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Extraction of V_{ud} from superallowed Fermi β decay by the Wilkinson techniques

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ABSTRACT

The up–down element V_{ud} of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix is derived from superallowed Fermi β decays between isospin $\mathbf{T}=1$ states using the semi-empirical techniques pioneered by D.H. Wilkinson. Rather than relying directly on the absolute magnitude of theoretical nuclear structure calculations to correct each decay for isospin symmetry breaking effects, these methods extrapolate the experimental *ft* values to the charge independent limit at $Z \approx 0$ where isospin-dependent effects are negligible. The value $V_{ud}=0.97430(29)$ derived in this work agrees with the value $V_{ud}=0.97425(22)$ obtained from the most recent shell-model calculations of isospin symmetry breaking that have expanded the shell-model spaces by including select core orbitals. These values are not in agreement with previous shell-model calculations in more restrictive model spaces or the values obtained from recent self-consistent relativistic Hartree and Hartree–Fock calculations based on the random-phase approximation that all underestimate the magnitude of isospin symmetry breaking.

1. Introduction

Unitarity tests of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix provide demanding constraints on the existence of new physics beyond the Standard Model description of electroweak interactions. While the sum of the squares of the CKM elements along any row or column could be used for this test, it is presently the top-row test that is by far the most demanding. The most precise value for $V_{ud}=0.97425(22)$ [1] is presently derived from superallowed Fermi nuclear β decay. Combining this value with $V_{us}=0.2255(19)$ and $V_{ub}=3.93(36)\times10^{-3}$ from the 2008 review of the Particle Data Group [2], the top-row test of CKM unitarity presently yields,

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1.0000(10) \tag{1}$$

and satisfies the Standard Model unitarity requirement at the level of 0.1%. Although less precise by a factor of 5, the next most precise test of CKM unitarity is obtained from the first column and combines V_{ud} with $V_{cd}=0.230(11)$ and $V_{td}=8.1(6)\times10^{-3}$ [2],

$$|V_{ud}|^2+|V_{cd}|^2+|V_{td}|^2=1.002(5). \tag{2}$$

This result also satisfies the unitarity condition but is presently limited by the uncertainty associated with V_{cd}. Additional tests of CKM unitarity such as those involving the second row or column are presently more than 2 orders of magnitude less precise than the first row test of Eq. (1). Searches for new physics beyond the Standard Model that would imply a small deviation from CKM unitarity must therefore focus on the elements in the first row or column of the CKM matrix. Because the up–down element V_{ud} makes up more than 95% of these unitarity sums and is common to both of these tests, this fundamental parameter continues to be the focus of intense theoretical and experimental scrutiny.

Derivation of V_{ud} from superallowed Fermi β decay between 0^+ isobaric analogue states relies on high-precision experimental measurements of the *ft* values for several decays ranging from ^{10}C to ^{74}Rb. As a consequence of the conserved vector current (CVC) hypothesis, which stipulates that the vector coupling constant for semileptonic weak interactions G_V is not renormalized in the nuclear medium [3], these experimental *ft* values for decays between $\mathbf{T}=1$ isobaric analog states, once corrected (denoted $\mathcal{F}t$) for radiative and isospin symmetry breaking effects, can be expressed as [1],

$$\mathcal{F}t=ft(1+\delta_R')(1+\delta_{NS}-\delta_C)=\frac{K}{2G_F^2V_{ud}^2(1+\Delta_R^V)}=\text{constant} \tag{3}$$

where $K/(\hbar c)^6=2\pi^3\hbar\ln2/(m_ec^2)^5=8120.2787(11)\times10^{-10}\,\text{GeV}^{-4}\text{s}$ is a constant and $G_F/(\hbar c)^3=1.16637(1)\times10^{-5}\,\text{GeV}^{-2}$ [2] is the Fermi coupling constant derived from purely leptonic muon decay. Radiative corrections are denoted δ_R', δ_{NS}, and Δ_R^V, while δ_C represents a correction that accounts for the fact that isospin is not a perfect

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symmetry of the nuclear Hamiltonian but is broken by Coulomb and charge-dependent nuclear forces.

Recently the theoretical corrections for isospin symmetry breaking have undergone considerable revision due to the realization that shell-model core orbitals play a significant role [4]. This change in the nuclear-structure dependent corrections, applied to a fixed set of 13 experimental superallowed *ft* values, results in a decrease to the world average $\mathcal{F}t$ value by 2.5 s, or 3.2 statistical standard deviations and corresponds to a 1.8σ increase to the value of V_{ud} derived from the superallowed data (discussed in detail below). Owing to the significant role that these theoretical correction terms play in the extraction of V_{ud}, and the implications for tests of possible extensions to the Standard Model, independent scrutiny of these new calculations is essential and work to this effect is being actively pursued [8–12]. The study of Ref. [8] points out that the calculations of δ_C in Ref. [4] implicitly use the concept of analogue spin (or *W*-spin), rather than the formally correct operator for isospin **T**. More recently, these authors have demonstrated that second-order renormalization effects caused by radial excitations, and neglected by Ref. [4], may contribute significantly to the calculation of the Fermi matrix element [11].

There have also been several new sets of isospin symmetry breaking calculations that are not based on standard shell-model approaches. Many of these are summarized in Fig. 1 and are compared to the results from shell-model calculations. The collective model of Ref. [9] involving the isovector monopole resonance consistently employs the isospin formalism and yields δ_C values smaller than those of Ref. [4] by factors of 2–4. Recent δ_C calculations based on self-consistent relativistic Hartree (RH) and Hartree–Fock (RHF) that utilize the random-phase approximation (RPA) [10], also yield corrections that are reduced relative to Ref. [4] by factors of 2–3. Another recent study [12] applied Pyatov's restoration formalism to eliminate the isovector contribution of the shell-model potential to the calculation of isospin symmetry breaking. Their calculations also support this factor of 2 reduction relative to Ref. [4]. If such a reduction factor were to be adopted, the resulting increase to V_{ud} would be nearly 6 times the present quoted uncertainty and would, in turn, suggest a violation of CKM unitarity by more than 2.5 standard deviations. Given these recent theoretical developments, and the significance of these calculations to the extraction of V_{ud}, the absolute values of the corrections for isospin symmetry breaking for superallowed Fermi β decay data must continue to be scrutinized.

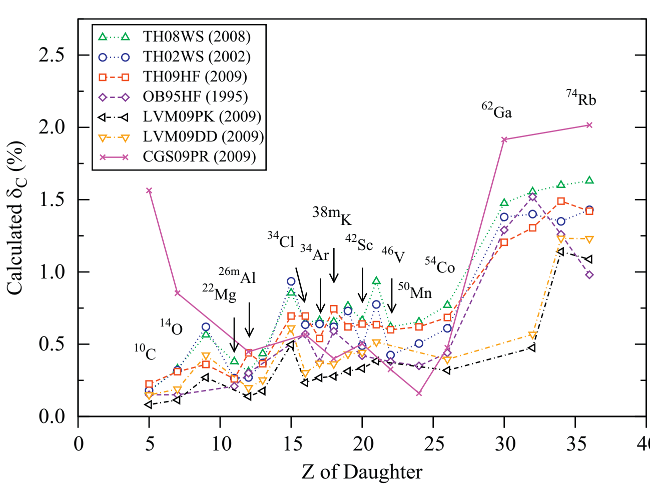

Fig. 1. (Color online) Summary of theoretical isospin symmetry breaking corrections. Calculated values shown are taken from Refs. [1] (TH09HF), [4] (TH08WS), [10] (LVM09PK, LVM09DD), [12] (CGS09PR), [18] (TH02WS), and [22–24] (OB95HF). For clarity, only the PK01 and DDME1 results (abbreviated here as PK and DD, respectively), have been included from Ref. [10].

One important advantage that superallowed Fermi β decay has over alternative approaches to extracting V_{ud} is that there are, to date, 13 separate cases whose experimental *ft* values have all been measured to at least 0.3% and, in many cases, significantly better. As these 13 cases span ^{10}C to ^{74}Rb, a considerable range of mass and charge, they provide a demanding test of the CVC hypothesis that implies that the corrected $\mathcal{F}t$ values for all of these cases should be equal within the experimental and theoretical uncertainties. Any theoretical model proposing to extract V_{ud} from CVC and the average $\mathcal{F}t$ value must first demonstrate the ability to remove the significant ($\sim 0.2-1.6\%$) case-by-case variations in the experimental *ft* values. This internal self consistency of the theoretical calculations represents a demanding and necessary test of the theoretical model. However, if a particular model can be shown to unequivocally meet this condition, is this sufficient to guarantee the accuracy in the average $\mathcal{F}t$ value and the corresponding value for V_{ud}? Hardy and Towner have argued that the excellent reduced χ^2 value of 0.29 [1] obtained from the average of the 13 corrected superallowed $\mathcal{F}t$ values provides compelling support for the *absolute* δ_C corrections and the resulting $\overline{\mathcal{F}t}$ value since "it would require a pathological fault indeed in the theory to allow the observed nucleus-to-nucleus variations in δ_C and δ_{NS} to be reproduced in such detail while failing to obtain the absolute values to comparable precision" [6,7]. However, the recent literature seems to suggest otherwise. For example, using the current experimental data set for the 13 most precisely determined cases and applying the previous [18] δ_C corrections of Towner and Hardy yields a reduced χ^2 value of 1.11. This set of isospin symmetry breaking corrections thus provides a convincing test of the CVC hypothesis but the average value $\overline{\mathcal{F}t} = 3074.6(7)$ s differs by 3.2 standard deviations from the result $\overline{\mathcal{F}t} = 3072.1(8)$ s obtained in the most recent survey [1] that use the modern set of isospin symmetry breaking calculations [4]. Recent RH/RHF RPA calculations of Ref. [10] also achieved reduced χ^2 values of ~ 1.0 for a subset of nine corrected $\mathcal{F}t$ values, yet obtained the average value $\overline{\mathcal{F}t} = 3080.1(7)$ s. Compared with $\overline{\mathcal{F}t} = 3072.1(8)$ s [1], this result is larger by 8.0 s or 10 standard deviations! These results raise the possibility that, while a particular set of theoretical calculations may correctly describe the case-by-case nuclear shell fluctuations of the δ_C values, the absolute magnitude of these corrections may not be described with comparable accuracy.

In the present work, the question of the accuracy of the absolute $\overline{\mathcal{F}t}$ value derived from these theoretical isospin symmetry breaking corrections, and the corresponding value deduced for V_{ud} from the set of 13 precisely determined superallowed Fermi β decays is addressed using the alternative approaches pioneered by Wilkinson [13–17]. Rather than relying directly on the absolute values of the theoretical isospin symmetry breaking calculations, these techniques instead determine the $\overline{\mathcal{F}t}$ value from empirical extrapolations of the data to the charge-independent limit $Z \approx 0$ where Coulomb effects and isospin symmetry breaking are negligible. The three methods proposed in Wilkinson's original work are applied, and critiqued, in light of recent experimental developments that now provide a modern set of 13 high-precision superallowed *ft* values.

2. Present status of superallowed Fermi β decay

2.1. Radiative and isospin symmetry breaking corrections

Radiative corrections to the experimental *ft* values are required to account for bremsstrahlung processes and virtual photon exchanges between the β particle liberated in the decay and the daughter nucleus, as well as hadronic loop effects [18–21]. For the case of $\mathbf{T}=1$ superallowed Fermi β decay, these corrections amount to ~3.8% and are thus significant in establishing corrected $\mathcal{F}t$ values at the 0.1% level. The overall radiative correction can be expressed as [4],

$$\Delta_R = \Delta_R^V + \delta_R' + \delta_{NS} \quad (4)$$

where the first term $\Delta_R^V = 2.361(38)\%$ [4,21] is nucleus independent, the second δ_R' depends on the number of protons in the daughter nucleus Z and the mean decay energy of the particular transition but is independent of the details of its nuclear structure, while the third δ_{NS} is nuclear-structure dependent. These corrections have traditionally been separated into nucleus-dependent and independent terms using the approximation [4],

$$1+\Delta_R \approx (1+\Delta_R^V)(1+\delta_R) \quad (5)$$

where $\delta_R = \delta_R' + \delta_{NS}$. To maintain this convention, Towner and Hardy [4] have slightly rearranged the radiative corrections of Ref. [21] to move a small transition-dependent term from the previous definition of Δ_R^V [21] to the new definition of δ_R' [4], leaving the overall radiative corrections Δ_R of Eq. (4) and the corresponding value for V_{ud} derived from the superallowed data unchanged. However, with this movement of terms between Δ_R^V and δ_R', the present definition for the world average $\mathcal{F}t$ value derived from Eq. (3) is now larger than the corresponding value obtained using the previous definitions for the radiative corrections by approximately 0.6 s. Direct comparisons between $\overline{\mathcal{F}t}$ values from previous surveys and present ones are therefore no longer strictly valid (although comparisons of V_{ud} are). The radiative corrections taken from the modern definition of Ref. [4] are used throughout this work and are summarized in Table 1 for the 13 superallowed decays considered here.

Detailed nuclear-structure calculations of the corrections to the ft values to account for isospin symmetry breaking have historically been performed with either the model of Towner and Hardy [4,18] or that of Ormand and Brown [22–24]. Both calculations divide the correction term δ_C into two components, $\delta_C = \delta_{C1} + \delta_{C2}$, where the first term accounts for different configuration mixing in the parent and daughter states, and the second corrects for the imperfect radial wavefunction overlap that arises from differences in the proton and neutron potentials and separation energies. The isospin-mixing term δ_{C1} is calculated in both models from configuration mixing within a shell-model calculation using charge-dependent interactions. To calculate the overlap term δ_{C2}, radial wavefunctions are derived in the model of Towner and Hardy [4,18] from a Woods–Saxon plus Coulomb potential, while Ormand and Brown [22–24] have employed a self-consistent Hartree–Fock calculation with Skyrme-type interactions. A small but systematic difference existed between these two methods, and led to the historic adoption of a ±0.85 s [5] systematic uncertainty in the world average $\mathcal{F}t$ value associated with the different nuclear structure models.

Recently, Towner and Hardy have demonstrated that although the dominant contributions to δ_{C2} come from valence orbitals, more deeply bound core orbitals can also play a significant role [4]. Guided by experimental spectroscopic factors from single nucleon pickup reactions (where available) and shell-model calculations of spectroscopic amplitudes, they identified which closed-shell orbitals were the most important to include and have recently published a new set of isospin symmetry breaking corrections (denoted here by TH08WS) [4]. Compared with their previous calculations that did not consider these orbitals [18] (TH02WS) the *relative* nucleus-to-nucleus fluctuations are similar (see Fig. 4). However, the *absolute* magnitude of these new δ_{C2} corrections are significantly larger (by nearly a factor of 2 in some cases) in the *pf* shell, indicating the importance of *sd* shell holes.

Due to the significant impact that the expanded model spaces have on the evaluation of δ_{C2}, a comparison between the new calculations by Towner and Hardy (TH08WS) to the previous Hartree–Fock calculations of Ormand and Brown (OB95HF) is no longer justifiable. The systematic uncertainty in the $\overline{\mathcal{F}t}$ value deduced from such a comparison would be entirely dominated by the differences in the choice of model space, as opposed to probing the inherent differences in the Woods–Saxon and Hartree–Fock wavefunctions. To alleviate this, Towner and Hardy have recently performed a new set of Hartree–Fock calculations (TH09HF) [1] that use the same model spaces as their Woods–Saxon equivalents. They have also corrected the calculation of the radial overlap term δ_{C2} in the Hartree–Fock procedure to describe the proper asymptotic form of the Coulomb potential at large radii [1]. This new parameterization, which leads to a general increase in the radial mismatch factors δ_{C2} over the previous Hartree–Fock calculations [22–24], is now directly comparable with the corresponding Woods–Saxon calculations.

The isospin symmetry breaking corrections from Towner and Hardy's Hartree–Fock (TH09HF) [1] and Woods–Saxon calculations (TH08WS) [4], are compared, in Table 2, to the Hartree–Fock calculations of Ormand and Brown (OB95HF) [22–24] and the

Table 1
Nucleus dependent radiative corrections used in the present analysis as given by the recent calculations of Ref. [4].

Parent	δ_R' (%)	δ_{NS} (%)	δ_R (%)
^{10}C	1.679(4)	−0.345(35)	1.334(35)
^{14}O	1.543(8)	−0.245(50)	1.298(51)
^{22}Mg	1.466(17)	−0.225(20)	1.241(26)
^{26m}Al	1.478(20)	+0.005(20)	1.483(28)
^{34}Cl	1.443(32)	−0.085(15)	1.358(35)
^{34}Ar	1.412(35)	−0.180(15)	1.232(38)
^{38m}K	1.440(39)	−0.100(15)	1.340(42)
^{42}Sc	1.453(47)	+0.035(20)	1.488(51)
^{46}V	1.445(54)	−0.035(10)	1.410(55)
^{50}Mn	1.444(62)	−0.040(10)	1.404(63)
^{54}Co	1.443(71)	−0.035(10)	1.408(72)
^{62}Ga	1.459(87)	−0.045(20)	1.414(89)
^{74}Rb	1.50(12)	−0.075(30)	1.42(12)

Table 2
Isospin symmetry breaking corrections used in the present analysis (TH09HF) [1], (TH08WS) [4], (TH02WS) [18], and (OB95HF) [22–24]. The mean value of $\overline{\delta_C}$ was calculated, for each case, from the average of all four δ_C values. The uncertainty in $\overline{\delta_C}$ was adopted from the TH08WS set of calculations. Derivation of the "experimental" set of isospin symmetry breaking corrections is described in Section 5.

Parent	δ_C (%) (OB95HF)	δ_C (%) (TH02WS)	δ_C (%) (TH08WS)	δ_C (%) (TH09HF)	$\overline{\delta_C}$ (%)	δ_C (%) (EXP)
^{10}C	0.150	0.180(18)	0.175(18)	0.225(36)	0.183(18)	0.33(16)
^{14}O	0.150	0.320(25)	0.330(25)	0.310(36)	0.278(25)	0.33(11)
^{22}Mg	0.210	0.265(14)	0.380(22)	0.260(56)	0.279(22)	0.59(24)
^{26m}Al	0.300	0.270(14)	0.310(18)	0.440(51)	0.330(18)	0.34(6)
^{34}Cl	0.570	0.635(36)	0.650(46)	0.695(56)	0.638(46)	0.62(17)
^{34}Ar	0.380	0.640(41)	0.665(56)	0.540(61)	0.555(56)	0.60(27)
^{38m}K	0.590	0.620(45)	0.655(59)	0.745(63)	0.653(59)	0.69(7)
^{42}Sc	0.420	0.490(42)	0.665(56)	0.640(56)	0.554(56)	0.69(8)
^{46}V	0.380	0.425(32)	0.620(63)	0.600(63)	0.506(63)	0.68(8)
^{50}Mn	0.350	0.505(36)	0.655(54)	0.620(59)	0.533(54)	0.64(9)
^{54}Co	0.440	0.610(43)	0.770(67)	0.685(63)	0.626(67)	0.72(9)
^{62}Ga	1.290	1.38(16)	1.48(21)	1.21(17)	1.34(21)	1.48(11)
^{74}Rb	0.980	1.43(40)	1.63(31)	1.42(17)	1.37(31)	1.83(28)

previous Woods–Saxon calculations by Towner and Hardy (TH02WS) [18] that did not include core orbitals.

2.2. Experimental ft and corrected $\mathcal{F}t$ values

Presently, there are 13 superallowed β decay *ft* values between ^{10}C and ^{74}Rb that have been determined experimentally to precision exceeding 0.3%, eight of which are known to better than 0.05% [1]. The experimental *ft* values for these 13 cases are listed in Table 3 and were taken from the latest evaluation by Towner and Hardy [1]. The experimental *ft* value for ^{10}C has been updated to include a recent half-life measurement [25].

Corrected $\mathcal{F}t$ values for these 13 cases were calculated from Eq. (3) using the experimental *ft* values from Table 3 and applying the radiative and isospin symmetry breaking corrections from Tables 1 and 2, respectively. Four sets of corrected $\mathcal{F}t$ values were obtained corresponding to the four sets of isospin symmetry breaking calculations and are tabulated for comparison in Table 3. A weighted average and reduced χ^2 value for each set was also computed and these are compared at the bottom of Table 3. As all of these corrected $\mathcal{F}t$ values share the same experimental data and radiative correction terms, a direct comparison between them demonstrates the model-dependent differences associated solely with the corrections for isospin symmetry breaking. Comparing the values of $\overline{\mathcal{F}t} = 3076.2(7)$ s obtained with the OB95HF description of isospin symmetry breaking to $\overline{\mathcal{F}t} = 3074.6(7)$ s derived using the TH02WS calculations reveals the 1.65 s difference which, as described above, was the source of the ± 0.85 s systematic uncertainty that was previously adopted [4,5] to reflect model dependent differences between these calculations. With the addition of select core orbitals to the shell-model spaces the value of $\overline{\mathcal{F}t} = 3072.1(8)$ s is deduced from the TH08WS calculations, while $\overline{\mathcal{F}t} = 3071.5(9)$ s is derived from the TH09HF calculations. The difference between these latter values is 0.55 s and would suggest that the model-dependent systematic uncertainty between these calculation methods should be reduced by a factor of 3 to ± 0.27 s [1].

Comparing the $\overline{\mathcal{F}t}$ values deduced from the Woods–Saxon calculations TH02WS and TH08WS, which differ only in the choice of shell-model space, leads to a difference of 2.5 s or 3.2 statistical standard deviations. This 2.5 s decrease to the $\overline{\mathcal{F}t}$ value has been adopted in the recent world survey [1], and corresponds to a 1.8σ increase to the value of V_{ud} determined from these 13 superallowed decays. These significant changes are not experimental in origin but are due entirely to refinements in the theoretical calculations of the isospin symmetry breaking correction terms. Careful scrutiny of these corrections is thus essential.

Table 3
Experimental *ft* values for the 13 most precisely determined superallowed emitters and the corresponding $\mathcal{F}t$ values for the four calculations of isospin symmetry breaking considered, (TH09HF) [1], (TH08WS) [4], (TH02WS) [18], and (OB95HF) [22–24].

Parent	*ft* (s)	$\mathcal{F}t$ (s) (OB95HF)	$\mathcal{F}t$ (s) (TH02WS)	$\mathcal{F}t$ (s) (TH08WS)	$\mathcal{F}t$ (s) (TH09HF)
^{10}C	3041.3(44)	3077.0(46)	3076.1(46)	3076.3(46)	3074.7(47)
^{14}O	3042.3(27)	3077.0(33)	3071.8(33)	3071.5(33)	3072.1(34)
^{22}Mg	3052.0(72)	3083.3(73)	3081.6(73)	3078.0(74)	3081.7(75)
^{26m}Al	3036.9(9)	3072.7(14)	3073.6(14)	3072.4(14)	3068.4(20)
^{34}Cl	3049.4(11)	3073.1(19)	3071.1(19)	3070.6(21)	3069.2(23)
^{34}Ar	3052.7(82)	3078.6(84)	3070.4(85)	3069.6(85)	3073.5(86)
^{38m}K	3051.9(10)	3074.5(21)	3073.6(21)	3072.5(24)	3069.7(25)
^{42}Sc	3047.6(14)	3079.9(25)	3077.8(25)	3072.4(27)	3073.1(27)
^{46}V	3049.5(9)	3080.8(21)	3079.4(21)	3073.3(27)	3073.9(27)
^{50}Mn	3048.4(12)	3080.3(25)	3075.6(25)	3070.9(28)	3072.0(29)
^{54}Co	3050.8(11)	3080.1(28)	3074.8(28)	3069.9(32)	3072.5(31)
^{62}Ga	3074.1(15)	3077.3(57)	3074.5(57)	3071.5(72)	3080.0(61)
^{74}Rb	3084.9(78)	3098.1(153)	3084.0(154)	3077.7(130)	3084.3(102)
$\overline{\mathcal{F}t}$ (s)		3076.22(72)	3074.56(72)	3072.06(79)	3071.51(88)
χ^2/ν		2.06	1.11	0.27	0.92

3. Wilkinson's methods for extracting V_{ud}

The average $\mathcal{F}t$ values derived from Eq. (3) using the four complete sets of calculations for isospin symmetry breaking that are presently available, clearly exhibit both model and model-space dependencies that significantly exceed the statistical uncertainties obtained experimentally. Furthermore, with the possible exception of the OB95HF set of corrections, the reduced χ^2 values listed in Table 3 cannot, by themselves, be used to discriminate between these calculations or assess the accuracy in the absolute $\overline{\mathcal{F}t}$ values derived from them. For these reasons, alternative methods first suggested by Wilkinson [13] are revisited in order to establish values for $\overline{\mathcal{F}t}$ and V_{ud} through extrapolation to the isospin-independent limit rather than directly relying on the absolute magnitude of these corrections. The analysis presented here follows Wilkinson's recent work [15–17] that utilized three methods for determining V_{ud} and is extended to include ^{22}Mg, ^{34}Ar, and the heavier cases ^{62}Ga and ^{74}Rb, as these have only recently been determined experimentally with sufficient precision [1,4,5].

3.1. Method 1

The first method proposed by Wilkinson completely eliminates the theoretical isospin symmetry breaking corrections from the analysis by defining the corrected *ft* value as,[1]

$$(ft)_1^* = ft(1+\delta_R) \tag{6}$$

where we adopt the radiative corrections $\delta_R = \delta_R' + \delta_{NS}$ from the recent evaluation [4], as tabulated in Table 1, and the experimental *ft* values from Table 3. The $(ft)_1^*$ values derived from Eq. (6) are listed in Table 4 and plotted in Fig. 2. As the case-by-case corrections for isospin symmetry breaking have not been included into their definition, the $(ft)_1^*$ values are not expected to be constant as a function of *Z*, the number of protons in the daughter nucleus, and thus a weighted average of these data cannot be used. Rather, to determine V_{ud} one requires the intercept of this plot at $Z \approx 0$ where isospin symmetry breaking and Coulomb effects are negligible. The charge-independent limit is taken at Z=0.5, rather than Z=0, since this is the effective charge where the total mass splitting within isospin multiplets vanishes [15]. Isospin symmetry breaking corrections for N ~ Z nuclei (**T**=0) are expected to scale approximately as Z^2 [9,26] thus the intercept can be obtained from a quadratic fit to these data. Although not the focus of this work, the radiative corrections that have been adopted here are also known to scale as Z^2 to lowest order [27,28]. One therefore expects that, to first order and to the level of precision with which the *ft* values are experimentally determined, a quadratic fit is sufficient to globally describe the overall trend of these data while accurately describing the underlying physics of isospin symmetry breaking. Historically, Wilkinson has employed both quadratic [15] and linear [16,17] fits to the nine previously well-measured cases. The linear fit, although not motivated physically, was employed out of necessity because the quadratic coefficient could not be adequately constrained without precise *ft*

[1] Our definition of the $(ft)^*$ value differs from that originally defined by Wilkinson [14] by the overall constant factor $1+\Delta_R^V = 1.0236$ [4] due to our adoption of the approximate separation of radiative corrections [4] expressed in Eq. (5).

Table 4
Corrected $(ft)^*$ values obtained from Wilkinson's three methods as described in Section 3 of this work. Radiative corrections, isospin symmetry breaking corrections, and input experimental ft values are listed in Tables 1, 2 and 3, respectively. The final rows are the quadratic fit intercepts at Z=0.5 and the reduced χ^2 values as discussed in the text.

Parent	$(ft)_1^*$ (s)	$(ft)_2^*$ (s)	$(ft)_3^*$ (s) (OB95HF)	$(ft)_3^*$ (s) (TH02WS)	$(ft)_3^*$ (s) (TH08WS)	$(ft)_3^*$ (s) (TH09HF)
^{10}C	3081.9(45)	3076.2(46)	3077.9(46)	3077.1(46)	3077.4(46)	3075.7(47)
^{14}O	3081.8(32)	3073.2(32)	3078.5(32)	3073.6(32)	3073.5(32)	3073.9(33)
^{22}Mg	3089.9(74)	3081.3(74)	3086.8(74)	3086.0(74)	3083.1(74)	3086.3(75)
^{26m}Al	3081.9(13)	3071.7(14)	3076.7(13)	3078.7(13)	3078.3(14)	3073.7(20)
^{34}Cl	3090.8(16)	3071.1(21)	3080.5(19)	3080.5(19)	3081.5(21)	3078.9(23)
^{34}Ar	3090.3(84)	3073.2(85)	3087.1(85)	3081.2(85)	3082.0(86)	3084.5(86)
^{38m}K	3092.8(16)	3072.6(24)	3084.0(21)	3085.6(21)	3086.4(24)	3082.1(25)
^{42}Sc	3092.9(21)	3075.8(27)	3091.6(25)	3092.6(25)	3089.5(27)	3088.4(27)
^{46}V	3092.5(19)	3076.9(27)	3095.0(21)	3097.4(21)	3094.2(27)	3092.5(27)
^{50}Mn	3091.2(22)	3074.7(28)	3097.3(25)	3097.1(25)	3095.8(28)	3094.2(29)
^{54}Co	3093.7(25)	3074.3(32)	3100.1(28)	3100.2(28)	3099.2(32)	3098.6(31)
^{62}Ga	3117.6(31)	3075.9(72)	3104.3(58)	3108.8(58)	3111.1(72)	3115.2(61)
^{74}Rb	3128.8(88)	3086(13)	3137(15)	3134(15)	3135(13)	3135.6(10)
$(\mathcal{F}t)^*$	3079.6(12)	3071.5(14)	3071.9(14)	3072.3(14)	3072.3(14)	3068.8(16)
χ^2/ν	3.86	0.51	1.10	0.92	0.28	0.77

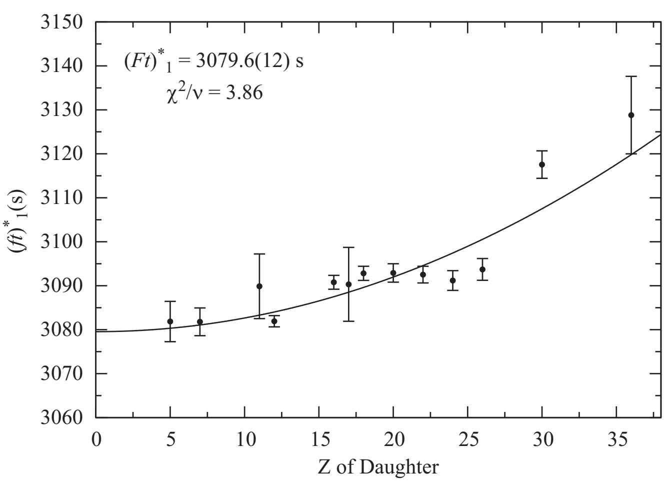

Fig. 2. Plot of the $(ft)_1^*$ data points that do not include theoretical corrections for isospin symmetry breaking and the resulting quadratic fit giving the global trend of the data. The adopted value for the charge independent intercept at $Z=0.5$ for this method is $(\mathcal{F}t)_1^*=3079.6(12)$ s.

values for ^{62}Ga and ^{74}Rb [16]. With the recent addition of these high-precision experimental ft values to the world data set, the linear fit can now be abandoned in favor of the physically motivated quadratic.

A quadratic fit to the present 13 superallowed $(ft)_1^*$ values is shown in Fig. 2 and, in general, gives a good description of the global trend for the entire data set. Extrapolation of this function from the lightest case of ^{10}C at $Z=5$ to the intercept at $Z=0.5$ yields the $\mathcal{F}t$ value from Wilkinson's first Method, denoted $(\mathcal{F}t)_1^*$,

$$(\mathcal{F}t)_1^*=3079.6(12)\,\text{s}. \tag{7}$$

While this method of determining the $(\overline{\mathcal{F}t})^*$ value is conceptually appealing in that it is entirely independent of the theoretical corrections for isospin symmetry breaking, the relatively poor reduced χ^2 value of 3.86 results from the fact that a smoothly varying quadratic fit cannot possibly describe the case-by-case nuclear structure variations that this method does not attempt to correct for. Even with the current extended set of 13 high-precision superallowed ft values, the range of Z values is still insufficient for this simple approach to completely separate the smooth Z^2 dependence of the isospin symmetry breaking corrections from the superimposed nuclear shell structure effects. The extrapolation of this quadratic trend, shown in Fig. 2, is thus particularly sensitive to the uncorrected $(ft)_1^*$ values for ^{10}C and ^{14}O. The validity of this simple approach and the accuracy with which the intercept of these data can be obtained from the empirical behavior of this quadratic fit for $Z<5$, must be called into question.

In order to reduce the sensitivity of the extrapolation to the case-by-case shell structure variations, two additional methods were proposed by Wilkinson that incorporated some of the aspects of the calculated isospin symmetry breaking effects while being careful to minimize the dependence on the absolute scale derived from any particular model.

3.2. Method 2

The second method was based on the observation that the systematic differences between the Hartree–Fock and Woods–Saxon calculations for isospin symmetry breaking demonstrated only moderate Z^2 dependence. As seen in Fig. 1 and Table 1, these calculations agree qualitatively on the relative nucleus-to-nucleus variations over the entire range of 13 superallowed emitters considered. The main discrepancy between them is an approximate constant shift in their absolute magnitude within a given nuclear shell. The second method uses this nucleus-to-nucleus agreement and minimizes the systematic Z dependence by employing a case-by-case average of the isospin symmetry breaking corrections $\overline{\delta_C}$. The corrected $(ft)_2^*$ values are thus defined as,

$$(ft)_2^*=ft(1+\delta_R)(1-\overline{\delta_C}) \tag{8}$$

where the radiative corrections are the same as defined above for Method 1. In the present work, the $\overline{\delta_C}$ values were taken from the average of all four calculations considered (OB95HF, TH02WS, TH08WS, TH09HF) as these are the only four calculations for which a complete set of values exist for each of the 13 superallowed decays discussed here. Furthermore, for isospin symmetry breaking corrections that are based on shell-model approaches, these four sets span the limits both in terms of suitable model space and the Woods–Saxon versus Hartree–Fock approaches to evaluating δ_{C2}. Because the uncertainties in the

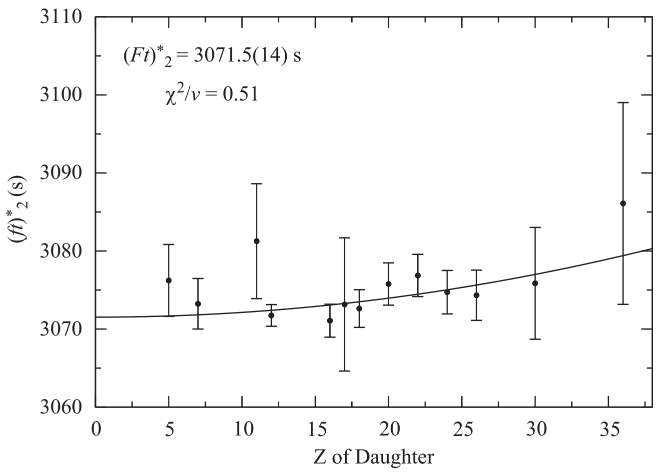

Fig. 3. Plot of the $(ft)^*_2$ data points derived using the averaged $\overline{\delta_C}$ corrections and the resulting linear and quadratic fits giving the global trend of the data. The adopted value for the charge independent intercept at $Z = 0.5$ yields $(\mathcal{F}t)^*_2 = 3071.5(14)$ s.

isospin symmetry breaking corrections for each of these calculations are not independent, the uncertainties on the $\overline{\delta_C}$ values are adopted from the study of TH08WS. The average δ_C values are tabulated in Table 2 and the resulting $(ft)^*_2$ values derived from Eq. (8) are listed in Table 4 and plotted in Fig. 3.

Compared to Method 1, the use of an average $\overline{\delta_C}$ correction largely suppresses the case-by-case scatter in the experimental *ft* values, as shown in Fig. 3. Rather than simply average the corrected $(ft)^*_2$ values, as is traditionally done when using each of the sets of theoretical δ_C corrections independently, an extrapolation to the $Z = 0.5$ isospin-independent limit is again performed to account for the possibility of remaining Z^2 dependent effects that may not be included in any of the theoretical calculations. A quadratic fit to these data yields a reduced χ^2 value of 0.51 and, from the intercept, the average $\mathcal{F}t$ value for Wilkinson's Method 2,

$$(\mathcal{F}t)^*_2 = 3071.5(14)\,\text{s}. \tag{9}$$

Although this method has the advantage of removing much of the scatter in the corrected *ft* values associated with the underlying shell structure that was neglected in Method 1, its main drawback comes from the fact that both the nucleus-to-nucleus fluctuations *and* the absolute magnitude of the isospin symmetry breaking corrections are incorporated into the result. Method 2 can therefore not be considered to be an independent test of the absolute magnitude of these theoretical corrections, which is the primary purpose of the analysis presented here.

With clear disadvantages in the results obtained from the first two methods, a third method was devised by Wilkinson that was designed to include the important nuclear shell effects for which all theoretical model calculations are consistent, while eliminating the dependence on the absolute magnitude of the correction terms.

3.3. Method 3

Wilkinson's third method was designed to take advantage of the individual strengths of the previous two Methods. Incorporating only the relative nucleus-to-nucleus variations inherent in the theoretical models while suppressing the absolute magnitude of the calculations is accomplished in Method 3 by defining the corrected $(ft)^*_3$ values as

$$(ft)^*_3 = ft(1+\delta_R)(1-\delta_{Cf}) \tag{10}$$

where δ_{Cf} are the nuclear shell structure fluctuations of the calculated isospin symmetry breaking corrections δ_C about a smooth quadratic fit δ_{Cs} to the calculated values themselves

$$\delta_{Cf} = \delta_C - \delta_{Cs}. \tag{11}$$

Because the δ_{Cf} values are quadratic fit residuals they contain only the *relative* nuclear shell structure information contained within the δ_C calculations and lack any assumptions regarding the validity of their *absolute* magnitude.

All four of the complete calculated δ_C sets (OB95HF, TH02WS, TH08WS, TH09HF) were fit to quadratic functions with the intercept fixed, as above, to $\delta_C = 0$ at $Z = 0.5$. The resulting δ_{Cf} and δ_{Cs} values obtained from all four of these models is presented in Fig. 4. For comparison, δ_{Cf} fluctuations are also shown for the subset of nine δ_C calculations from Ref. [10]. Although these calculations exhibit large differences in absolute magnitude, the residuals show remarkable consistency in the nucleus-to-nucleus shell structure about these smoothly varying quadratic trends. The nucleus-to-nucleus fluctuations are similarly described by all four of the shell-model calculations for isospin symmetry breaking across the entire range of superallowed emitters from ^{10}C to ^{74}Rb and the world average $\mathcal{F}t$ values derived from the application of these correction terms yield a range from $\overline{\mathcal{F}t}(\text{TH09HF}) = 3071.51(88)$ s to $\overline{\mathcal{F}t}(\text{OB95HF}) = 3076.22(72)$ s (see Table 3). This systematic difference between them is thus dominated by the absolute magnitude of the underlying quadratic component of the isospin symmetry breaking correction terms as opposed to the case-by-case nuclear shell structure effects superimposed on this smooth trend.

The corrected $(ft)^*_3$ values, determined from Eq. (10), using the fluctuations from all four shell-model calculations are listed in Table 4 and are plotted in Fig. 5. Extrapolating quadratic fits to the

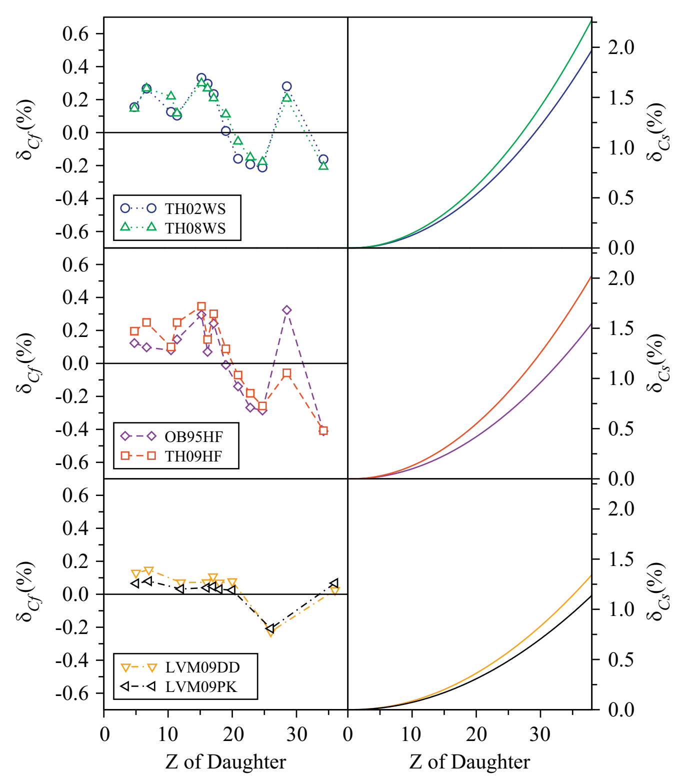

Fig. 4. (Color online) (Left) Fit residuals δ_{Cf} from quadratic fits to the isospin symmetry breaking corrections from Table 2 and (Right) smooth quadratic fits δ_{Cs} that represent the underlying absolute magnitude of these theoretical calculations. For Method 3, the δ_{Cs} values are removed for each decay and corrections to the experimental *ft* values are made using only the δ_{Cf} residuals. Uncertainties have been omitted for clarity.

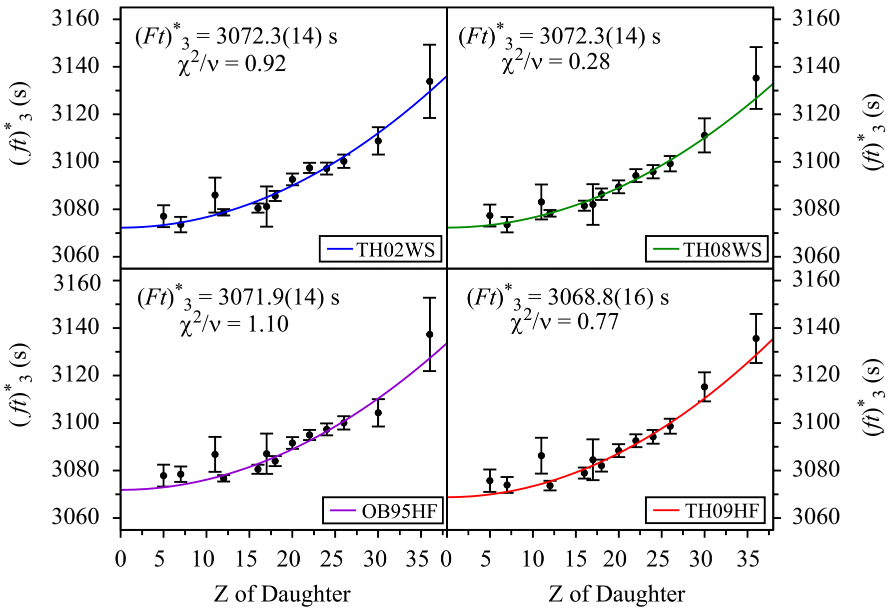

Fig. 5. (Color online) Plot of the $(ft)_3^*$ data points derived using the $\overline{\delta_{Cf}}$ fluctuations from the four complete sets of isospin symmetry breaking corrections and the resulting quadratic fits giving the global trend of the data. The intercepts at $Z = 0.5$ and the reduced χ^2 values are provided for each fit.

corrected ft values obtained using the fluctuations of the Woods–Saxon calculations yield numerically identical results $(\mathcal{F}t)_3^* = 3072.3(14)$ s with reduced χ^2 values of 0.92 and 0.28 for the TH02WS and TH08WS calculations, respectively. This is a significant result in of itself. By first correcting each ft value using the δ_C values from both the TH02WS and TH08WS, the resulting average $\mathcal{F}t$ values, $\overline{\mathcal{F}t}(\text{TH02WS}) = 3074.6(7)$ s and $\overline{\mathcal{F}t}(\text{TH08WS}) = 3072.1(8)$ s are not in agreement (see Table 3). However, it has been argued that the main difference between these two calculations is not the case-by-case fluctuations (these are nearly identical for these two sets, see Fig. 4) but is primarily due to the difference in the absolute magnitude, which is larger for TH08WS due to the use of an increased model space compared with TH02WS. Because Wilkinson's third method subtracts the absolute scale to utilize only the fluctuations, both sets of calculations therefore yield identical results. The fact that Method 3 relies only on the shell fluctuations of the isospin symmetry breaking calculations yet returns a result for TH08WS $(\mathcal{F}t)_3^* = 3072.3(14)$ s that is in excellent agreement with the average value $\overline{\mathcal{F}t}(\text{TH08WS}) = 3072.1(8)$ s [1] obtained from the standard procedure of applying the full set of δ_C corrections is also an important test of self consistency. This comparison provides independent support for the conclusion that the TH08WS set of δ_C calculations that employ a larger shell-model space, no longer underestimate the *absolute* scale of the isospin symmetry breaking corrections at the level of precision probed by the experimental data and radiative corrections. On the other hand, for the older TH02WS set of calculations that lacked these important core orbitals, Method 3 has accurately accounted for these deficiencies and yielded a result that is in agreement with both $\overline{\mathcal{F}t}$ and $(\mathcal{F}t)_3^*$ from the modern set of TH08WS calculations.

Applying Method 3 to the Hartree–Fock calculations for isospin symmetry breaking also yield very similar results $(\mathcal{F}t)_3^* = 3071.9(14)$ s (with a reduced $\chi^2 = 1.10$) and $(\mathcal{F}t)_3^* = 3068.8(16)$ s (reduced $\chi^2 = 0.77$) for the OB95HF and TH09HF calculations, respectively. The OB95HF result is also in excellent agreement with TH08WS even though the absolute magnitude of these two sets of calculations are significantly different. The TH09HF result, although it almost overlaps with the OB95HF, TH02WS, and TH08WS values within uncertainty, is ~ 3 s smaller and is significant given the fact that all of these methods use exactly the same input experimental ft values. The reason for this discrepancy is almost entirely due to the relative shell fluctuation of the isospin symmetry breaking correction for the most precisely measured superallowed decay of ^{26m}Al. Compared to $\delta_C \sim 0.3\%$ derived by OB95HF, TH02WS, and TH08WS for ^{26m}Al, the value $\delta_C = 0.44(5)\%$ [1] obtained with TH09HF is nearly 50% larger. Due to the high-precision obtained experimentally for this case, the difference in this single fluctuation for ^{26m}Al that is unique to the set of TH09HF calculations is enough to shift the corresponding intercept by ~ 3.0 s. However, it should be stressed that Method 3 is not uniquely sensitive to this single δ_{Cf} value for ^{26m}Al. The traditional approach of calculating the average of 13 corrected cases is also affected since the $\mathcal{F}t$ value for ^{26m}Al is presently the most-precisely determined [1]. Further investigation into the origin of this difference in δ_C obtained for the particular case of ^{26m}Al in the TH09HF set of isospin symmetry breaking calculations is highly desirable given the impact of this single highest-precision ft value on the determination of V_{ud}.

The excellent overall agreement between the results of Method 3 using the residuals δ_{Cf} to correct each experimental ft value, is a reflection of the similarity in the relative nucleus-to-nucleus shell structure variations included in all four models of isospin symmetry breaking that are based on shell-model approaches. An average of the Method 3 results for the four sets of calculations is adopted as the final result for Wilkinson's third method,

$$(\mathcal{F}t)_3^* = 3071.5(14)\,\text{s}. \tag{12}$$

This value is numerically identical to the one derived using Method 2 (Eq. (9)), although Method 3 is conceptually more appealing for providing an independent test of the absolute values of the isospin symmetry breaking corrections in superallowed Fermi β decay. This method uses the results of the shell-model calculations to describe only the case-by-case shell fluctuations of the isospin symmetry breaking corrections without making any assumption regarding their absolute magnitude. It is the relative nuclear shell fluctuations that are consistently described by all four of the models as evidenced in both Fig. 4 and the acceptable reduced χ^2 values of $\lesssim 1$ for the CVC test obtained by many of these calculations (Table 3). Incorporating only these case-by-case shell fluctuations yields a smoothly varying set of $(ft)^*$ values as a function of Z, as illustrated in Fig. 5, from which the isospin-independent limit at $Z = 0.5$ can be determined with minimal extrapolation uncertainty. This intercept ultimately provides an important and independent test of the *absolute* magnitude of the theoretical corrections for isospin symmetry breaking in superallowed Fermi β decays.

4. $\overline{(\mathcal{F}t)}^*$, V_{ud}, and tests of CKM unitarity

Previous work by Wilkinson [15–17] derived the fundamental Standard Model parameter V_{ud} from an adopted $\overline{(\mathcal{F}t)}^*$ value that was obtained from an average of the results from each of the three analysis methods. As discussed above, Method 1 is truly independent of all of the theoretical calculations for isospin symmetry breaking. However, the result $(\mathcal{F}t)^*_1 = 3079.6(12)$ s is limited by the fact that no attempt was made to describe the nuclear shell-structure fluctuations that are clearly inherent in the experimental ft values (and is evident in the reduced χ^2 value of 3.86). Given the limited number of data points, the intercept derived from the extrapolation of a smoothly varying quadratic fit is particularly sensitive to the case-by-case shell fluctuations and does not provide a completely unbiased estimate for V_{ud}. Method 2 incorporated these shell-structure effects by taking an average of four sets of shell-model calculations for isospin symmetry breaking. This method led to a significant suppression of the case-by-case fluctuations (reduced χ^2 value of 0.51), however, the resulting intercept $(\mathcal{F}t)^*_2 = 3071.5(14)$ s is not independent of the absolute magnitude of the isospin symmetry breaking corrections. Because the primary purpose of this analysis is to assess the accuracy of the absolute magnitude of these modern shell-model correction terms that now include core orbitals (TH08WS,TH09HF) relative to older shell model (TH02WS,OB95HF) and alternative approaches [9,10,12], Method 2 cannot be utilized. Wilkinson's Method 3 avoids both of these difficulties by using shell-model calculations to provide only the relative fluctuations of the isospin symmetry breaking corrections without making any assumptions as to their absolute magnitude. The result of this third method $(\mathcal{F}t)^*_3 = 3071.5(14)$ s thus provides an independent and compelling test of their absolute scale and is the value adopted in this work.

The average $(\mathcal{F}t)^*$ value obtained from Method 3 (Eq. (12)) is plotted as a horizontal band in Fig. 6 and is compared to the

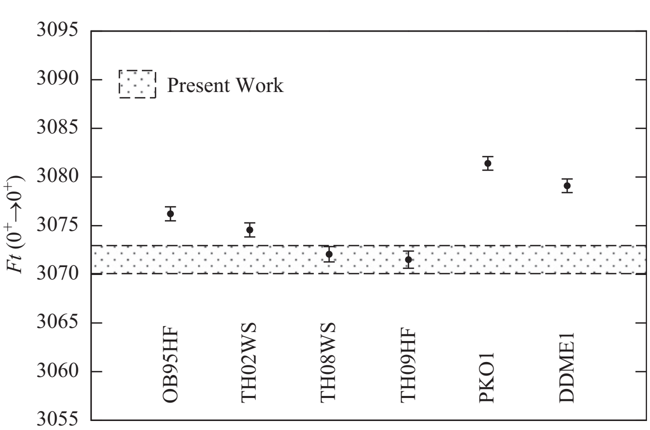

Fig. 6. Comparison of $\overline{\mathcal{F}t}$ derived in this work (shaded band) with the corresponding values obtained using the isospin symmetry breaking corrections of OB95HF [22–24], TH02WS [18], TH08WS [4], TH09HF [1], and the RPA approach using the effective interactions PKO1 and DDME-1 [10].

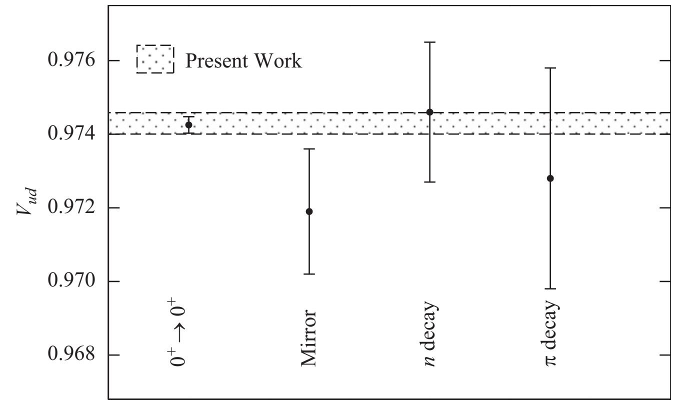

Fig. 7. Comparison of V_{ud} derived in this work (shaded band) to the adopted values for V_{ud} based on superallowed $0^+ \rightarrow 0^+$ decays with the isospin symmetry breaking corrections TH08WS and TH09HF [1], nuclear mirror transitions [29], neutron decay [29], and pion decay [30].

Table 5
Comparison of V_{ud} deduced in this work to the four models of isospin symmetry breaking considered. Tests of CKM unitarity are performed with the corresponding V_{ud} and the presently adopted values for V_{us} and V_{ub} from Ref. [2].

Method	$\overline{\mathcal{F}t}$ (s)	V_{ud}	CKM
Present work	3071.5(14)	0.97430(29)	1.0001(10)
OB95HF	3076.2(7)	0.97355(21)	0.9987(10)
TH02WS	3074.6(7)	0.97381(21)	0.9992(10)
TH08WS	3072.1(8)	0.97421(22)	0.9999(10)
TH09HF	3071.5(9)	0.97430(23)	1.0001(10)

average $\mathcal{F}t$ value derived from the four shell-model calculations for isospin symmetry breaking (OB95HF, TH02WS, TH08WS, TH09HF), and the values obtained by recent RPA calculations derived in Ref. [10] using the PKO1 and DDME1 effective interactions. There is very good agreement between the value derived in this work and the $\overline{\mathcal{F}t}$ values derived from the recent shell-model calculations of TH08WS and TH09HF for isospin symmetry breaking that include select core orbitals. Previous shell-model calculations of TH02WS and OB95HF that use a more restrictive model space, and the recent RPA calculations of Ref. [10] all result in $\overline{\mathcal{F}t}$ values that are too large suggesting that isospin symmetry breaking is, in general, being underestimated by these approaches. The result derived using Wilkinson's Method 3 thus provides an independent confirmation for the most recent shell-model calculations (TH08WS,TH09HF) of Towner and Hardy [1,4].

Using the average $\overline{(\mathcal{F}t)}^*$ value from Method 3, and the constants K, G_F, and Δ_R^V, the value of V_{ud} derived from Eq. (3) is

$$V_{ud} = 0.97430(29). \quad (13)$$

This result is in excellent agreement with the adopted value $V_{ud} = 0.97425(22)$ obtained in the latest world superallowed data survey [1] that use only the TH08WS and TH09HF δ_C calculations. In Fig. 7, the value of V_{ud} derived in this work is compared with the adopted value from the recent world survey of superallowed data [1], and the present values obtained from $T = \frac{1}{2}$ nuclear mirror transitions [29], neutron decay [29], and pion decay [30], respectively.

Combining the value of V_{ud} from Eq. (13) with the values of V_{us} and V_{ub} from the Particle Data Group [2] yields the top-row test of CKM unitarity,

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0001(10) \quad (14)$$

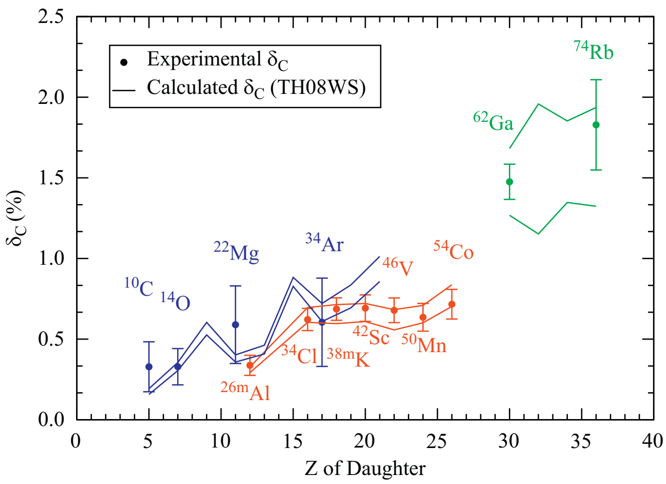

Fig. 8. (Color online) "Experimental" isospin symmetry breaking corrections determined in this work from the $\overline{(\mathcal{F}t)}^{*}$ value (points). Overlayed for comparison are bands representing the TH08WS [4] isospin symmetry breaking calculations.

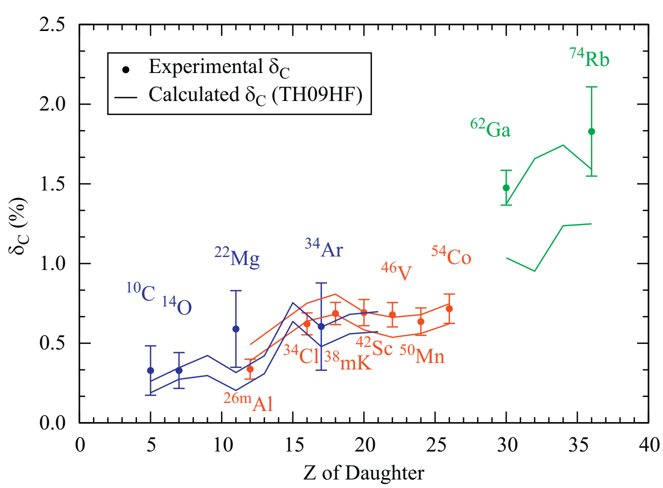

Fig. 9. (Color online) "Experimental" isospin symmetry breaking corrections determined in this work from the $\overline{(\mathcal{F}t)}^{*}$ value (points). Overlayed for comparison are bands representing the TH09HF [1] isospin symmetry breaking calculations.

a value that agrees with the Standard Model prediction at the level of 0.1%. A summary of the values for V_{ud} and the corresponding top-row tests of CKM unitarity obtained from all four sets of isospin symmetry breaking corrections is provided in Table 5.

5. Tests of isospin symmetry breaking corrections

With the world average $\overline{(\mathcal{F}t)}^{*}$ value derived from Method 3 that is independent of the absolute magnitude of the theoretical isospin symmetry breaking corrections, this result can be used to constrain the isospin symmetry breaking corrections for each of the 13 superallowed decays by rearranging Eq. (3) to obtain

$$(1-\delta_C)=\frac{\overline{(\mathcal{F}t)}^{*}}{ft(1+\delta_R')}-\delta_{NS}. \quad (15)$$

From Eq. (15) an "experimental" δ_C value can be determined for each decay that utilizes only the radiative corrections listed in Table 1, the experimental ft values of Table 3, and the overall $\overline{(\mathcal{F}t)}^{*}$ value from Eq. (12). The results are listed in Table 2 and are compared to the specific examples of the TH08WS and TH09HF calculations in Figs. 8 and 9, respectively. The fact that the $\overline{(\mathcal{F}t)}^{*}$ value has been derived independently of the magnitude of these corrections ensures that this absolute comparison is a stringent test of the accuracy in the theoretical calculations. Comparing the experimental δ_C values derived in this analysis (data points in Figs. 8 and 9), to the calculated values (colored bands) there is very good agreement with the δ_C corrections determined in the present analysis with the Woods–Saxon (TH08WS) and Hartree–Fock (TH09HF) models. These calculations not only agree with the absolute magnitude of the corrections but follow the nucleus-to-nucleus variations extremely well with the possible exception of the ^{26m}Al value that appears to be more accurately predicted by the TH08WS calculation. Similar comparisons for the TH02WS set (not shown) follow the case-by-case fluctuations extremely well but globally underpredict the absolute magnitude of the effect of isospin symmetry breaking. This comparison provides a critical test of both the shell-structure fluctuations and the absolute values of the corrections in these models and suggests that the inclusion of select core orbitals into the shell-model calculations, as proposed in Ref. [4], is essential to extract an accurate value for V_{ud} from the world superallowed data set. The current analysis also indicates that the further inclusion of additional orbitals that are not presently in the TH08WS and TH09HF calculations will not make significant contributions to the isospin symmetry breaking corrections at the level of precision probed by the current experimental data and radiative corrections.

The set of isospin symmetry breaking corrections derived from the experimental data in this work can be used to further constrain alternative theoretical developments. Recent calculations include relativistic Hartree–Fock calculations that utilized the random phase approximation [10,31], perturbation theory that used the concept of the isovector monopole resonance [9], modified shell-model calculations based on Pyatov's restoration [12], and for the case of ^{10}C, an *ab initio* no-core shell model (NCSM) approach [32]. These calculations do not all rely on the model-dependent separation [1,8] of δ_C into the two components $\delta_{C1}+\delta_{C2}$ that is presently required to make the shell-model calculations tractable. However, with the exception of the NCSM ^{10}C result, these theoretical models are, in general, underpredicting the magnitude of isosopin symmetry breaking for the $\mathbf{T}=1$ superallowed emitters.

6. Conclusion

The up–down element of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix has been derived from superallowed Fermi β decay between isospin $\mathbf{T}=1$ states using Wilkinson's semi-empirical techniques [13–17] and the most recent set of evaluated superallowed β decay data [1]. The result, $V_{ud}=0.97430(29)$ agrees with the value $V_{ud}=0.97425(22)$ [1] calculated by first applying nucleus-dependent corrections for isospin symmetry breaking that are based on Woods–Saxon and Hartree–Fock shell-model wavefunctions. Wilkinson's Method 3 adopted here employs only the consistent description of the nucleus-to-nucleus shell fluctuations contained within all of the shell-model calculations and suppresses their absolute scale. This technique provides an important and independent approach to evaluate V_{ud} from the superallowed decay data set in light of the recent and significant shifts to this quantity that have resulted from model-space changes in modern shell-model calculations for isospin symmetry breaking. The results presented here provide an independent confirmation of these most recent improvements made by Towner and Hardy [1,4] and suggests that previous shell-model calculations employing restricted model spaces and the recent self-consistent calculations based on perturbation theory

[9] and RPA approaches [10] are consistently underpredicting the magnitude of these corrections.

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