17}\text{Ne}$ . Combining the computed  $E_p$ 's with that mass excess yields the  $^{18}\text{Na}(\text{gs})$  mass excesses listed in Table V. Also listed there are values from two separate analyses [12, 13] of the latest experiment [12]. Calculated results are 25.152, 25.137, and 25.132 MeV, respectively, for exp+wc, exp+LSF, and exp+sm. Also listed is the weak-coupling value of 25.23 MeV from Ref. [13]. In  $^{18}\text{N}$ , the probable 2<sup>-</sup> first excited state is 115 keV above the 1<sup>-</sup> gs. Our calculations give the same ordering in  $^{18}\text{Na}$ , with a splitting of 135–200 keV in the various models.

A recent experiment found [12] the  $^{18}\text{Na}(\text{gs})$  mass excess to be either 25.04(17) or 24.19(16) MeV. If the higher value is correct, the lower energy corresponds to decay of excited state(s) to excited state(s). A separate analysis [13] of the spectrum of Ref. [12] suggests that the 25.04-MeV peak is actually a doublet, with the two components separated by 240(50) keV. The gs is then at 24.88 MeV +  $\Delta$ , where  $\Delta$  is estimated [13] to be 0.18(12) MeV, so that from that analysis, the experimental  $^{18}\text{Na}(\text{gs})$  mass excess is 25.06(13) MeV. The computed results are in embarrassingly good agreement with the measured value. It would appear that the status of the Coulomb energy calculations is such that

TABLE V. Calculated and measured mass excess of  $^{18}\text{Na}(\text{gs})$ .

Mass excess (MeV)	
Calculated	
$^{17}\text{Ne}$ exp + wc <sup>a</sup>	25.152
$^{17}\text{Ne}$ exp + LSF <sup>a</sup>	25.137
$^{17}\text{Ne}$ exp + sm <sup>a</sup>	25.132
Weak coupling <sup>b</sup>	25.23
Measured	
Reference [12]	25.04(17) or 24.19(16)
Reference [13]	25.06(13)

<sup>a</sup>Using  $^{17}\text{Ne}$  mass excess of 16461 (for which the uncertainty is 27 keV).

<sup>b</sup>From Table II of Ref. [13].

improved experimental values of both  $^{17}\text{Ne}$  and  $^{18}\text{Na}$  mass excesses are needed. The present comparisons suggest that

such calculations for nuclei further removed from stability (e.g.,  $^{19}\text{Mg}$ ) might be worthwhile.

- 
- [1] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. **A564**, 1 (1993).
- [2] H. T. Fortune, G. E. Moore, L. Bland, M. E. Cobern, S. Mordechai, R. Middleton, and R. D. Lawson, Phys. Rev. C **20**, 1228 (1979).
- [3] G. Mairle *et al.*, Nucl. Phys. **A280**, 97 (1977).
- [4] E. K. Warburton and B. A. Brown, Phys. Rev. C **46**, 923 (1992).
- [5] R. D. Lawson, F. J. D. Serduke, and H. T. Fortune, Phys. Rev. C **14**, 1245 (1974).
- [6] R. Sherr and H. T. Fortune, Phys. Rev. C **58**, 3292 (1998).
- [7] V. Guimaraes *et al.*, Phys. Rev. C **58**, 116 (1998).
- [8] G. Audi, O. Bersillon, J. Blachet, and A. H. Wapstra, Nucl. Phys. **A729**, 3 (2003).
- [9] A. H. Wapstra and G. Audi, Nucl. Phys. **A432**, 1 (1983).
- [10] H. T. Fortune and R. Sherr, Phys. Lett. **B503**, 70 (2001).
- [11] A. Ozawa *et al.*, Phys. Lett. **B334**, 18 (1994); N. K. Timofeyuk, P. Descouvemont, and D. Baye, Nucl. Phys. **A600**, 1 (1996); S. Nakamura, Y. P. Guimaraes, and S. Kubono, Phys. Lett. **B416**, 1 (1998).
- [12] T. Zerguerras *et al.*, Eur. Phys. J. A **20**, 389 (2004).
- [13] H. T. Fortune and R. Sherr, Phys. Rev. C **72**, 034304 (2005).