Letter | Editors' Suggestion

Isospin mixing and the cubic isobaric multiplet mass equation in the lowest $T = 2, A = 32$ quintet

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The isobaric multiplet mass equation (IMME) is known to break down in the first $T = 2, A = 32$ isospin quintet. In this work we combine high-resolution experimental data with state-of-the-art shell-model calculations to investigate isospin mixing as a possible cause for this violation. The experimental data are used to validate isospin-mixing matrix elements calculated with newly developed shell-model Hamiltonians. Our analysis shows that isospin mixing with nonanalog $T = 1$ states contributes to the IMME breakdown, making the requirement of an anomalous cubic term inevitable for the multiplet.

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If nuclear isospin T were a conserved quantity, the members of an isobaric multiplet would be $(2T + 1)$-fold degenerate. However, it is known [1] that this degeneracy is broken by two-body charge-dependent interactions, which can be described at tree level as the sum of an isoscalar, isovector, and isotensor operator of rank 2. To first order, the energy spacings between the multiplet members can be obtained from the expectation value of the charge-dependent perturbation. On applying the Wigner-Eckart theorem to the perturbing Hamiltonian, the mass splittings are described by the isobaric multiplet mass equation (IMME) [2,3]

$$M(T_z) = a + bT_z + cT_z^2, \tag{1}$$

where each member of the multiplet is characterized by its isospin projection $T_z = (N - Z)/2$.

The general success of the IMME over a large mass range made it a reliable tool to address a variety of research problems. For example, it was used to test recent advances in nuclear theory [4–6], map the proton dripline [7], identify candidates for two-proton radioactivity [8,9], search for physics beyond the standard model [10], infer rapid proton capture (rp) nuclear reaction rates relevant for studies of novae and x-ray bursts [11–13], assess global nuclear mass model predictions [14] and constrain calculations relevant for CKM unitarity tests [15].

In this context, the lowest isospin $T = 2$ quintet for $A = 32$ (with spin and parity $J^\pi = 0^+$) is an interesting case. The β decay of ^{32}Ar, the most proton-rich member of the quintet, was previously used for searches of exotic scalar [10] and tensor [20] weak interactions as well as for benchmarking isospin symmetry breaking (ISB) corrections [17] important for obtaining a precise value of V_{ud}, the up-down element of the CKM quark-mixing matrix [15]. In fact, the $A = 32$ quintet is one of the most extensively studied and precisely measured multiplets to date [18,21–25]. It remains an anomalous case, for which the IMME breaks down significantly [26]. A satisfactory fit to the measured masses is only obtained with an additional cubic dT_z^3 term, with $d = 0.89(11)$ keV (cf. Table I). This is the smallest and most precisely determined violation of the IMME observed so far. Unlike other multiplets, where apparent violations of the IMME were resolved through subsequent measurements [27–32], the $A = 32$ anomaly has persisted over several years, despite high-precision remeasurements of ground state masses [21,22,33]

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TABLE I. Cubic IMME fit to measured mass excesses of the lowest $T = 2$ quintet in $A = 32$. The fit yields $d = 0.89(11)$ keV, with $P(\chi^2, \nu) = 0.95$.

Isobar	T_z	M_{exp} (keV)[a]	M_{IMME} (keV)
^{32}Ar	−2	−2200.4(1.8)	−2200.35(158)
^{32}Cl	−1	−8288.4(7)[b]	−8288.43(47)
^{32}S	0	−13967.58(28)[c]	−13967.57(25)
^{32}P	+1	−19232.44(7)[d]	−19232.43(7)
^{32}Si	+2	−24077.69(30)	−24077.69(30)

[a]Ground state masses are taken from Ref. [16].
[b]$E_x = 5046.3(4)$ keV from Ref. [17].
[c]$E_x = 12047.96(28)$ keV from Ref. [18].
[d]$E_x = 5072.44(6)$ keV from Ref. [19].

as well as excitation energies [18,28]. A recent compilation [26] showed the $A = 32$ quintet to be a unique case, in which the χ^2 value for a cubic fit yields 95% probability that it is the correct model to describe the data. Since there are no known fundamental reasons that preclude a cubic IMME term, it is interesting that the magnitude of the extracted d coefficient for this case agrees well with theoretical estimates that used a simple nonperturbative model [34] or a three-body second-order Coulomb interaction [35], both of which allow a nonvanishing cubic term, with $|d| \approx 1$ keV. Alternatively, the role of isospin mixing with nonanalog 0^+ states was also theoretically investigated in the recent past [24,25].

We delve into the above aspect here, via an analysis of high-resolution experimental data and a comparison with calculations that use recently developed shell-model Hamiltonians [36]. For the former, we mainly rely on data from a previous ^{32}Ar β decay experiment at CERN-ISOLDE [10], that acquired β-delayed protons from unbound states in the daughter ^{32}Cl ($S_p \approx 1581$ keV) with high resolution (full widths at half-maximum of ≈6 keV). The primary goal of the ISOLDE experiment was to search for scalar currents in the weak interaction, by determining the $\beta\nu$ angular correlation ($a_{\beta\nu}$) for the decay, via a precise analysis of the shape of the superallowed β-delayed proton peak [10]. Part of the proton spectrum is shown in Fig. 1.

The high-resolution nature of the ISOLDE data allow an identification of potential isospin admixtures to the $T = 2$ isobaric analog state (IAS) in ^{32}Cl. The nature of each β transition is encoded in the shapes of the proton groups, which would be different if the transitions were Fermi ($0^+ \rightarrow 0^+$), with $a_{\beta\nu} = 1$ or Gamow-Teller ($0^+ \rightarrow 1^+$), with $a_{\beta\nu} = -1/3$. We analyzed these data using the R-matrix formalism described in Refs. [37,38]. In the analysis, the proton peaks were grouped as p_0, p_1, p_2, or p_3 depending on whether the proton emission left the residual ^{31}S nucleus in its ground state or any of its first three excited states at 1249, 2234, and 3077 keV (see Fig. 9 in Ref. [17]). Interference was allowed between all levels that had the same quantum numbers, transition type (Fermi or Gamow-Teller), and final states in ^{31}S. The R-matrix fits folded in the detector response function and lepton recoil effects (described in Ref. [10]), and were parameterized using various J^π values for the daughter ^{32}Cl states and associated $a_{\beta\nu}$ coefficients. The fits yielded relative intensities, ^{32}Cl excitation energies and intrinsic widths. They were repeated for different values of $a_{\beta\nu}$, spin-parity combinations and p_0, p_1, p_2, p_3 assignments for the daughter levels to obtain best agreement with experimental data. A few important features of the analysis are described below.

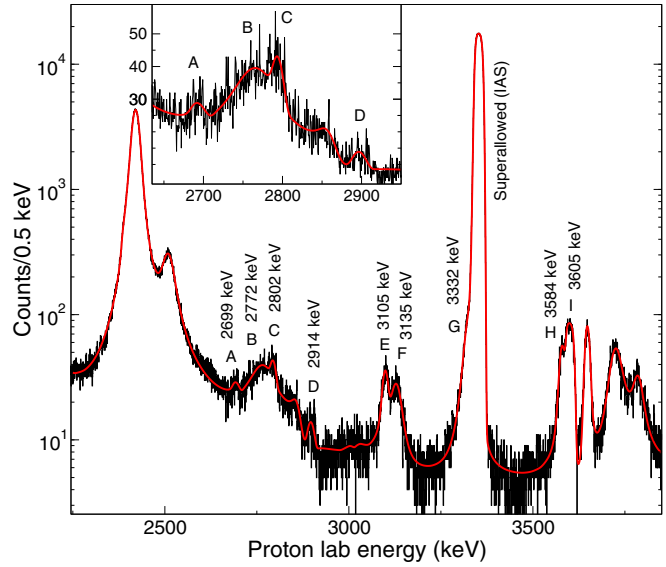

FIG. 1. ^{32}Ar β-delayed proton spectrum from the ISOLDE experiment [10] and its corresponding R-matrix fit. The inset shows a magnified portion of the spectrum.

Peaks C, E, and H were assumed to be from the p_1 group. These assignments were based on data reported by independent ^{32}Ar β-delayed proton-γ coincidence measurements [17,39]. We observe that a reasonably good R-matrix fit is attained (Fig. 1) with the parameters listed in Table II. The fit assumes that peak B arises from a Fermi transition, while the others (apart from peak I) are exclusively from Gamow-Teller decays. Based purely on χ^2 values from independent fits, peak I could be either from a Fermi or Gamow-Teller decay.

We compared these results with ^{32}S(^{3}He, t) data that were independently obtained at the MLL tandem accelerator

TABLE II. R-matrix fit results for the ISOLDE data. I_p^{rel} is the intensity relative to the p_0 superallowed proton group. The last column lists corresponding states observed via the ^{32}S(^{3}He, t) reaction.

		^{32}Ar β decay [10]				^{32}S(^{3}He, t)
Peak	Group	$a_{\beta\nu}$	E_x (keV)	Γ (keV)	I_p^{rel} (%)	E_x (keV)
A	p_0	−1/3	4366(4)	<1	0.23(3)	4356(5)
B	p_0	1	4443(3)	77(15)	0.8(1)	...
C	p_1	−1/3	5721(4)	11(3)	0.10(3)	...
D	p_0	−1/3	4588(4)	30(4)	0.20(3)	4584(5)
E	p_1	−1/3	6034(2)	13(3)	0.14(2)	..
F	p_0	−1/3	4817(2)	26(5)	0.26(3)	4815(5)
G	p_0	−1/3	5020(2)	21(2)	0.49(6)	5020(5)
H	p_1	−1/3	6530(2)	10(3)	0.25(3)	...
I	p_0	−1/3 or 1	5302(2)	⩽1	0.45(4)	...
			$\chi^2/\nu = 0.80$			

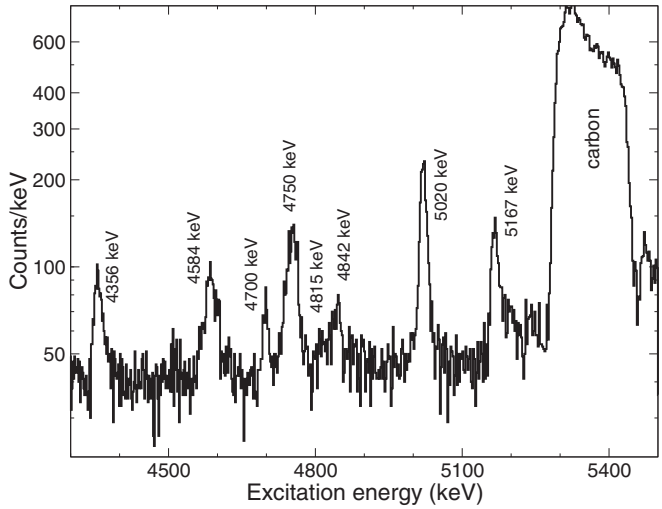

FIG. 2. Triton spectrum from ^{32}S(^{3}He, t) at $\theta_{\rm lab} = 10^\circ$.

facility in Garching, Germany. The experiment used ≈300 enA of 33 MeV ^{3}He^{++} ions, incident on a 120 μg/cm^2-thick natural ZnS target. The tritons exiting the target were momentum analyzed using the high-resolution Q3D magnetic spectrograph [40,41]. A sample triton spectrum in the energy range of interest is shown in Fig. 2. These data provided an important confirmation of the p_0 assignments for peaks A, D, F, and G in our R-matrix analysis. Additionally, since the ^{32}S(^{3}He, t) charge-exchange reaction predominantly populates $J^\pi = 1^+$, $T = 1$ levels at forward angles,[1] the states observed at these energies in both ^{32}Ar β decay and the ^{32}S(^{3}He, t) reaction can be ruled out as sources of $J^\pi = 0^+$ isospin impurity. This comparative analysis leaves only the 4443 and 5302 keV levels (cf. Table II) as potential admixed states. We find from the β decay data that the p_1 intensity for the latter is around 1.2 times larger than its p_0 group. In comparison, the p_1 intensity for the IAS is roughly 80 times smaller than the p_0. This is due to the low penetrability of $l = 2$ protons from the $J^\pi = 0^+$ IAS. The above discrepancy makes it highly unlikely for the 5302 keV state to have spin-parity 0^+, which rules it out as a source of isospin mixing.

We next used the measured β-delayed proton intensities in Table II, together with shell-model calculations of isospin mixing to investigate the matter further. For the latter we used newly developed isospin nonconserving (INC) USDC and USDI interactions, described extensively in Ref. [36]. The INC parameters in the new USD Hamiltonians were obtained from a fit to several mirror displacement energies and stringently tested via a comparison with experimental data [36]. The isospin-mixing matrix elements calculated with these Hamiltonians were robustly validated [36] with results from independent high-precision ^{31,32}Cl β decay experiments [42–44], where large isospin-mixing in the daughter ^{31,32}S states were observed. More recently, such calculations were used together with a ^{32}Ar β decay measurement [39], that acquired valuable proton-γ coincidence data, albeit with lower proton energy resolution. Reference [39] identified two possible sources of $T = 1$ isospin mixing at 4799 and 4561 keV. However, their measured proton branches were significantly lower than calculated values. We show below that the higher-resolution ISOLDE data justifies ruling out these proposed levels, while providing a viable alternative for the admixed $T = 1, 0^+$ state, which is consistent with both theory predictions as well as experimental observations.

TABLE III. Calculated energy differences between the $T = 2$ IAS and the nearest 0^+, $T = 1$ state in ^{32}Cl, ^{32}S, and ^{32}P. The isospin mixing matrix element in ^{32}Cl is listed for comparison.

	ΔE (keV)			v (keV)
Interaction	^{32}Cl	^{32}S	^{32}P	^{32}Cl
USDC	−226	−186	−237	40
USDI	−308	−266	−326	41
USDCm	−324	−239	−293	46
USDIm	−405	−321	−383	47
USDB-CD	−440	−378	−427	22
Expt (this work)	−603			39.0(24)

Our shell-model calculations show that the isospin mixing within the $T_z = 1$, 0, and −1 members of the quintet occurs primarily with a single $T = 1$ state, located a few hundred keV below the $T = 2$ IAS in each isobar. The results are summarized in Table III, which lists the energy differences ($\Delta E = E_i - E_{\rm IAS}$) between the admixed $T = 1$ and $T = 2$ states for each nucleus, and the calculated isospin-mixing matrix element (v) for ^{32}Cl. The evaluated mixing matrix elements for each of the three nuclei are plotted in Fig. 3. We note that the mixing matrix elements obtained with the older USDB-CD interactions [45] are nearly a factor of two smaller than the ones obtained with the newer interactions, for all three isobars. This is consistent with previous observations for ^{31,32}Cl β decay [36].

The predicted $J^\pi; T = 0^+; 1$ level in ^{32}Cl can be identified by obtaining an experimental value of v from the data in Fig. 1

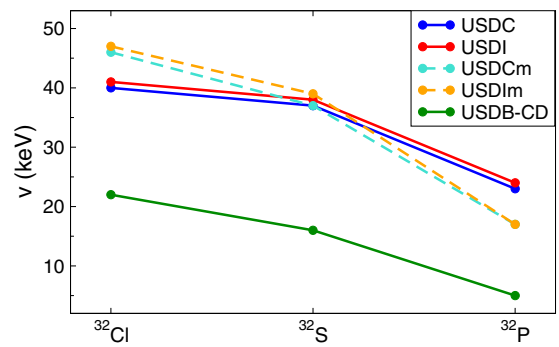

FIG. 3. Evaluated isospin mixing matrix elements (v) using various interactions.

[1]This assumes no anomalous isospin-mixing mechanisms within ^{32}S.

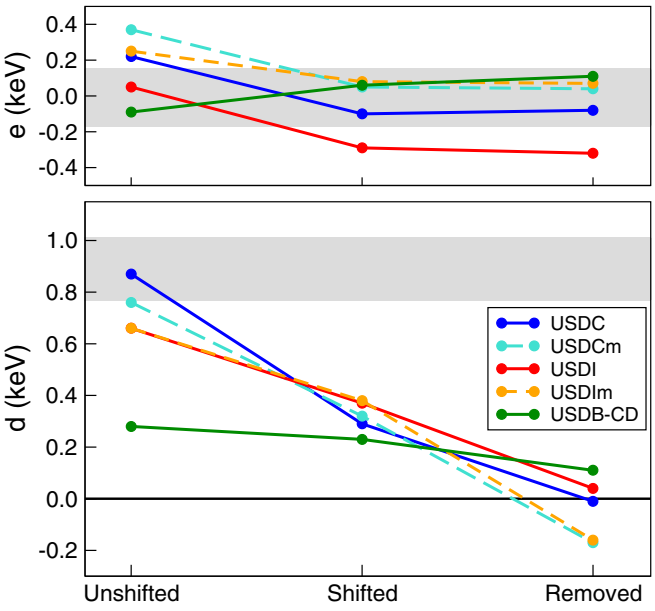

FIG. 4. Extracted cubic and quartic coefficients. The three groups of results are obtained (i) at face value, (ii) by shifting the energies of the $T = 2$ states in ^{32}Cl, ^{32}S and ^{32}P to match the 603 keV energy difference observed in ^{32}Cl, and (iii) on removing the $T = 1$ isospin mixing. The shaded areas correspond to experimental values.

and Table II. For two-state mixing, $v_{\rm expt}$ is simply

$$v_{\rm expt} = \Delta E_{\rm expt}\left[\frac{B(F)_{\rm admix}}{B(F)_{\rm SA}}\right]^{1/2}, \tag{2}$$

where the ratio in the square bracket is the (Fermi) strength to the admixed $T = 1$ state, relative to the superallowed decay. This is easily determined from the measured $I_p^{\rm rel}$ values in Table II, the ratio of calculated phase-space factors, a small ISB correction [17] and the p_0 contribution to the total superallowed intensity. On applying this prescription to the only candidate 0^+ level at 4443 keV, we obtain a $v_{\rm expt} = 39.0(24)$ keV, in excellent agreement with the calculations. The results in Table III, together with our aforementioned observations and the experimental values listed in Table II allow a credible identification of the 4443 keV level as the predicted admixed $T = 1$ state. The discrepancy between theory and experiment for ΔE should not be surprising, given the ≈150 keV root-mean-square (rms) deviation for energies in USD interactions [36].

We next investigated additional cubic (dT_z^3) and quartic (eT_z^4) terms to the IMME due to such isospin mixing. One can determine the exact solutions for the d and e coefficients by modifying Eq. (1) to incorporate such terms, such that

$$\begin{aligned} d &= \tfrac{1}{12}(M_2 - 2M_1 + 2M_{-1} - M_{-2}) \quad \text{and} \\ e &= \tfrac{1}{24}(M_2 - 4M_1 + 6M_0 - 4M_{-1} + M_{-2}), \end{aligned} \tag{3}$$

where the M_{T_z} are isobar masses in the quintet. The results for d and e using the calculated values of v and ΔE are shown in Fig. 4, and labeled as "unshifted". We repeated these evaluations by shifting the $T = 2$ states in ^{32}Cl, ^{32}S, and ^{32}P by the amount needed to reproduce our experimentally determined 603 keV energy difference in ^{32}Cl. The same ΔE was used for the three isobars due to the lack of similar experimental information for ^{32}S and ^{32}P. The shifts were accomplished by adding a T^2 term to the Hamiltonian that shifts the $T = 2$ states relative to the others, without changing the isospin mixing. As evident in Fig. 4, the shifts mildly affect the e coefficient (due to changes in the $T = 0$ mixing with the IAS in ^{32}S), but significantly decrease the calculated d coefficient to ≈0.3–0.4 keV for the new interactions. The single-state contributions from $T = 0$ and $T = 1$ levels are

$$\begin{aligned} d_i &= -\tfrac{1}{6}s_{\rm P} + \tfrac{1}{6}s_{\rm Cl} \\ e_i &= -\tfrac{1}{6}s_{\rm P} + \tfrac{1}{4}s_{\rm S} - \tfrac{1}{6}s_{\rm Cl}, \end{aligned} \tag{4}$$

where $s = -v^2/\Delta E$ is the shift in each IAS due to two-state mixing. Thus, one can remove the $T = 1$ mixing contribution for further investigation (labeled as "removed" in Fig. 4). We observe that on doing so, the extracted coefficients are mostly consistent with zero. The negative e coefficient from the USDI calculation is due to mixing with a $T = 0$ state in ^{32}S. However such $T = 0$ mixing would not explain the nonzero d coefficient required for the quintet, as evident from Eq. (4).

The above analysis validates the contention that isospin mixing with predicted $T = 1$ levels necessitates a small cubic term for the multiplet. Our extracted d coefficients for the shifted calculations from different USDC and USDI Hamiltonians agree reasonably well with one another, but are smaller than the experimental value $d = 0.89(11)$ keV, from Table I.

As further tests of our calculations, we also evaluated amplitudes for isospin-forbidden proton emission from the two admixed $J^\pi = 0^+$ levels in ^{32}Cl and the effect of the $T = 1$ isospin mixing on the superallowed Fermi decay of ^{32}Ar. Unlike the energy shift of the $T = 2$ IAS in ^{32}Cl, which is predominantly from isospin mixing with the predicted 0_2^+ $T = 1$ state below the IAS, isospin-forbidden proton emission from the IAS depends on $T = 1$ mixing with a large number of states in ^{32}Cl and isospin mixing within ^{31}S, which is dominated by mixing of the lowest $T = 3/2$ state into its ground state.

We calculated proton widths for p_0 and p_1 transitions from the 0_2^+ admixed $T = 1$ state and the $T = 2$ IAS in ^{32}Cl. The widths were evaluated using the simple expression

$$\Gamma_{\rm th} = (C^2S)(32/31)^2\Gamma_{\rm sp}, \tag{5}$$

where the $(32/31)^2$ factor is a center-of-mass correction [46], the C^2S are the shell-model spectroscopic factors, and $\Gamma_{\rm sp}$ are single-particle proton widths. Similar to Ref. [24], the $\Gamma_{\rm sp}$ were calculated from $p + ^{31}$S scattering on potentials obtained with an energy-density functional calculation with the Skx Skyrme-type interaction [47]. On the other hand, the measured proton widths [17] for the $T = 2$ IAS are known to be 18.2(5) eV and 0.233(7) eV for the p_0 and p_1 protons respectively. Together with the calculated single-particle proton decay widths, we use these results to obtain experimental values for the decay amplitudes $A = (C^2S)^{1/2}$. These are simply determined from the relation $\Gamma_{\rm expt} = A_{\rm expt}^2(32/31)^2\Gamma_{\rm sp}$. The

TABLE IV. Calculated proton emission amplitudes from states in ^{32}Cl, compared with experiment. The last column lists calculated isospin mixing corrections for ^{32}Ar superallowed Fermi decay.

	Proton emission amplitudes (A)			
Interaction (shifted calculation)	$T=2$ (p_0)	$T=2$ (p_1)	$T=1$ (p_0)	$\delta_C^{\rm cm}$ (%)
USDC	0.011	0.022	0.21	0.55
USDI	0.011	0.031	0.19	0.58
USDCm	0.0052	0.031	0.21	0.15
USDIm	0.0043	0.031	0.19	0.70
USDB-CD	0.0024	0.017	0.21	0.15
$\Gamma_{\rm sp}$ (keV)	990	17.5	590	
$\Gamma_{\rm expt}$ (keV)	0.0182(5)[a]	0.000233(7)[a]	77(15)[b]	
$A_{\rm expt}$	0.0041(1)[a]	0.0035(1)[a]	0.34(4)[b]	

[a]From Ref. [17].
[b]This work, using data from Ref. [10].

results for $A_{\rm expt}$ are shown in Table IV and compared with theory predictions, obtained using the shifted calculations.[2] We observe reasonable agreement between theory and experiment, except for the $T=2$ p_1 transition, whose calculated amplitudes are found to be much larger.

Finally, we also provide isospin-symmetry-breaking (ISB) corrections for ^{32}Ar $\rightarrow$ ^{32}Cl superallowed Fermi decay, due to the isospin mixing in ^{32}Cl. The $T=2$ $\rightarrow$ $T=2$ superallowed strength is reduced by a factor $(1-\delta_C)$, where δ_C is the total ISB correction [15]. Such corrections play a critical role in testing the unitarity of the CKM matrix and placing important constraints on beyond the standard model (BSM) physics [15]. The ISB correction is generally expressed as a sum of two separate contributions, $\delta_C = \delta_C^{\rm cm} + \delta_C^{\rm ro}$ [17], from configuration mixing and a overlap mismatch between the parent and daughter radial wave functions. The former are known to be very sensitive to the details of the model calculation [15]. Our calculated results for $\delta_C^{\rm cm}$ (from the $T=1$ mixing in ^{32}Cl) are listed the final column of Table IV. It may be noted that for the shifted USDCm and USDIm calculations, which show best agreement with the measured $T=2$ p_0 amplitude, we obtain $\delta_C^{\rm cm} = 0.15\%$ and 0.70%, respectively. From a previous evaluation of $\delta_C^{\rm ro} = 1.4\%$ [17], these yield $\delta_C = 1.6\%$ and 2.1%, in agreement with the experimentally extracted value, $\delta_C^{\rm expt} = 2.1(8)\%$ [17].

In summary, we used high-resolution experimental data to validate newly developed shell-model calculations of isospin mixing in ^{32}Cl. This analysis is used to investigate the observed IMME violation in the first $T=2$, $A=32$ quintet. We show that isospin mixing with shell-model-predicted $T=1$ states below the IAS necessarily result in a breakdown of the IMME, leading to the requirement of a small cubic term. However, this alone cannot explain the magnitude of the experimental d coefficient in Table I. Experimental investigations of intruder 0^+ levels, isospin mixing in ^{32}S and ^{32}P, continuum coupling of the proton unbound states in ^{32}Cl, and further mass measurements may be useful in this regard.

Our observations pertaining to ^{32}Ar $\rightarrow$ ^{32}Cl superallowed Fermi decay may also be useful to benchmark theory calculations [17] of model-dependent ISB corrections that are important for top-row CKM unitarity tests [15]. This is particularly relevant in light of recent evaluations of radiative corrections [48] that show an apparent violation of CKM unitarity at the $>3\sigma$ level [49,50].

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[2]For example, for the shifted USDCm calculation, the calculated amplitude can be decomposed as $A = 0.0041\,(0_1^+;\,T=1) + 0.0159\,(0_2^+;\,T=1) + 0.0009\,(0_3^+;\,T=1) + 0.0009$ (all other $T=1$) $-0.0166\,(^{31}{\rm S};\,T=3/2) = 0.0052$. The USDC result has a larger amplitude, mainly due to a 50% smaller destructive contribution from the $T=3/2$ state in ^{31}S.

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