Two-parameter Fermi function fits to experimental charge and point-proton densities for ^{208}Pb

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We use the model-independent experimental charge density for ^{208}Pb to determine a model-independent point-proton density. An improved two-parameter Fermi function representation for the this point-proton density provides input for the analysis of experiments for the neutron density interpreted in terms of neutron skin and halo properties of ^{208}Pb.

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Charge form factors for many nuclei are accurately measured by electron scattering. These data can be analyzed by model-independent methods, such as the Fourier–Bessel expansion fit, to extract numerical values for the charge density as a function of radius. These results serve as one of the fundamental tests of nuclear models. The neutron densities are not nearly so well known from experiment and must be characterized by gross properties such as their root-mean-square (rms) radius. In a recent experiment with coherent pion photoproduction, the data were analyzed to determine the parameters of a two-parameter Fermi (2pF) function for the neutron density [1].

In this brief report we show 2pF function used for point protons in Ref. [1] is not accurate at the level that it was used to interpret the data. We derive an improved 2pF form for point protons based upon a fit to the the model-independent analysis of charge scattering data. The consequence is that the conclusions concerning the neutron skin in Ref. [1] need to be modified. We will also show that the three-parameter Fermi distribution fit to the model-independent data does not provide a significant improvement.

The 2pF function is $\rho(r) = \rho_o/[1 + \exp(r - a)/c]$, where the diffuseness parameter a and the radius parameter c determine the shape of the density near the nuclear surface, and ρ_o is constrained to give the normalization $\int \rho(r)d\tau = (Z/N)$, where (Z/N) is the number of (protons/neutrons). When a is small compared to c (as is the case for ^{208}Pb) the moments of this distribution are given by $\langle r^L\rangle = [3c^L/(L + 3)][1 + \frac{1}{6}L(L + 5)\pi^2(a/c)^2]$.

The radius and diffuseness parameters give a conceptually intuitive objective measure of how the neutron skin arises from the density of neutron centers. The determination of these parameters is of interest because it allows the differences in the nucleon distribution between species to be attributed to either the differences in the radial extent of nuclear matter at saturation density or the rate of decrease in the density from saturation. There are distinct physical implications if the differences in nucleon species distribution is attributed to one of these parameters as opposed to the other [2].

The purpose of this short note is to propose improved point-proton distributions for ^{208}Pb for use in the analyses of data for the matter and neutron densities.

The experimental charge density for ^{208}Pb from the model-independent analysis of Euteneuer *et al.* [3] has a rms charge radius of 5.503 (2) fm. To obtain the point-proton density we need to unfold form factors related to the finite size of the proton, the neutron, and to the relativistic Darwin–Foldy and spin-orbit terms [4]. We use the experimental charge density to obtain the experimental form factor: $F_{\rm ch}(q) \sim \int \rho_{\rm ch}(r) j_0(qr)d\tau$, where $j_0(qr)$ is the spherical Bessel function for $\ell = 0$. The point-proton form factor is given by this divided by the finite-size correction: $F_p(q) = F_{\rm ch}(q)/F_{\rm fs}(q)$. The point-proton density is given by $\rho_p(r) \sim \int F_p(q) j_0(qr)d\tau$.

We start with the proton finite size as given by the Mainz electron-scattering form factor measurement with a rms radius of $\langle r_p^2\rangle^{1/2} = 0.879\,(8)$ fm [5]. For the proton form factor we use the standard dipole form. [The muonic hydrogen Lamb-shift measurement for the proton gives $\langle r_p^2\rangle^{1/2} = 0.84184\,(67)$ fm [6], which is outside of the error bars from the electron-scattering result. But this will not significantly change our conclusions.] We introduce the neutron finite-charge-size correction using a mean-square charge radius for the neutron of -0.115 fm^2 [7]. There is an error on the order of 0.02 fm^2 from the model dependence of the neutron data analysis [8]. The neutron finite-size correction is given by a two-Gaussian form factor [9]. For the point-neutron density we use the point-proton density scaled up by N/Z (the correction coming from a more realistic neutron distribution is very small). The rms radius of the point protons is $[(5.503)^2 - (0.879)^2 + (N/Z)0.115 - 0.033 + 0.012]^{1/2} = 5.444$ fm (or 5.450 fm with the muonic hydrogen data). The last two terms are the relativistic Darwin–Foldy and spin-orbit corrections, respectively, and the form factors for these are given in Ref. [4]. The resulting point-proton density is shown by the bold dots of Fig. 1. In addition to the density profile $\rho(r)$ this same quantity is multiplied by r^2 and r^4 in order to emphasize its volume and surface properties. The model-independent results from the independent analysis of Frois *et al.* [10] are within size of the dots in Fig. 1. (Another set of data referred to in the compilation of Fricke *et al.* [11] has not been published except in a thesis [12].)

When the 2pF form is fixed to reproduce this rms radius of 5.444 fm, the minimum for the volume element difference $r^2[\rho_p(r) - \rho_f(r)]^2$ summed up to $r = 10$ fm comes when $a_p = 0.497\,(20)$ fm [$c_p = 6.613\,(40)$ fm], where the errors are estimated based on the limitations of fitting a Fermi function shape to the data. The resulting point-proton distribution is shown by the black line on the left-hand side of Fig. 1. It is seen to reproduce the model-independent density (bold dots) rather precisely, except for the interior oscillations. But as shown in

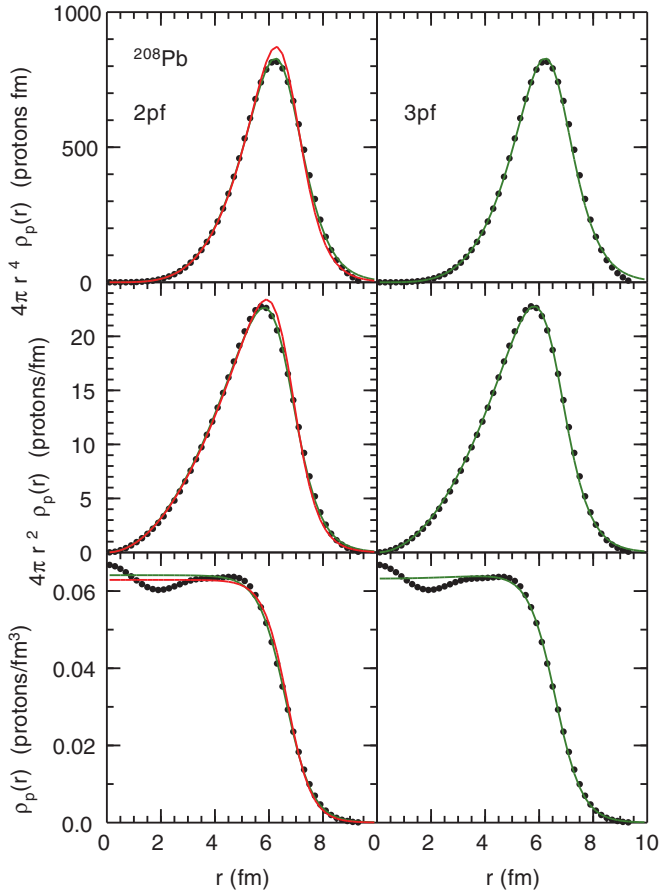

FIG. 1. (Color online) The panels in the left column plot the experimental point-proton distribution (bold dots) with the fitted 2pF (green line) and the 2pF used in Ref. [1] (red line). The column on the right plots the same experimental point-proton distribution (bold dots) with the fitted 3pF (green line). The top row is the radial integrand for the rms radius calculation. The middle row is the radial integrand for normalization. The bottom row is the unweighted density.

the middle panel, the interior oscillations in the integrand of the volume integral are insignificant.

In the analysis of coherent pion photoproduction data a 2pF form for the point-proton density with fixed parameters was assumed, and then a 2pF form for the point-neutron density was fit to the data [1]. The point-proton parameters used were $a_p = 0.447$ fm and $c_p = 6.680$ fm. This is shown by the red line on the left-hand side of Fig. 1. These parameters were taken from Klos *et al.* [13]. Klos *et al.* refer to the charge distribution from the compilation of Fricke *et al.* [11], together with an approximation given by Oset *et al.* [14] to transform the charge-density parameters to those for point-protons based on taking into account the proton finite-size in terms of its rms radius. However, the compilation of Fricke *et al.* does not give values 2pF parameters for the experimental charge density of ^{208}Pb. The result with our 2pF parameter set (black lines on the left-hand side of Fig. 1) is in much better agreement with experiment compared with the parameters used in Ref. [1], shown by the red lines.

The sum of the square of the differences of the volume-element-weighted density elements was selected as the residual metric because it represents the square of the number of misplaced nucleons. It is easy to see this from the normalization condition (i.e., integral of the volume element multiplied by density is the number of nucleons). The square of this quantity is introduced to make the quantity positive definite. Note that this quantity is the square of the L_2 norm of the residual vector.

It is natural to ask if the 2pF describes the charge distribution in sufficient detail to attribute the shape of the nucleon distribution to the bulk radius or diffuseness. A common alternative to the 2pF is the three-parameter Fermi distribution (3pF). This parametrization allows for another term to allow the central density to have some quadratic dependence allowing the density a hollowed out "wine bottle" shape. The 3pF has the functional form $\rho(r) = \rho_o(1 + wr^2/c^2)/[1 + \exp(r - a)/c]$. When the procedure used to fit the 2pF is applied to the 3pF the resulting parameters are $a_p = 0.499\,(20)$ fm, $c_p = 6.583\,(40)$ fm, and $w_p = 0.0393$. The 3pF distribution with these parameters is plotted on the right panels in Fig. 1 because it is indistinguishable by eye when compared to the plot of the 2pF case. This is not surprising since the shared parameters change by less than 1% and the new w parameter is small compared to the radius parameter. The smallness of the w parameter can be understood by noting the central density of the distribution does not contribute much to the residual because most of the volume where the density is appreciable is far from where the w parameter contributes to the shape of the distribution. Thus, it is sufficient to use the 2pF form that also has the advantage of simple interpretation in terms of neutron-halo properties [2].

With the old point-proton parameters of $a_p = 0.447$ fm and $c_p = 6.680$ fm. The neutron diffuseness obtained from the pion data was $a_n = 0.55\,(2)$ fm. This was used to obtain $\Delta a = a_n - a_p = 0.55\,(2) - 0.447 = 0.10\,(2)$ fm. This large value was interpreted as showing a halo-type character for the neutron density distribution ^{208}Pb [2].

We recommend using $a_p = 0.497\,(20)$ fm together with the associated values of $c_p = 6.613\,(40)$ fm and $\rho_{po} = 0.0641\,(7)$ protons/fm^3. This would nominally reduce the Δa value by a factor of two down to 0.05 (3) fm. But the photoproduction data needs to be reanalyzed with our new point-proton distribution or, even better, by using the experimental point-proton distribution as shown in Fig. 1.

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