Theoretical analysis of isospin mixing with the $\beta$ decay of $^{56}\text{Zn}$

N. A. Smirnova* and B. Blank
CENBG, CNRS/IN2P3 and Université de Bordeaux, Chemin du Solarium, 33175 Gradignan cedex, France

B. A. Brown
Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA

W. A. Richter
iThemba LABS, P.O. Box 722, Somerset West 7129, South Africa
and Department of Physics, University of the Western Cape, Private Bag X17, Bellville 7535, South Africa

N. Benouaret
Faculté de Physique, Université des Sciences et de la Technologie Houari Boumediene, USTHB, El-Alia 16111, Bab-Ezzouar-Alger, Algeria

Y. H. Lam
Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

(Received 4 March 2016; published 8 April 2016)

We present a shell-model analysis of the $\beta$ decay of $^{56}\text{Zn}$. The calculations are performed using isospin-nonconserving Hamiltonians constructed on the basis of the GXPF1A and KB3G interactions. Our theoretical results reproduce the essential features of the decay of $^{56}\text{Zn}$ and explain the surprising competition between $\beta$-delayed proton and $\gamma$-ray emission from the isobaric analog state.

DOI: 10.1103/PhysRevC.93.044305

Production of proton-rich nuclei in the last decades and advances in the development of experimental techniques made it possible to explore nuclear structure via exotic decay modes, such as direct or $\beta$-delayed emission of protons, diprotons, or $\alpha$ particles [1,2]. In this context, $\beta$-delayed proton emission is of particular interest because it involves transitions proceeding via isospin-symmetry breaking and thus provides a perfect testing ground for the determination of isospin mixing in nuclear states and puts constraints on theoretical modeling.

In this article, we propose a theoretical study of the decay of the proton-rich $pf$ shell nucleus $^{56}\text{Zn}$. First experimental spectroscopic data were provided by Dossat et al. [3] and a comprehensive study of $\beta$-delayed proton and $\gamma$ decay of $^{56}\text{Zn}$ has been reported recently in Ref. [4]. A striking feature of this decay is that proton and $\gamma$ branches of almost equivalent intensity have been observed, despite a large amount of isospin mixing in the isobaric analog state (IAS). Our work proposes a detailed theoretical analysis of this phenomenon.

A partial scheme of the $\beta$ decay of the $0^+, T = 2$ ground state of $^{56}\text{Zn}$ is shown in Fig. 1. The most intense branch populates the isobaric analog $0^+$ state at around 3.5 MeV excitation energy in the daughter nucleus, $^{56}\text{Cu}$. Although this state is situated well above the proton separation threshold, its decay by proton emission to low-lying $T = 1/2$ states of $^{55}\text{Ni}$ is forbidden by isospin symmetry. Thus, if the isospin-symmetry were exact, the IAS would decay via $\gamma$ emission only. The observation of proton emission from the IAS is possible solely if isospin symmetry is broken either in the proton-emitting state or the final state. The IAS in the odd-odd daughter nucleus lying in a region of relatively high-level density is expected to have much more isospin impurity than the ground state of the final nucleus.

Let us suppose that isospin mixing of the IAS can be modeled as due to the admixture of a single close-lying $0^+, T = 1$ state. Then we can express the IAS as $|\text{IAS}⟩ = \sqrt{1 - \alpha^2}|T = 2⟩ + \alpha|T = 1⟩$. Thus, the spectroscopic factor for proton emission from the IAS can be obtained as

$$S_{\text{IAS}} = \alpha^2 S^{T=1},$$

where $S^{T=1}$ is the allowed spectroscopic factor of the admixed state.

Indeed, it has been found experimentally [4] that the IAS of the $^{56}\text{Zn}$ ground state in $^{56}\text{Cu}$ is strongly mixed with another $0^+$ (mainly $T = 1$) state lying about 85 keV below. This conclusion was drawn on the basis of the Fermi strength $B(\gamma)$ from the $^{56}\text{Zn}$ ground state, which turned out to be split between the two $0^+$ states, resulting in $\alpha^2 = 33(10)\%$ of isospin mixing in the IAS [4]. A puzzling feature is that despite this high isospin impurity of the IAS and a large probability for proton emission with an energy of $E_p = 2948(10)$ keV, its proton decay does not represent the dominant decay, but is observed in competition with $\gamma$-ray emission with similar intensities, $I_p = 18.8(10)\%$ and $I_\gamma = 19.2(50)\%$.

Within a two-level mixing approach and using experimental values [4] for the square root of the ratio of the $B(\gamma)$ values to the two $0^+$ states in $^{56}\text{Cu}$ at 3432(150) keV and 3508(150) keV, $R = B(\gamma)_{3432}/B(\gamma)_{3508} = 0.69(20)$, and for the observed energy splitting of $\Delta E = 85(10)$ keV, we can...
To understand the decay features and to cross-check theoretical descriptions, we have performed large-scale shell-model calculations in the full pf shell using the NUSHELLX@MSU [6] and ANTOINE [7] shell-model codes with two different charge-dependent Hamiltonians. The first one, cdGX1A, is based on the GXPF1A [8] interaction with the addition of Coulomb, strong charge-asymmetry, and charge-independence-breaking interactions from Ref. [9] and updated isovector single-particle energies from Ref. [10]. The other Hamiltonian, cdKB3G, was constructed on the basis of the KB3G interaction [11], with the addition of the Coulomb interaction and isovector single-particle energies scaled as $\sqrt{\hbar \omega (A)}$.

First, we calculated theoretical spectra at low energies of the nuclei of interest, $^{56}\text{Cu}$ and $^{55}\text{Ni}$, as well as the ground-state properties of $^{56}\text{Zn}$ to check the validity of the predictions. Table I summarizes the theoretical $\beta$-decay half-life of $^{56}\text{Zn}$ and the excitation energy of the $0^+$ IAS and of the admixed $0^+$ state in $^{56}\text{Cu}$, and their electromagnetic- and proton-decay characteristics. The $\gamma$-decay widths were calculated using standard effective charges $e_p = 1.5e$ and $e_n = 0.5e$ and optimized empirical $g$ factors as given in Ref. [8] for the cdGX1A interaction and in Ref. [11] for the cdKB3G interaction. The experimental energy $E_{\gamma} = 1835$ keV of the most intense $\gamma$ transition from the IAS to the $1^+_{1}$ state at 1691 keV was used and, therefore, we suppose $E_{\gamma} = 1750$ keV for a possible decay from the $0^+$ state which is 85 keV below

\begin{table}[h]
\centering
\begin{tabular}{l|c|c|c|c}
 & Expt. & cdGX1A & cdKB3G \\
\hline
$^{56}\text{Zn}$ & & & \\
$T_{1/2}$ (ms) & 32.9(8) & 35(4) & 24(4) \\
$^{56}\text{Cu}, 0^+, IAS$ & & & \\
$E_{\text{IAS}}$ (MeV) & 3.508(140) & 3.505 & 3.827 \\
$S_{\text{IAS}}$ & $1.5 \times 10^{-3}$ & 1.5 $\times 10^{-3}$ & 3.1 $\times 10^{-3}$ \\
$\Gamma_{\text{IAS}}$ & 0.13(4) & 1.6(1) & 3.2(2) \\
$\Gamma_{\text{IAS}}$ & 0.16 & 0.11 & \\
$^{56}\text{Cu}, 0^+, T = 1$ & & & \\
$E^{+1}$ (MeV) & 3.423(140) & 2.910 & 3.456 \\
$S^{+1}$ & $4.4 \times 10^{-3}$ & 9.4 $\times 10^{-3}$ & 9.4 $\times 10^{-3}$ \\
$\Gamma_{+1}$ & 0.32(8) & 3.6(2) & 7.7(3) \\
$\Gamma_{+1}$ & 0.04 & 0.02 & \\
$\alpha^2$ (%) & 33(10) & 11 & 34 \\
\end{tabular}
\end{table}

We deduced the isospin-mixing interaction matrix element $V$ as

$$V = \frac{\Delta E \times R}{1 + R^2} = 40^{+22}_{-9} \text{ keV}. \quad (2)$$

To understand the decay features and to cross-check theoretical descriptions, we have performed large-scale shell-model calculations in the full pf shell using the NUSHELLX@MSU [6] and ANTOINE [7] shell-model codes with two different charge-dependent Hamiltonians. The first one, cdGX1A, is based on the GXPF1A [8] interaction with the addition of Coulomb, strong charge-asymmetry, and charge-independence-breaking interactions from Ref. [9] and updated isovector single-particle energies from Ref. [10]. The other Hamiltonian, cdKB3G, was constructed on the basis of the KB3G interaction [11], with the addition of the Coulomb interaction and isovector single-particle energies scaled as $\sqrt{\hbar \omega (A)}$.
the IAS. A quenching factor $q_F = 0.74$ was applied to the Gamow-Teller (GT) operator to calculate the $\beta$-decay strength distribution. The theoretical half-lives were evaluated on the basis of the 20 lowest $1^+$ states due to a very large model space needed for the calculations. We note a good agreement between both sets of calculations and existing experimental data (see Fig. 1).

The experimentally known levels of $^{56}$Cu are well reproduced by the charge-dependent versions of the GXPF1A and KB3G interactions. The IAS of the $^{56}$Zn ground state is found to be the 4th (cdGX1A) and the 3rd $0^+$ state (cdKB3G) in $^{56}$Cu. Both interactions confirm that this state is mixed with a lower-lying $0^+$, $T = 1$ state by strong isospin-nonconserving (INC) matrix elements of $V_{\text{INC}} = 20$ keV (cdGX1A) and $V_{\text{INC}} = 48$ keV (cdKB3G). In first-order perturbation theory, the magnitude of mixing is proportional to $(V/\Delta E)^2$, where $V$ is the mixing matrix element and $\Delta E$ is the energy difference between the admixed states. The latter value calculated in an odd-odd nucleus may carry a significant uncertainty, which influences the amount of the isospin mixing [12].

Table II summarizes excitation energies of the five lowest $0^+$ states in $^{56}$Cu, obtained from the two interactions. For each state, different from the IAS, within a two-level mixing approximation we have also estimated an INC matrix element $(V_{\text{INC}})$ between a given state and the IAS. This can be done using Eq. (2) and a theoretical distribution of the Fermi strength, the corresponding mixing matrix element is well reproduced, the shell-model diagonalization are

<table>
<thead>
<tr>
<th>$S_p$</th>
<th>$S_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\Delta E_{\text{inc}}/\Delta E_{\text{exp}}$</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\alpha^2$</td>
<td>0.23%</td>
</tr>
<tr>
<td>$\beta^2$</td>
<td>0.23%</td>
</tr>
</tbody>
</table>

In the calculations with cdGX1A the next $0^+_1$ state comes out to be almost degenerate with the IAS, leading to a strong mixing between the two states by, however, a very small mixing matrix element of $V_{\text{INC}} = 3$ keV with respect to the experiment value. Hence, to estimate the interaction matrix elements $V_{\text{INC}}$ between the first three $0^+$ states and the IAS, we used an unperturbed $T = 2$ state.

In the same table, we also present the spectroscopic factors for an $f_{7/2}$ proton emission from the five lowest $0^+$ states in $^{56}$Cu to the $7/2^-$ ground state of $^{55}$Ni obtained from a large-scale shell-model diagonalization. It is seen that, except for the first $0^+$ state, all other states have rather small spectroscopic factors. In particular, the allowed spectroscopic factors of the $T = 1$ states admixed to the IAS by a strong mixing matrix element are small, $S_{T=1} = 4.4 \times 10^{-3}$ and $S_{T=1} = 9.4 \times 10^{-3}$ for cdGX1A and cdKB3G, respectively.

Spectroscopic factors of the IAS that come out of the shell-model diagonalization are $S_{\text{diag}}^{\text{IAS}} = 2.1 \times 10^{-3}$ (cdGX1A) and $S_{\text{diag}}^{\text{diag}} = 0.2 \times 10^{-3}$ (cdKB3G). Let us remark that these values arise from quantities of mixing different from the experimental one. A relatively large spectroscopic factor from cdGX1A is mainly due to the mixing with the closely lying $0^+_2$, as we mentioned above. A small value of the cdKB3G spectroscopic factor comes from a small amount of isospin mixing as obtained in diagonalization. Taking into account the uncertainty of the positions of the unknown $T = 1$ states we cannot use these results directly.

Therefore, we propose to estimate the spectroscopic factors for the isospin-forbidden proton emission from the IAS as about 1/3 of the $S_{T=1}$ values, based on the experimentally determined amount of mixing. This gives $S_{\text{exp}}^{\text{IAS}} = 1.5 \times 10^{-3}$ (cdGX1A) and $S_{\text{exp}}^{\text{IAS}} = 3.1 \times 10^{-3}$ (cdKB3G), shown in Table I.

Interestingly, these numbers can be compared with experimentally deduced values. To show this let us note that branching ratios $S_{\text{exp}}^{\text{IAS}} / S_{\text{exp}}^{\text{IAS}}$ as follows:

$$I_{\gamma}^{\text{IAS}} / I_{\gamma}^{\text{IAS}} = \Gamma_{\gamma}^{\text{IAS}} / \Gamma_{\gamma}^{\text{IAS}}.$$ 

The total proton width can be expressed as a single-particle proton width multiplied by the corresponding spectroscopic factor [13], i.e., $\Gamma_p = \Gamma_{\gamma} S$. The single-particle proton width has been calculated from the proton scattering cross section in a Woods-Saxon potential with the potential depth adjusted to reproduce known proton energies. Thus, if we provide the electromagnetic width of a decaying state from the shell model, this relation can be used to extract from experimental data a spectroscopic factor for an isospin-forbidden proton emission from the IAS [14]:

$$S_{\text{exp}}^{\gamma} = \Gamma_{\gamma}^{\text{IAS}} / \Gamma_{\gamma}^{\text{IAS}}.$$ 

The calculated $\gamma$ decay width of the IAS is $\Gamma_{\gamma}^{\text{IAS}} = 0.135(30)$ eV (an average from the two interactions). The $\gamma$ width of the lower $T = 1$ state (accounting for a phase-space factor when it is shifted closer to the $T = 2$ state) is about 0.03(1) eV (an average from the two interactions again). The calculated single-particle proton decay width is $\Gamma_{\gamma}^{\text{sp}} = 1040(50)$ eV. Assuming a two-level mixing and that
the $\gamma$ width of the $T = 1$ state is small compared to that of the $T = 2$ state, we can extract a proton decay width of the IAS, $\Gamma_p = 0.13(5)$ eV, and a spectroscopic factor, $S_{\text{IAS}}^{\text{exp}} = 0.13(5) \times 10^{-3}$. From the experimentally determined amount of mixing, 34(10)$\%$, the allowed spectroscopic factor for the $T = 1$ state can be estimated as $S_{\text{exp}}^{\text{th}} = 0.4(1) \times 10^{-3}$. This value is a factor of 10 smaller than the theoretical estimates, which can be partially explained by the fact that a two-level mixing model might not be fully justified and that the errors on these small spectroscopic factors of $T = 1$ states might be large. However, the main conclusion of the analysis is clear: the hindrance of the proton decay from the IAS is thus due to a very small overlap between the admixed $0^+$, $T = 1$ state of $^{56}\text{Cu}$ and the ground state of $^{55}\text{Ni}$ plus an $f_{7/2}$ proton. Proton emission from the admixed $0^+$, $T = 1$ state is allowed by the isospin quantum number selection rule; however, it is hindered by nuclear structure effects. So, a large admixture of that state to the IAS makes the proton width of the IAS comparable to but not much larger than its $\gamma$-decay width.

The properties of the $0^+$, $T = 1$ state that is admixed to the IAS are also well understood. From the experimental proton decay energy $E_p = 2683(10)$ keV, we get the proton single-particle width of this state of $\Gamma_p = 819(30)$ eV. Thus, using the experimentally deduced spectroscopic factor, we get the total proton width of $\Gamma_p = 0.32(8)$ eV (the factor of 10 difference from the theoretical width is due to the difference in spectroscopic factors, as pointed out above). The shell-model electromagnetic width is much smaller, about 0.02–0.04 eV. This result is in agreement with the fact that the electromagnetic decay of the state at 3423 keV has not been observed in the past experiments.

Finally, we have examined the GT strength distribution from $^{56}\text{Zn}$ $\beta$ decay. The GT feeding of the $1^+_2$ state is found to be around 23% (cdGX1A) and 25% (cdKB3G), perfectly matching the measured proton branching ratio of 23.8(11)$\%$. The shell model predicts a few other excited $1^+$ states below the IAS, while only two additional states have been seen experimentally [4]. The summed GT strength to those theoretical $1^+$ states is about 17% (cdGX1A) and 24% (cdKB3G), which can be compared to the experimental proton branching ratio for the $1^+_2$ states of 21.7(12)$\%$. In conclusion, we have studied $\beta$-delayed proton and $\gamma$ emission from the $0^+$, $T = 2$ pf-shell precursor $^{56}\text{Zn}$. This theoretical analysis allows us to understand the highly retarded proton decay of the IAS in $^{56}\text{Cu}$, largely mixed with a close-lying $0^+$, $T = 1$ state. The observed hindrance is due to a very small spectroscopic factor of the admixed state indicating a small overlap between this excited $0^+$ state of $^{56}\text{Cu}$ and the ground state of $^{55}\text{Ni}$ plus an $f_{7/2}$ proton. We demonstrated that the spectroscopic factor for an isospin-forbidden proton emission can be deduced with good accuracy, if the experimental data are supplemented by a simple shell-model input.

The work was supported by the CFT (IN2P3/CNRS, France), AP Théorie 2015–2016; by the Ministry of Foreign Affairs and International Development of France in the framework of PHC Xu GuangQi 2015 under Project No. 34457VA; by the U.S. National Science Foundation under Grant No. PHY-1404442; and by the National Research Foundation of South Africa under Grant No. 76898. N.B. is grateful to the University of Bordeaux for a visiting scientist grant.