Spectroscopy of ^{20}Mg: The isobaric mass multiplet equation for the 2^+ states of the $A = 20$, $T = 2$ quintet and distant mirror nuclei

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We report on the first determination of the 2_1^+ energy of ^{20}Mg, the most neutron-deficient Mg isotope known to exist. The result, $E(2_1^+) = 1598(10)$ keV, obtained from in-beam γ-ray spectroscopy following the two-neutron removal from a ^{22}Mg secondary beam, is discussed in the framework of the isobaric mass multiplet equation (IMME). Resulting predictions for the excitation energies of the $T = 2$, 2^+ states in the ^{20}F and ^{20}Na isobars are presented. The mirror energy difference, $E(2_1^+, {}^{20}\mathrm{Mg}) - E(2_1^+, {}^{20}\mathrm{O}) = -77(10)$ keV, is compared to a recent prediction within the nuclear shell model based on the "USD^m - gap Z14<" modification of the universal *sd* (USD) effective interaction.

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I. INTRODUCTION

Experimental evidence for the equality of the interaction between like and unlike nucleons [1] justified the concept of isospin introduced by Heisenberg [2]. If the nucleon-nucleon interaction were to be independent of charge, the $2T + 1$ members of isospin multiplets would have identical binding energies. With the assumption that only two-body forces are responsible for the charge-dependent effects in nuclei, the mass excess ME(A, T, T_z) of members of an isospin multiplet is determined in first-order perturbation theory by the quadratic equation [3,4]:

$$\mathrm{ME}(A, T, T_z) = a + bT_z + cT_z^2, \tag{1}$$

where $T_z = (N - Z)/2$ is the projection of the isospin T with $-T \leqslant T_z \leqslant T$. This simple equation has been proven very successful for the description of the mass excess within isobaric multiplets [5,6]. More than 430 multiplets have been studied and significant departures are only found in light nuclei where the states of interest are particle unbound.

Deviations from this quadratic form of the so-called isobaric mass multiplet equation (IMME) are expected from isospin mixing in cases where two states with different isospin T are close in energy and from modifications to the wave functions of particle-unbound members of the multiplet. With the availability of experimental information on isospin quartets ($T = 3/2$) or quintets ($T = 2$), the necessity of cubic and quartic terms, dT_z^3 and eT_z^4, has been studied. It was shown that in absence of charge-dependent interactions of tensor rank higher than 2, the parameter d of a cubic extension to Eq. (1) would be small with $|d| < 1$ keV [7,8].

The knowledge on isobaric $T = 2$ quintets is very limited. There are six complete $0^+_{T=2}$ quintets ($A = 8$, 20, 24, 28, 32, and 36) [6]. A general method to access $T = 2$ states in $T_z = -1$ nuclei has not been established. For $2^+_{T=2}$ states, so far only members of the $A = 16$ quintuplet have been discussed [5]. For higher masses, excited states in the most exotic members of quintets—nuclei with $T_z = -2$ in the proximity of the proton dripline—have only become accessible with the advent of radioactive beam facilities (Fig. 1).

In the present article, we report on the observation of the first excited 2^+ state of ^{20}Mg, the $T_z = -2$ member of the $T = 2$ isospin quintet for the $A = 20$ nuclei. ^{20}Mg is the most neutron-deficient Mg isotope known to exist. The mass excess of the 2^+ excitation is discussed in the framework of the IMME and the mirror energy difference (MED), $\Delta E_M = E_x(2^+, T_z = -2) - E_x(2^+, T_z = +2)$, is compared to a shell-model prediction based on a modification, "USD^m - gap Z14<" [9], of the universal *sd* (USD) effective interaction [10].

II. EXPERIMENT

The proton-dripline nucleus ^{20}Mg was populated in the two-neutron removal reaction ^{9}B(^{22}Mg,^{20}Mg+γ)X. An exotic cocktail beam consisting of the $N = 10$ isotones of Ne, Na, Mg, Al, and Si was produced by fragmentation of a 150 MeV/nucleon ^{36}Ar primary beam delivered by the Coupled Cyclotron Facility of the National Superconducting Cyclotron Laboratory. A ^{9}Be production target of 893 mg/cm^2 thickness was located at the midacceptance target position of the large-acceptance A1900 fragment separator [11]. The separator was operated at 0.5% momentum acceptance.

The secondary beam that contained 33% of ^{22}Mg interacted with a 188(4)-mg/cm^2-thick ^{9}Be reaction target positioned at the pivot point of the large-acceptance S800 spectrograph [12]. The reaction residues were identified on an event-by-event basis from time-of-flight (TOF) information, the energy loss measured in the ionization chamber, and the position and angle information provided by the cathode readout drift chambers of the S800 focal-plane detector system [12]. The difference in the TOF measured between two plastic scintillators before the reaction target provided the particle identification of the incoming beam (Fig. 2). Software gates applied on

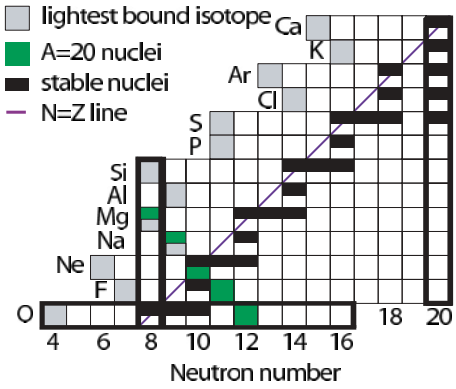

FIG. 1. (Color online) Section of the nuclear chart. The $A = 20$ isobaric $T = 2$ quintet is marked by green squares. ^{20}Mg, the $T_z = -2$ member, is located at the proton dripline.

the incoming projectiles allowed for a clean identification of the reaction products of the different constituents of the beam.

The secondary ^{9}Be target was surrounded by SeGA, an array of 32-fold segmented HPGe detectors [13]. Sixteen detectors were arranged in two rings (90° and 37° central angles with respect to the beam axis). The 37° ring was equipped with seven detectors, whereas nine detectors occupied positions at 90°. The energy calibration of the array was performed with ^{152}Eu and ^{226}Ra sources. The γ rays emitted by nuclei moving at more than 30% of the speed of light are detected in the laboratory reference frame with significant, angle-dependent Doppler shifts. Event-by-event Doppler reconstruction taking advantage of the high degree of segmentation of SeGA was used to transform the detected γ-ray energies into the reference frame of the projectile-like reaction residues.

In coincidence with ^{20}Mg two-neutron knockout residues, a γ-ray transitions at 1598(10) keV was detected in SeGA (see upper panel of Fig. 3). We identify this as the transition from the first excited 2^+ state to the ground state. The proton separation energy is with $S_p = 2646$ keV fairly low and the 2^+_1 state might be the only excited state that is stable against

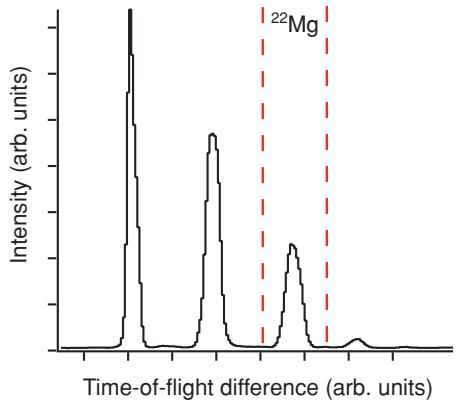

FIG. 2. (Color online) TOF difference taken between two plastic scintillators positioned before the reaction target. The constituents of the cocktail beam are well separated in TOF. A software gate is then applied to unambiguously select reaction residues produced by one incoming species only, here ^{22}Mg.

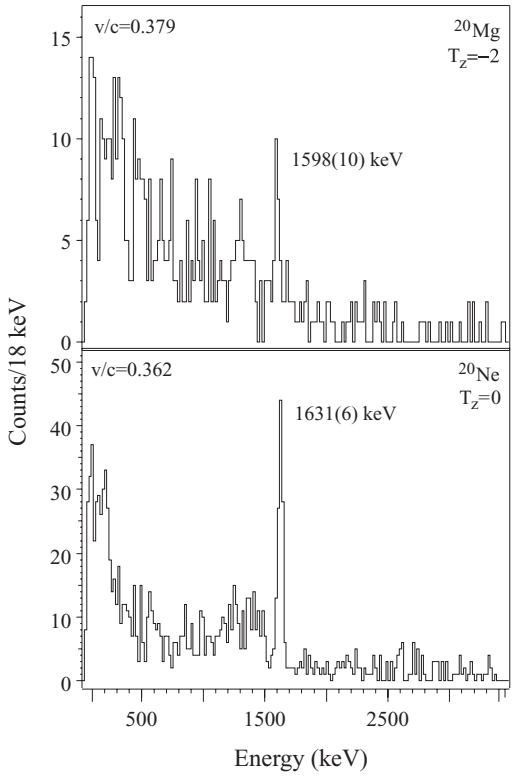

FIG. 3. Doppler-reconstructed γ-ray spectrum detected in coincidence with ^{20}Mg reaction residues (upper panel) and ^{20}Ne (lower panel). The photopeaks and corresponding Compton edges are clearly visible at about 1600 and 1300 keV, respectively. The background between 400 and 600 keV partly stems from prompt γ-ray transitions emitted at rest, e.g., 511 keV and transitions produced by inelastic neutron scattering on the detector material—^{72,74}Ge$(n, n'\gamma)$—that are smeared out by the Doppler reconstruction.

proton emission. For comparison, we show the first excited 2^+ state of ^{20}Ne at similar excitation energy as detected in coincidence with the one-proton removal residues from ^{21}Na, which was one of the constituents of the cocktail beam.

III. DISCUSSION

Table I summarizes the mass excess and the excitation energies for the $T = 2, 0^+$, and 2^+ states of the $A = 20$ isobaric quintet. The mass excesses of the ground states are taken from Ref. [14]; excitation energies, spin, and isospin assignments are taken from the compilation of Tilley *et al.* [15]. The energy of the first excited 2^+ state of ^{20}Mg stems from the present work. The $T = 2, 2^+$ state of ^{20}Na has not yet been identified.

For the $T = 2, 0^+$ states of the $A = 20$ quintet the quadratic IMME describes the data well as pointed out in Ref. [6] and shown in the upper panel of Fig. 4 using the most recent mass evaluation [14]. With the $2^+_{T=2}$ states known in ^{20}O, ^{20}F, and ^{20}Ne, the parameters a, b, and c of the IMME can be determined (parameter set [C] in Table I). However, this choice of parameters predicts the excitation energy of the $T = 2, 2^+$ of ^{20}Mg to be 2916 keV, almost a factor of 2

TABLE I. Properties of the $A = 20$ isobaric quintet members and corresponding IMME coefficients a, b, and c [Eq. (1)]. Given are the mass excess ME of the $T = 20^+$ and 2^+ states and the excitation energies [14,15]; parameter sets [A] and [B] resulting from fits of the data to the quadratic IMME; parameter sets [C] and [D] derived from respective subsets of the data and the result of a cubic extension of the IMME [E].

^{A}Z	T_z	$ME(T = 2)$ (keV)	E_x (keV)		J^π	
^{20}Mg	−2	17570(27)	0.0		0^+	
		19168(29)	1598(10)		2^+	
^{20}Na	−1	13383(15)	6534(13)		0^+	
					2^+	
^{20}Ne	0	9691.0(27)	16732.9(27)		0^+	
		11388(7)	18430(7)		2^+	
^{20}F	+1	6501.6(30)	6519(3)		0^+	
		8032(100)	8050(100)		2^+	
^{20}O	+2	3797.5(11)	0.0		0^+	
		5471.2(11)	1673.68(15)		2^+	
J^π	a (keV)	b (keV)	c (keV)	d (keV)	χ^2	
0^+	9692.5(20)	−3438.9(36)	245.7(17)	–	0.69	[A]
2^+	11387(11)	−3425(12)	233.3(66)	–	2.7	[B]
2^{+a}	11388	−3753.6	397.6	–		[C]
2^{+b}	11388	−3724.2	232.9	–		[D]
2^{+c}	11388	−3643.8	232.9	54.9		[E]

[a] $T = 2$, 2^+ states of ^{20}O, ^{20}F, and ^{20}Ne.
[b] $T = 2$, 2^+ states of ^{20}O, ^{20}Ne, and ^{20}Mg.
[c] Cubic IMME using ^{20}O, ^{20}F, ^{20}Ne, and ^{20}Mg.

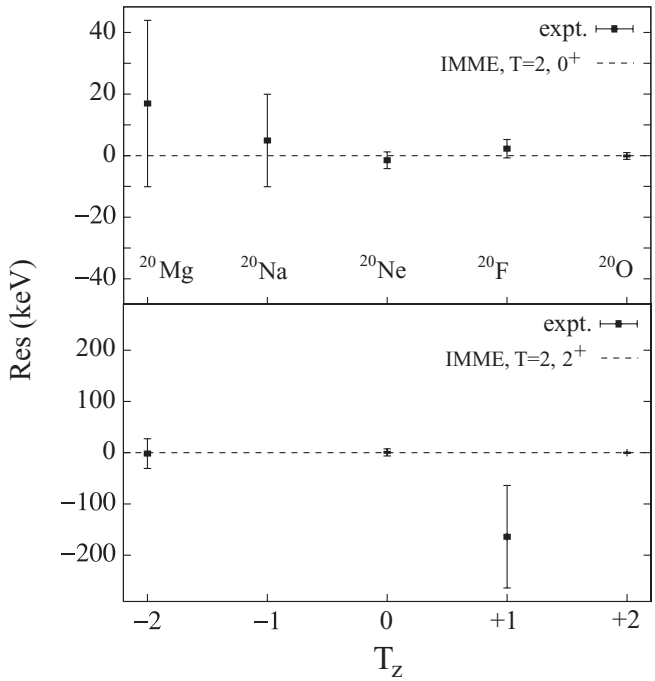

FIG. 4. Residuals $Res = ME(T = 2, J^+)_{\text{exp}} - ME(T = 2, J^+)_{\text{IMME}}$ of the $T = 2$, 0^+ (upper panel) and $T = 2$, 2^+ (lower pane) states of the $A = 20$ isobaric quintet. The parameter sets and reduced χ^2 values of the fit are listed as parameter sets [A] and [B] in Table I. The $2^+_{T=2}$ state in ^{20}F deviates considerably from the fit (see text for a discussion).

higher than the experimental result. Including the new result for the mass excess of the 2^+ state at $T_z = -2$, ^{20}Mg, a fit reveals that the $2^+_{T=2}$ state of ^{20}F deviates beyond 1σ from a description with the quadratic IMME (see the lower panel of Fig. 4 corresponding to parameter set [B] in Table I). We note that a possible cubic extension of the IMME with the term dT_z^3 would result in $d = 55(33)$ keV[1] (parameter set [E])—an unreasonably large value for d in the absence of charge-dependent interactions of tensor rank higher than 2 [7,8]. The energy of the $2^+_{T=2}$ state of ^{20}F has been measured in a ^{22}Ne$(p,^3$He$)^{20}$F two-nucleon transfer reaction in 1964 [16]. The peak attributed to the $2^+_{T=2}$ excitation is obscured by background and the mass excess ME for this state carries the largest uncertainty of all members of the multiplet. Now, in turn, the $2^+_{T=2}$ states of ^{20}O, ^{20}Ne, and ^{20}Mg determine parameter set [D] for the quadratic IMME and can be employed to predict the excitation energies of the $2^+_{T=2}$ states in ^{20}F and ^{20}Na to 8211 and 8197 keV, respectively. For ^{20}F the prediction is only 61 keV beyond the uncertainty quoted in Ref. [16]. For ^{20}Na nothing is known about excited states above 6534 keV [15] and the experimental confirmation of our prediction has to remain a challenge for future experiments.

The energies of distant mirror pairs have recently been used as a sensitive probe of nuclear structure changes in the *sd* shell [9]. Prompted by the identification of the 2^+_1 state

[1] $\delta d = 1/3 \times \delta E(^{20}\text{F}, 2^+_{T=2}) = 33$ keV from finite difference derivatives.

of ^{36}Ca, the evolution of the mirror energy differences (MED), $\Delta E_M = E_x(J^+, T_z = -T) - E_x(J^+, T_z = +T)$, of $T = 1$ and $T = 2$ pairs in the *sd* shell has been used by Doornenbal *et al.* to modify the USD effective interaction to account for the observed trends by (i) replacing the fitted isospin symmetric single-particle energies of the USD interaction by the experimental values of the $A = 17, T = 1/2$ mirror pair and (ii) monopole corrections, including a quenching of the proton $d^2_{5/2}$ two-body matrix elements in the lower *sd* shell [9]. With the so modified interaction – "USD^m - gap Z14<" for the lower *sd* shell—that now includes a total reduction of the $Z = 14$ proton subshell gap relative to the $N = 14$ mirror gap by 0.32 MeV, Doornenbal *et al.* succeed in describing the trend of ΔE_M very well, in particular its changes in sign.

The largest contribution to mirror asymmetry comes from the reduced Coulomb energy associated with single-particle states with small separation energies and large rms radii (the Thomas-Ehrman effect). For this mass region the $\ell = 0$ state can have a large rms radius relative to $\ell = 2$ due to the lack of a centrifugal barrier for the former. The Thomas-Ehrman effect appears in its simplest form for the downward shift of the $1/2^+$ state in ^{17}F compared to ^{17}O by 375 keV. The work of Ref. [9] makes an attempt to parametrize this effect as it applies to multiparticle configurations in the *sd* shell. The experimental ΔE_M for the $A = 20, T = 2$ pair was unknown at that time and the modified effective shell-model interaction was used to predict $\Delta E_M = E_x(2^+, ^{20}\mathrm{Mg}) - E_x(2^+, ^{20}\mathrm{O}) = -50$ keV [9]. The $E(2^+_1)$ energy of ^{20}Mg in our experiment results in a corresponding MED of $\Delta E_M^{\mathrm{exp}} = -77(10)$ keV. It is remarkable that the USD^m interaction presented in Ref. [9] that does not include the additional reduction of the $Z = 14$ proton subshell gap, predicts the mirror energy difference ΔE_M for ^{20}Mg and ^{20}O to be small and positive. This indicates predictive power of the modified interaction and underlines the sensitivity of MEDs to details of the nuclear shell structure. In particular, Doornenbal *et al.* speculate that the required reduction of the $d^2_{5/2}$ two-body matrix elements in the lower *sd* shell is tied to the proximity of the particle continuum.

IV. SUMMARY

In summary, we have identified the first excited 2^+ state of ^{20}Mg, the $T_z = -2$ member of the $A = 20, T = 2$ isobaric quintet. A fit to the quadratic IMME revealed that the $2^+_{T=2}$ state identified in ^{20}F 40 years ago might be more uncertain in energy than originally quoted. A prediction based on the quadratic IMME using the mass excess of the more accurately known $2^+_{T=2}$ states of ^{20}O, ^{20}Ne, and ^{20}Mg places the $2^+_{T=2}$ analog states in ^{20}F and ^{20}Na at excitation energies of 8211 and 8197 keV, respectively. The mirror energy difference, $\Delta E_M = E_x(2^+, ^{20}\mathrm{Mg}) - E_x(2^+, ^{20}\mathrm{O}) = -77(10)$ keV, of the distant mirror pair ^{20}O ($T_z = +2$) and ^{20}Mg ($T_z = -2$) was found close to the prediction of about $\Delta E_M = -50$ keV based on a modification of the USD effective shell-model interaction—"USD^m - gap Z14<"—that includes a total reduction of the $Z = 14$ proton subshell gap relative to the $N = 14$ mirror gap by 0.32 MeV.

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