

**Proton single-particle energies in  $^{23}\text{F}$** 

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We compare recent experimental results for the spectroscopic properties of  $^{23}\text{F}$  with theoretical results based on the new Hamiltonians USDA and USDB for the  $sd$ -shell model space. The implications for the proton single-particle energies in  $^{23}\text{F}$  are discussed. Conclusions are drawn about the level scheme of  $^{23}\text{F}$  and the accuracy of the new Hamiltonians from these comparisons.

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**I. INTRODUCTION**

Nucleon transfer reaction experiments on nuclei far from stability give information on the single-particle properties of these nuclei. The single-particle properties are essential for the basic test of nuclear models used for making predictions and extrapolations for the properties of nuclei even further from stability. In this paper we consider results obtained in a recent experiment by Michimasa *et al.* [1] for proton transfer onto  $^{22}\text{O}$  to make low-lying excited states in  $^{23}\text{F}$ , together with information from other reactions leading to  $^{23}\text{F}$ . In the experimental paper, some results were compared to the universal  $sd$ -shell (USD) Hamiltonian [2,3] that has provided realistic  $sd$ -shell ( $0d_{5/2}$ ,  $0d_{3/2}$ ,  $1s_{1/2}$ ) wave functions for use in nuclear structure models, nuclear spectroscopy and nuclear astrophysics for over two decades. However, the derivation of the USD Hamiltonian has been revised using an updated and complete set of available energy data. The resultant Hamiltonians USDA and USDB [4] lead to a new level of precision for realistic shell-model wave functions. Therefore, in addition to USD, we use the USDA and USDB to compare with the experimental results. We find that the lowest fragments of proton single-particle strength found in these experiments are in agreement with the new Hamiltonians, but that there is a large fragmentation of single-particle strength to higher levels, unobserved in the experiment, that push the single-particle centroid energy to much higher energy.

The primary sources of experimental data are Figs. 3 and 5 of Ref. [1]. Figure 3 shows the level scheme derived from the analysis of  $\gamma$  spectra, along with the relative cross section at each level for each reaction. Figure 5 compares this scheme to results obtained with the USD Hamiltonian. The theoretical calculations for energy levels, gamma decay, and spectroscopic factors in this paper were obtained in the  $sd$ -shell model space with the USD, USDA, and USDB Hamiltonians using the shell-model code OXBASH [5]. The electromagnetic matrix elements were obtained with free-nucleon  $g$ -factors for  $M1$ , effective charges of  $e_p = 1.5$  and  $e_n = 0.5$  for  $E2$  and harmonic oscillator radial wave functions. This information was used to infer which of the experimental levels could be matched with the  $sd$ -shell predictions. Nine of the 16 levels listed in the Michimasa paper have been identified by their energy and spin based on this analysis below. The complete level scheme up to 7.0 MeV for each Hamiltonian

is included in Figs. 1, 2, and 3, with a comparison to the experimental level scheme. The lowest five levels could be matched with the  $sd$ -shell calculations. Near 4 MeV the experimental level density is higher than theory, indicating that some of these states may have negative parity. Above 4.5 MeV the theoretical level density is much higher than experiment, indicating an incomplete experimental level scheme, and unique associations between experiment and theory could not be made.

As discussed in [4], USDA and USDB are determined from a least squares fit to 608 energy data by varying linear combinations of two-body matrix elements (TBME) that are relatively well determined by the data and by using the renormalized  $G$  matrix for the  $sd$ -shell model space (RGSD) for the other linear combinations. For USDA (USDB), 30 (56) linear combinations were varied to achieve an rms deviation of 170 (126) keV for the 608 energy data with resulting TBME that differ from the RGSD by an rms of 290 (375) keV. USDA is a “conservative” Hamiltonian with a good but not best fit to energy data, and with TBME closest to the RGSD. USDB provides the best fit with TBME that differ more from RGSD. Comparison of USDA and USDB results can provide a measure of the theoretical error for various calculated quantities. This paper provides one of the first examples for such a comparison.

The proton transfer reaction,  $^{22}\text{O} (0^+) \rightarrow ^{23}\text{F}$  reaction, should be strong only to low-lying  $sd$ -shell states that can be made with the addition of a proton in the state with  $(nlj)$  equal to  $1s_{1/2}$ ,  $0d_{3/2}$ , or  $0d_{5/2}$  to spin zero, leading to states with spin-parity  $1/2^+$ ,  $3/2^+$  or  $5/2^+$ , respectively. The neutron knockout reaction,  $^{24}\text{F} (3^+) \rightarrow ^{23}\text{F}$ , can lead to  $sd$ -shell states with spin-parity  $1/2^+$  to  $11/2^+$ , or to  $p$ -shell states with  $3/2^-$  to  $9/2^-$ . The two-nucleon knockout,  $^{25}\text{Ne} (1/2^+) \rightarrow ^{23}\text{F}$  is most complicated with possible final states of  $1/2^+$  to  $11/2^+$  and  $1/2^-$  to  $9/2^-$ .

The USDB Hamiltonian results will be used for a discussion of the gamma decay and nucleon transfer spectroscopic factors, with a comparison to USD and USDA results when the differences are large. The first level is the ground state, already assigned as the  $5/2^+$  state [6]. The binding energy of  $^{23}\text{F}$  relative to  $^{16}\text{O}$  is 47.65 MeV to be compared with the theoretical values of 47.67, 47.76, and 47.75 MeV, for USD, USDA, and USDB, respectively (the theoretical values

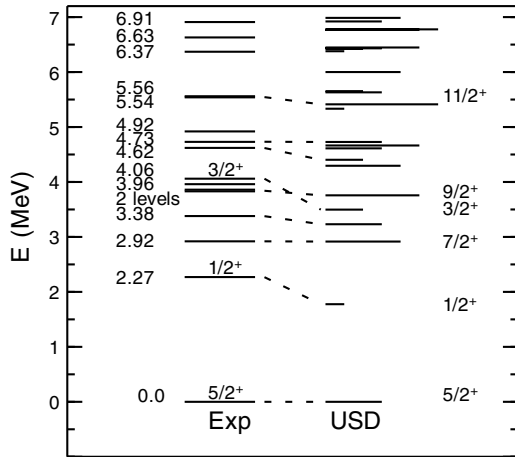


FIG. 1. Comparison between the experimental and USD level schemes, with dotted lines connecting corresponding energy levels. The length of the line on the right-hand side indicates the spin of the theoretical level, with the lowest energy state for each spin labeled. The “2 levels” are at 3.83 and 3.86 MeV, with the dotted line connecting the experimental level at 3.83 MeV to a theoretical level. The 3.86 MeV level could not be identified.

include the 3.48 MeV Coulomb correction for  $Z = 9$  relative to  $Z = 8$  [4]).

### II. ENERGY LEVELS

The first excited state is at 2.27 MeV, and is correctly labeled as the lowest energy  $1/2^+$  state by the authors. The energy for this level is double the rms difference in energy for the USDB Hamiltonian, but the spin assignment of the state is supported by the large proton-transfer cross section, and because the neutron knockout reaction has a near zero cross section. As discussed in [1], the hindered neutron knockout reaction is evidence of the single particle nature of the 2.27 MeV state, and is reasoned to be the  $s_{1/2}$  single particle state. The calculated spectroscopic factor of 0.07 for the knockout reaction is consistent with the small cross section.

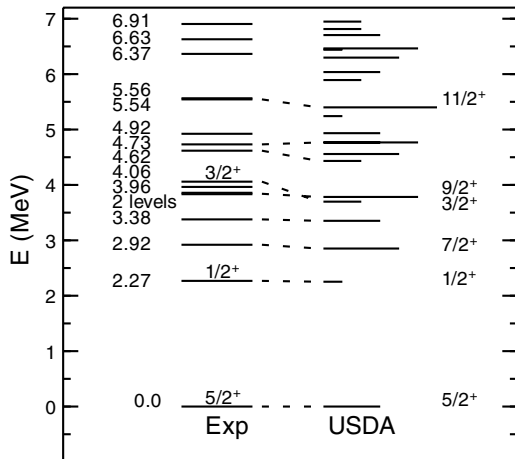


FIG. 2. Comparison between the experimental and USDA level schemes. See caption to Fig. 1.

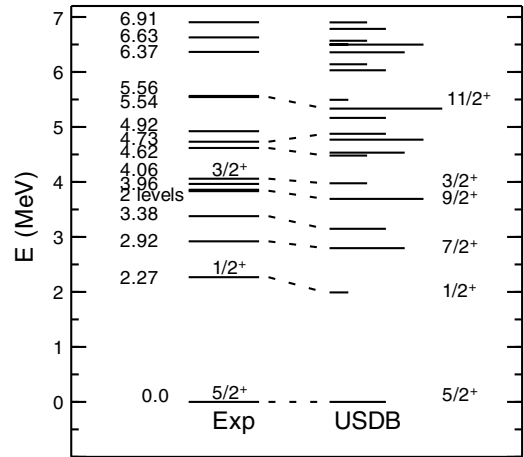


FIG. 3. Comparison between the experimental and USDB level schemes. See caption to Fig. 1.

The next state is at 2.92 MeV. It is not observed in the proton transfer, suggesting that the spin is larger than  $5/2$ . The experimental energy is in good agreement with the first  $7/2^+$  level. Also, the calculated spectroscopic factors for the other three reactions are large for this state, which agrees with the experimental results. This state decays only to the ground state and not to the  $1/2^+$  state experimentally, agreeing with the calculated decay scheme. This analysis enables the experimental state to be associated with the first  $7/2^+$  state.

The next experimental level has an energy of 3.38 MeV. It has a large proton-transfer cross section, as well as significant cross sections for the other reactions. The excitation energy and transfer-reaction properties correspond to the second  $5/2^+$  state. This level is expected to decay through two channels: 88% to the ground state, and 12% to the  $7/2^+$  state. The  $\gamma$  ray for the second transition is not listed in the experimental scheme. Experimentally, the energy should be 0.46 MeV. As seen in the  $\gamma$  ray spectra in Fig. 1 of [1], the lower limit of the energy of the spectra is 0.5 MeV, which explains why the transition may have been missed in experiment.

The next level has an energy of 3.83 MeV, which was determined from a 0.91 MeV  $\gamma$  ray in coincidence with the 2.92 MeV  $\gamma$  ray to the ground state. The nearby theoretical state is the  $9/2^+$  level at 3.69 MeV, which decays to the  $7/2^+$  state by a 0.90 MeV  $\gamma$  ray. All four reactions have small cross sections for this level, consistent with theory except for the proton transfer, which should have zero cross section. This might be explained by the population of higher energy  $5/2^+$  states that decay into the 3.83 MeV state. For instance, the  $5/2^+$  4.88 MeV state decays to the  $7/2^+$  4.53 MeV state (among others), which decays to the ground state and the first  $9/2^+$  state. Thus, the  $9/2^+$  state can be populated indirectly by the transfer reaction. The decay scheme from the  $9/2^+$  state consists of 61% decay to the ground state and 39% decay to the  $7/2^+$  state by the 0.90 MeV  $\gamma$  ray. The decay to the ground state is within the gamma-energy range of the  $\gamma$  spectra of the experiment, and is expected to be measured experimentally. However, this transition is not given in the decay spectra of [1].

The next state at 3.86 MeV decays to the ground state, and has peaks in all four reactions. There are few nearby

theoretical levels, none which can be easily associated with this level. It is possible that the state corresponds to an odd parity state.

The next level at 3.96 MeV is only observed in the neutron knockout reaction, and is measured by its decay to the  $1/2^+$  state. Again, there are few surrounding theoretical levels, and none have a decay to the  $1/2^+$  state. Therefore, this state cannot be assigned to a theoretical  $sd$ -shell level. Most likely, this state corresponds to an odd parity level.

The 4.06 MeV level is the next level, and is measured by its decay to the ground state. Only the proton transfer reaction contributes to this level. The authors suggest that this state corresponds to a  $d_{3/2}$  proton single transfer to a  $3/2^+$  state, and the strength of the transfer reaction and hindrance of the neutron knockout reaction support this assignment. The energy is close to the USDB prediction, while the USDA energy is 350 keV lower than experiment. All three Hamiltonians predict 100% decay from this state to the ground state, as observed.

The next state at 4.62 MeV is only observed in the transfer and two-nucleon knockout reactions. It decays to the first excited  $5/2^+$  state by a 1.24 MeV  $\gamma$  ray. This information is consistent with the second theoretical  $3/2^+$  state. The gamma branching ratios are sensitive to the Hamiltonian with the branches to the ground (first excited)  $5/2^+$  states of 77 (14), 24 (55), and 4 (72)% for the USD, USDA, and USDB, respectively. Since the decay to the ground state is not seen experimentally, and at 4.62 MeV is still within the range of the  $\gamma$  spectra, the USDA and USDB Hamiltonians agree better with the experiment than the USD.

The next state decays to the ground state, and the 4.73 MeV  $\gamma$  ray is the highest energy  $\gamma$  ray measured in the experiment. Higher energy decays are expected from the theoretical decay scheme, which could possibly be determined with more precise experimental methods. The relatively large transfer reaction cross section suggests that  $J \leq 5/2^+$ . The nearby theoretical levels that fit this requirement are the third and fourth  $5/2^+$  states. The third level has a larger gamma branch to the ground state and has a larger spectroscopic factor for the proton transfer reaction than the fourth  $5/2^+$  state (although still smaller than observed). As the excitation energy increases the three theoretical models give increasingly different gamma branches. Even with these variations, the 4.73 MeV state is most likely associated with the third  $5/2^+$  theoretical state.

Next is the 4.92 MeV state, which decays to the first  $7/2^+$  state. There are few nearby states, and only the second  $9/2^+$  state is expected to decay to the  $7/2^+$  state. However, a  $9/2^+$  assignment is inconsistent with its observation in the proton transfer, and in this case, we cannot find a higher level populated in proton transfer that would gamma decay to this state. Thus, this level cannot be associated with any of the  $sd$  model-space states.

The 5.54 MeV experimental state decays to the 3.83 MeV ( $9/2^+$ ) state. In the USDB scheme, there are four nearby states (within 0.5 MeV), but only the  $11/2^+$  state permits a decay to the lower  $9/2^+$  state. There is no proton transfer cross section, as expected for this spin. The calculated decay from this state consists of two transitions: 56% to the first  $9/2^+$  state (seen experimentally) and 43% to the second  $9/2^+$  state, which takes

place by a 0.56 MeV  $\gamma$  ray, at the edge of the range of the  $\gamma$  spectra. As seen in Fig. 1 of [1], there seems to be a peak around 0.6 MeV in the spectra, but is not labeled by energy. The USD and USDA schemes give similar results, suggesting that the experimental 5.54 MeV state can be associated with  $11/2^+$ . At higher excitation, the theoretical level density becomes higher than experiment, and the experimental data for the gamma decay to the ground state is missing, thus no definitive association between experiment and theory can be made.

The rms values of the difference between the theoretical and experimental excitation energies enable a comparison of the accuracy of the three Hamiltonians. Only states which have been identified with theoretical levels were used in the calculation. The rms values for USD, USDA, and USDB are 0.285, 0.156, and 0.172 MeV, respectively. As can be seen, the USDA and USDB values are similar and fairly consistent with their global rms values of 0.170 and 0.126 MeV, respectively. Both of the new Hamiltonians display a marked improvement over USD.

### III. PROTON TRANSFER SPECTROSCOPIC FACTORS

Table I shows the experimental and theoretical values for the strong proton transfer spectroscopic factors for all three Hamiltonians. (In Fig. 5 of [1], the authors list values of  $(2J+1)C^2S$  for two experimental levels and for the USD calculations. However, the theoretical values listed are actually for  $(2J+1)S$ ;  $C^2 = 6/7$  for this reaction.) The three Hamiltonians give similar results, with the largest difference being 20% for the  $3/2^+$  state. The ratio of the experimental and theoretical spectroscopic factors of about 0.50 is consistent with the global reduction factor observed in nucleon-transfer and nucleon knockout reactions shown in Fig. 2 of [7]. This reduction factor is attributed to configuration mixing beyond the  $sd$ -shell including those due to the short-range and tensor two-nucleon correlations [8].

Figure 4 shows proton-transfer  $C^2S_f$  values summed over all states up to  $E_f - E_i$  as a function of  $E_f - E_i$  for USDB, where  $E_f$  are the energies of states in  $^{23}\text{F}$  and  $E_i$  is the energy of the  $^{22}\text{O}$  ground state. (The calculated energy difference between  $^{22}\text{O}$  and  $^{23}\text{F}$  includes the 3.48 MeV Coulomb energy correction for  $Z = 9$  relative to  $Z = 8$ .) This plot shows that the lowest energy state for each spin contains only about 60–80% of the total strength. The remaining strength is fragmented over many final states up to 15 MeV higher. The larger fragments of strength near  $E_f - E_i = 0-5$  MeV correspond to the strength going to the  $T = T_> = 7/2$  isospin states that are the isobaric analogues of the low-lying states of  $^{23}\text{O}$ .

TABLE I. Comparison of theoretical and experimental spectroscopic factors  $C^2S$  for the proton capture reaction.

$E$ (MeV)	$J^\pi$	Exp.	USD	USDA	USDB
0.00	$5/2^+$	n/a	0.80	0.77	0.78
2.27	$1/2^+$	$0.36^{+0.10}_{-0.16}$	0.67	0.64	0.65
4.06	$3/2^+$	$0.24^{+0.07}_{-0.09}$	0.47	0.56	0.47

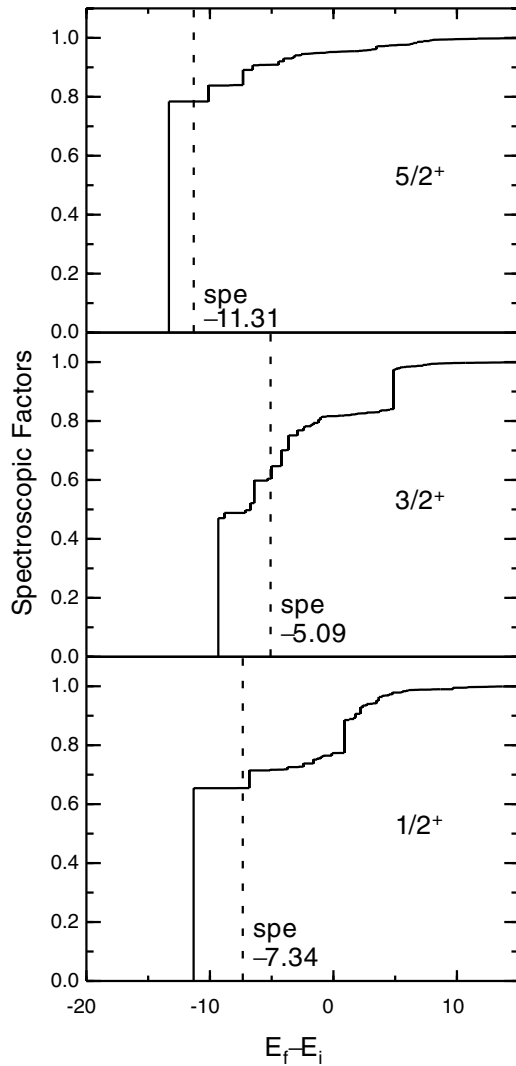


FIG. 4.  $C^2S_f$  vs  $E_f - E_i$  for the proton transfer using the USDB Hamiltonian. The dotted lines mark the single particle centroid energies.

The single-particle energy given by

$$\epsilon = \frac{\sum_f (E_f - E_i) C^2 S_f}{\sum_f C^2 S_f} \quad (1)$$

is shown on the right-hand side of Table II. The centroid energies are significantly higher than the lowest state energies. The largest change is for  $0d_{3/2}$ , which is shifted up by 4.19 MeV for USDB.

The best experimental information available for the fragmentation of proton single-particle strength in the fluorine isotopes is from the  $^{18}\text{O}(d, n)^{19}\text{F}$  experiment of [9]. The spectroscopic factors from this experiment are compared to the calculation with USDB in Fig. 5. The fragmentation to  $T_{>} = 5/2$  states is large with the lowest  $T = 5/2$  state being the strong  $5/2^+$  state at 7.54 MeV (at  $E_f - E_i = 0.54$  MeV in Fig. 5). The experimental data stop at an excitation energy of 14 MeV (where the experimental lines stop in Fig. 5). Most of the theoretical strength is below 14 MeV excitation with the exception of a large value for  $3/2^+$  near

TABLE II. Proton single-particle energies for  $^{23}\text{F}$  relative to  $^{22}\text{O}$  based on the lowest state for each spin and the centroid energy for each  $n, \ell, j$  value.

State $n\ell_j$	$\epsilon$ lowest			$\epsilon$ centroid			
	Exp.	USD	USDA	USDB	USD	USDA	USDB
$0d_{5/2}$	-13.20	-13.05	-13.28	-13.26	-11.16	-11.21	-11.31
$1s_{1/2}$	-10.93	-11.27	-11.03	-11.27	-7.58	-6.92	-7.34
$0d_{3/2}$	-9.14	-9.55	-9.59	-9.28	-5.04	-5.46	-5.09

14 MeV. Thus some  $d_{3/2}$  strength may be missed. The overall pattern of experimental strength distributions is in excellent agreement with theory except that the  $d_{3/2}$  strength above 8 MeV in excitation (above  $E_f - E_i = 1$  MeV in Fig. 5) is more fragmented in experiment than theory, presumably due to mixing with intruder states coming from two nucleons excited from the  $0p$  to  $0d1s$  or  $0d1s$  to  $0p1f$  shell.

For  $^{23}\text{F}$ , comparison of USDA and USDB gives an estimate of the theoretical error in the centroid energy of 0.10, 0.4 and 0.4 MeV for  $0d_{5/2}$ ,  $1s_{1/2}$  and  $0d_{3/2}$ , respectively. The  $0d_{5/2} - 0d_{3/2}$  proton spin-orbit splitting for  $^{23}\text{F}$  is thus estimated to

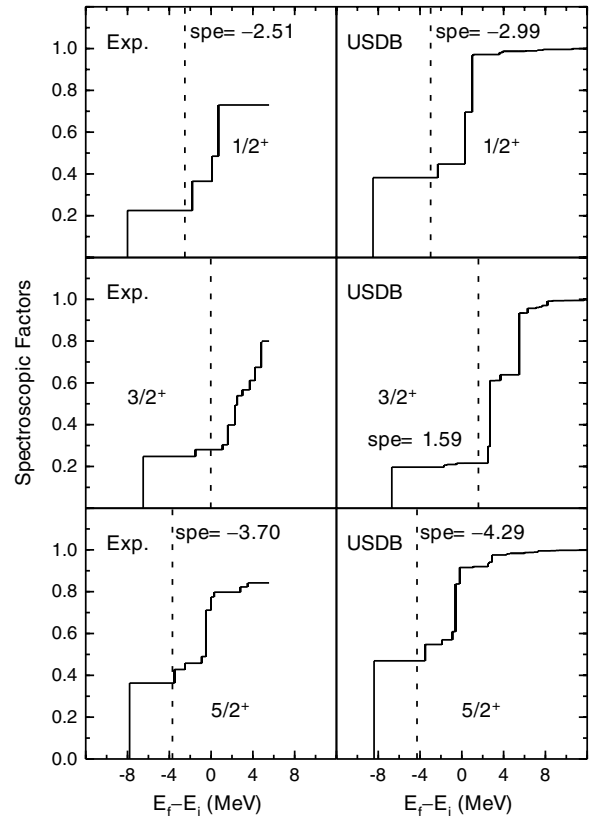


FIG. 5. A comparison of  $C^2S$  vs  $E_f - E_i$  for experimental [9] and USDB calculations for  $^{19}\text{F}$  as the product of a proton transfer reaction. The dotted lines mark the single particle centroid energies. The  $3/2^+$  experimental single-particle energy is not listed because the strength above  $E_f - E_i = 6$  MeV, near where there is a large theoretical spectroscopic factor, was not measured.

be  $6.2 \pm 0.4$  MeV. With USDB we can find the spin orbit splitting from the proton-drip line, 6.0 in  $^{17}\text{F}$ , to the neutron drip line, 6.9 MeV in  $^{29}\text{F}$ . Thus, within the uncertainty of the calculation, the spin-orbit splitting is constant as a function of neutron number.

#### IV. SUMMARY AND CONCLUSIONS

The level scheme of  $^{23}\text{F}$  in Fig. 3 of [1] has been compared to the theoretical level schemes obtained from the USD, USDA, and USDB Hamiltonians. Nine experimental energy levels have been identified by their spin, decay behavior, and probability of production in the four  $\alpha$  induced reactions. The USDA and USDB level schemes more accurately reproduce the experimental scheme for the states which can be determined, which suggests that the new

Hamiltonians are able to better approximate the behavior of nuclei. The proton-transfer spectroscopic factors are in good agreement with experiment, but the spectroscopic strength is fragmented, resulting in centroid single-particle energies that are significantly higher in energy. The fragmentation is in good agreement with observations for the  $^{18}\text{O}(d, n)^{19}\text{F}$  reaction. The proton  $0d_{3/2} - 0d_{5/2}$  spin-orbit splitting for  $^{23}\text{F}$  is estimated to be  $6.2 \pm 0.4$  MeV and is predicted to be nearly constant with neutron number from the proton drip line to the neutron drip line.

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