Magnetic moments of $T = 3/2$ mirror pairs

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(Received 23 August 2010; published 10 December 2010)

We predict values of the magnetic moments of $T = 3/2$ proton-rich fp-shell nuclei in the mass range $A = 43$–53, by using known values for their neutron-rich mirrors together with shell-model estimates for small quantities. We extend the analysis to those $T = 3/2$ sd-shell mirror pairs for which both the $T_z = -3/2$ and $T_z = +3/2$ magnetic moments have been measured. We find that these obey the same linear relation as previously deduced for $T = 1/2$ mirror pairs.

DOI: 10.1103/PhysRevC.82.064305 PACS number(s): 23.40.Hc, 21.10.Ky, 23.40.Bw, 27.40.+z

I. INTRODUCTION

Techniques involving radioactive ion beams have recently been used to determine the magnetic moments of β-unstable nuclei [1,2]. It is useful in these circumstances to have reliable estimates of the moments, conventionally obtained from large-basis shell-model calculations [3–5]. Focusing on $T = 1/2$ odd-even mirror pairs, we have previously investigated a different method of making these estimates, which either does away with the structure calculations altogether [6] or, more accurately, uses them to determine the small contributions [7] to the magnetic moments coming from the even type of nucleon [8]. Here we extend the method to the $T_z = \pm 3/2$ members of a $T = 3/2$ quadruplet, specifically to cases where the magnetic moment of the $T_z = +3/2$ neutron-rich member is accurately known and where predictions for the proton-rich member are required.

When isospin is conserved the magnetic moments of an odd-even mirror pair can be written (in units of μ_N) as [6]

$$\mu_p = g_p J + (G_p - g_p)(S_o - S_e) + [(G_p - g_p + G_n - g_n)S_e - (g_p - g_n)J_e] \quad (1)$$

and

$$\mu_n = g_n J + (G_n - g_n)(S_o - S_e) + [(G_p - g_p + G_n - g_n)S_e + (g_p - g_n)J_e], \quad (2)$$

where μ_p and μ_n are the magnetic moments of the odd-proton and odd-neutron members of the odd-even mirror pair. The free-nucleon values of the g factors are $g_p = 1.0$, $g_n = 0.0$, $G_p = 5.586$, and $G_n = -3.826$. $S_{e/o}$ and $J_{e/o}$ are the contributions from the even/odd type of nucleon to the z components of the total spin S and total angular momentum J of the mirror pair.

Eliminating $(S_o - S_e)$ from Eqs. (1) and (2) yields

$$(\mu_p + \Delta\mu_p) = \alpha(\mu_n + \Delta\mu_n) + \beta J, \quad (3)$$

where $\Delta\mu_p$ and $\Delta\mu_n$ are small terms [7,8] involving the even type of nucleon. Specifically,

$$\Delta\mu_p = -[(G_p - g_p + G_n - g_n)S_e - (g_p - g_n)J_e] \quad (4)$$

and

$$\Delta\mu_n = -[(G_p - g_p + G_n - g_n)S_e + (g_p - g_n)J_e]. \quad (5)$$

Dividing Eqs. (3), (4), and (5) by J and defining

$$\gamma_{p,n} = \mu_{p,n}/J \quad \text{and} \quad \Delta\gamma_{p,n} = \Delta\mu_{p,n}/J \quad (6)$$

yields

$$(\gamma_p + \Delta\gamma_p) = \alpha(\gamma_n + \Delta\gamma_n) + \beta, \quad (7)$$

where $\alpha = (G_p - g_p)/(G_n - g_n)$ and

$$\beta = g_p - \alpha g_n. \quad (8)$$

We assume that for nucleons in nuclei the free-space g factors can be replaced by a single set of effective values denoted by $\tilde{G}$ and $\tilde{g}$. This then results in α and β being replaced by $\tilde{\alpha}$ and $\tilde{\beta}$ in Eqs. (7) and (8). Thus, given the values of the global parameters $\tilde{\alpha}$ and $\tilde{\beta}$, together with the small terms $\Delta\gamma_p$ and $\Delta\gamma_n$ for a particular pair of mirror nuclei, we can obtain either γ_p or γ_n from a knowledge of the other. For example, an application to $T = 1/2$ mirror pairs yields $\tilde{\alpha} = -1.147 \pm 0.013$ and $\tilde{\beta} = 1.027 \pm 0.013$, which, when used in an application of Eq. (7) to the mirror pair ^{57}Cu and ^{57}Ni [8], results in $\gamma = +1.698 \pm 0.012$ for ^{57}Cu [8], in excellent agreement with the recently measured value of $\gamma = +1.721 \pm 0.005$ [1].

II. APPLICATION TO $T = 3/2$ MIRROR PAIRS

The technique described above for $T = 1/2$ mirror pairs is more generally applicable for $T = n/2$, with $n = 3, 5, \ldots$. For $T = 3/2$ it has already been used under the assumption that $\Delta\gamma_p = \Delta\gamma_n = 0$ [2,8]. Here we apply it to the $T = 3/2$ mirror pairs in the mass range $A = 43$–53, with values of $\Delta\gamma_p$ and $\Delta\gamma_n$ taken from a $0\hbar\omega$ shell-model

TABLE I. Values of γ_p, and γ_n from data on magnetic dipole moments [10]. The contributions $\Delta\gamma_p$ and $\Delta\gamma_n$ have been estimated from $0\hbar\omega$ shell-model calculations using Eqs. (4)–(6) and free-nucleon values for the g factors.

A, J^π	Nucleus	γ_p	Nucleus	γ_n	$\Delta\gamma_p$	$\Delta\gamma_n$
43,7/2$^-$	^{43}V		^{43}Ca	−0.3765(−)	+0.0000	0.0000
45,7/2$^-$	^{45}Sc	+1.3590(−)	^{45}Cr		+0.1312	−0.1691
47,5/2$^-$	^{47}Mn		^{47}Ti	−0.3154(−)	+0.0606	−0.0925
49,7/2$^-$	^{49}V	(+)1.28(1)	^{49}Fe		+0.1462	−0.1810
51,7/2$^-$	^{51}Co		^{51}Cr	−0.267(1)	+0.0675	−0.0992
53,7/2$^-$	^{53}Mn	(+)1.435(2)	^{53}Ni		+0.0442	−0.0751

TABLE II. Predicted magnetic moments for $T = 3/2$ proton-rich nuclei in the mass range $A = 43$–53.

Nucleus	μ (nm)
^{43}V	$+5.106 \pm 0.049$
^{45}Cr	-0.822 ± 0.052
^{47}Mn	$+3.586 \pm 0.035$
^{49}Fe	-0.585 ± 0.068
^{51}Co	$+4.828 \pm 0.049$
^{53}Ni	-1.117 ± 0.057

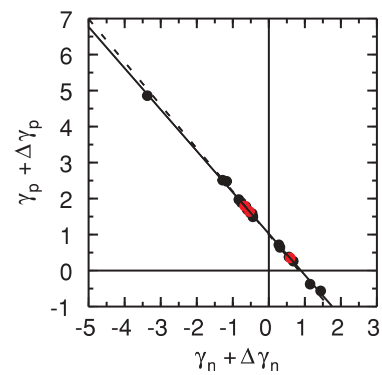

FIG. 1. (Color) $\gamma_p + \Delta\gamma_p$ versus $\gamma_n + \Delta\gamma_n$. The line is the result of fit (B) in Ref. [8]. The single-particle model using free-nucleon values for the coupling constants model is shown by the dashed line.

calculation with the GPFX1A Hamiltonian [9] using free-nucleon values for the g factors in Eqs. (4) and (5). Table I summarizes the information that, together with $\tilde{\alpha} = -1.147 \pm 0.013$ and $\tilde{\beta} = 1.027 \pm 0.013$, results in the predictions for the magnetic moments of the proton-rich nuclei given in Table II.

In Fig. 1 we reproduce a plot of $\gamma_p + \Delta\gamma_p$ versus $\gamma_n + \Delta\gamma_n$ for the $T = 1/2$ mirror nuclei from Ref. [8] (Table I). On the plot we have superposed, in red, points corresponding to the three sd-shell $T = 3/2$ mirror pairs for which measurements of both γ_p and γ_n are available (see Table III, where for completeness we have included pertinent theoretical results for all sd-shell $T = 3/2$ mirror pairs, using the new interaction USDB). We see that the points corresponding to the $T = 1/2$ mirror pairs and to the $T = 3/2$ mirror pairs all fall accurately on a universal $\tilde{\alpha} = -1.147$ and $\tilde{\beta} = 1.027$ line, in support of a single set of effective g factors. This important conclusion needs to be reinforced by the inclusion of more $T = 3/2$ data, but we note here that an application of Eq. (7) using the results of Table III for γ_n, $\Delta\gamma_p$, and $\Delta\gamma_n$ yields $\gamma_p = 1.569$, 1.560, and 0.237 for ^{21}F, ^{23}Al, and

TABLE III. Values of γ_p and γ_n from theory (USDB) and from data on magnetic dipole moments [2,10–12]. The contributions $\Delta\gamma_p$ and $\Delta\gamma_n$ have been estimated from $0\hbar\omega$ shell-model calculations using Eqs. (4)–(6) and free-nucleon values for the g factors.

A, J^π	Nucleus	γ_p(theor)	γ_p	Nucleus	γ_n(theor)	γ_n	$\Delta\gamma_p$	$\Delta\gamma_n$
19,5/2$^+$	^{19}Na	1.7343		^{19}O	−0.6126		0.0000	0.0000
21,5/2$^+$	^{21}F	1.5115	1.5678	^{21}Mg	−0.3392	−0.393	0.2233	−0.2739
23,5/2$^+$	^{23}Al	1.5462	1.56(9)	^{23}Ne	−0.4198	−0.4318	0.0796	−0.1023
25,5/2$^+$	^{25}Na	1.3468		^{25}Si	−0.2348		0.1332	−0.1657
27,1/2$^+$	^{27}P	2.0738		^{27}Mg	−0.8238		0.1501	−0.1975
29,5/2$^+$	^{29}Al	1.4370		^{29}S	−0.3186		0.1210	−0.1469
31,3/2$^+$	^{31}Cl	0.5051		^{31}Si	0.4180		0.1179	−0.1035
33,1/2$^+$	^{33}P	2.8706		^{33}Ar	−1.5306		0.1155	−0.1265
35,3/2$^+$	^{35}K	0.0906	0.245(5)	^{35}S	0.7411	0.667(27)	0.1084	−0.0728
37,3/2$^+$	^{37}Cl	0.2346		^{37}Ca	0.6385		0.0000	0.0000

^{35}K, in excellent agreement with the measured values 1.5678, 1.56(9), and 0.245(5), respectively.

III. CONCLUSIONS

We have extended previous analyses of the correlations between magnetic dipole moments of $T = 1/2$ mirror pairs to $T = 3/2$ mirror pairs. This has been done by explicitly including contributions made by the even type of nucleon in these odd-even nuclei. A number of predictions of magnetic dipole moments of β-unstable proton-rich nuclei in the mass range $A = 43$–53 have been made.

ACKNOWLEDGMENTS

This work is partly supported by NSF Grant PHY-0758099, NSF/MRI Grant PHY-0619407, and the National Research Foundation of South Africa under Grant 2054166.

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