New Magic Nuclei Towards the Drip Lines

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Abstract

The predicted and experimental properties of new doubly-magic nuclei ^{22}O and ^{24}O are discussed. These together with previous observations lead to a new rule for magic numbers: if there is an oscillator magic number (2, 8, 20 or 40) for one kind of nucleon, then the other kind of nucleon has a magic number for the filling of every possible (n,ℓ,j) value. Predictions for the calcium isotopes are also mentioned.

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Doubly-magic nuclei are crucial for the understanding of nuclear structure. The vacuum of free nucleons can be used to calculate properties of nuclei up to about A=10 [1]. But we rely upon the effective vacuum provided by the doubly-magic nuclei to understand the properties of all heavier nuclei. This talk will focus on the predictions and observations of new doubly-magic nuclei - those that have a magic number for both protons and neutrons.

The magic nuclei are understood in terms of an effective mean field and its associated shell gaps. Starting with the mean-field basis, a shell-model calculation is based upon a subset of single-particle states together with their interaction via an effective two (or more) body hamiltonian. In many cases the single-particle energies can be inferred from experiment. For such calculations each doubly-magic nucleus provides a "reset vacuum" in the sense that the input depends on the specific single-particle and residual interaction properties observed for each case. The experimental energies for all magic nuclei can be semi-quantitatively described in terms of Hartree-Fock models [2]. These models are necessary for the extrapolation of single-particle properties for nuclei far from stability.

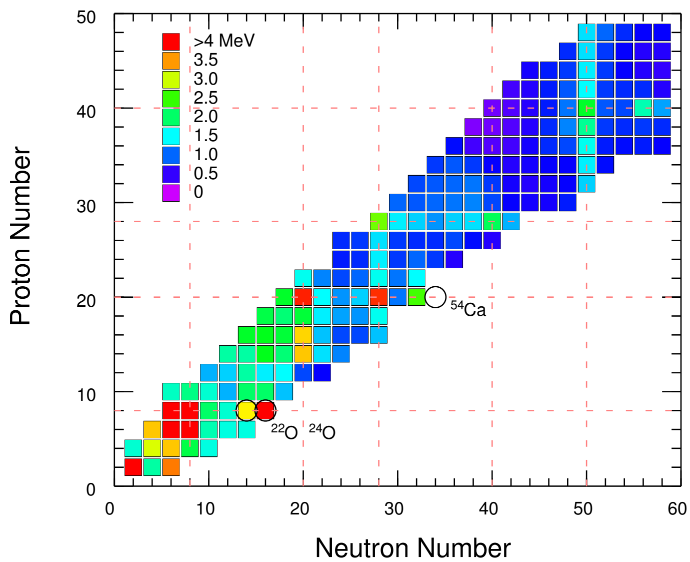

Figure 1: Top: Excitation energy of 2^+ states in even-even nuclei.

The closed-shell property of the doubly-magic nuclei provides a good zeroth-order wave function which can be systematically improved upon by using perturbation theory. The effective two-body interactions and electromagnetic operators can be based upon the free-nucleon properties corrected by perturbative calculations of the core-breaking contributions. Starting with a zero-particle zero-hole (0p-0h) closed shell, the structure of the closed shells themselves can be systematically improved by the mixing of 2p-2h, 3p-3h etc. The renormalized G matrix provides a starting point for the two-body hamiltonians, but the agreement with experiment can often be greatly improved by empirical modifications of the two-body matrix elements. I will discuss examples for the sd and pf shells.

The experimental signatures of magic numbers are the presence of a relative high energy for the first-excited state in even-even nuclei (usually 2^+) at the magic number, and a discontinuity in the one- and two-nucleon separation energies. These shell-gap effects in the binding energies give rise to the dramatic isotopic fluxuations in the r-process element abundances (they are related to shell effects in very neutron-rich nuclei which have not yet been studied experimentally). In this talk I will focus on neutron-rich nuclei that have recently been discovered to have highly excited 2^+ states. I show in Fig. 1 the excitation energies of 2^+ states in nuclei with $Z < 50$. The characteristic feature of most doubly-magic nuclei is an excitation energy for the 2^+ state which is significantly higher than all of the neighboring even-even nuclei. For those nuclei beyond $Z \geq 50$ (not shown in Fig. 1) there are only two such nuclei, ^{132}Sn and ^{208}Pb. There are many more such nuclei with $Z < 50$ in Fig. 1 which will be discussed here.

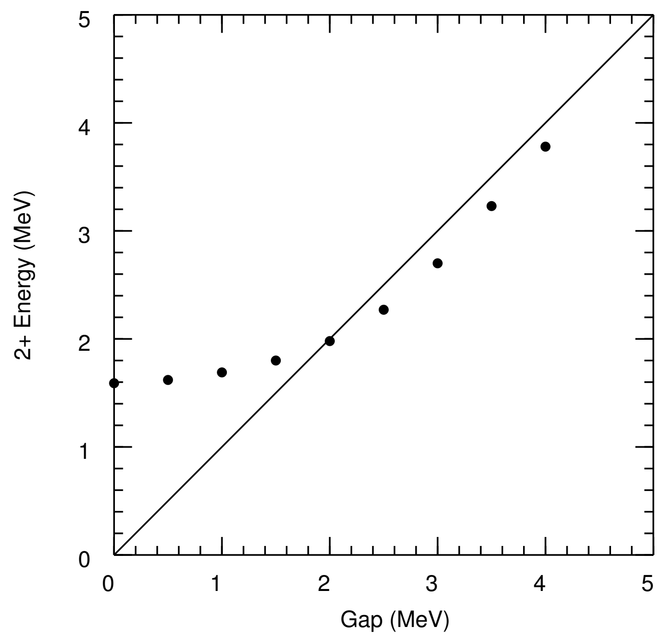

Figure 2: Excitation energy of the 2^+ state in ^{48}Ca as a function of the shell gap.

I start with a model calculation which illustrates the dependence on the 2^+ excitation energy on the shell gap. A full space calculation for spectra of the even-even nuclei with neutrons in the four orbits $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ is carried out with the Kuo-Brown G matrix [3] for the residual two-body interaction. Initially all single-particle energies are made degenerate, and then the gap between the $0f_{7/2}$ orbit and the other three is increased from 0 up to 5 MeV. The results for the energy of the 2^+ state in ^{48}Ca are shown in Fig. 2. For a small gap the 2^+ excitation energy is rather constant and is determined by the residual interaction (mainly the pairing part). As the gap is increased above a critical value of about 2 MeV, pairing is broken and the energy of the 2^+ state increases linearly with the gap, but is always lower than the gap due to the particle-hole interaction energy.

I will focus on the discussion of the two nuclei circled in Fig. 1, ^{22}O and ^{24}O, which were predicted about twenty years ago to be new doubly-magic nuclei, but only recently been observed experimentally. I will also discuss predictions for ^{54}Ca. The accumulation of data on magic nuclei leads to a new rule for magic numbers.

The comparisons to theory presented in this section are based upon the $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$ (sd) model space for $A-16$ neutrons outside of a closed core for ^{16}O. I will compare the results with two hamiltonians. Both of these start with the three single-particle energies as determined from the spectrum of ^{17}O. The first is a "no-parameter" calculation based on the renormalized Kuo-Brown G matrix [3] represented by 30 two-body matrix elements (TBME) with T=1. The second is the "universal sd" (USD) hamiltonian [4], [5] which consists of empirical values for these

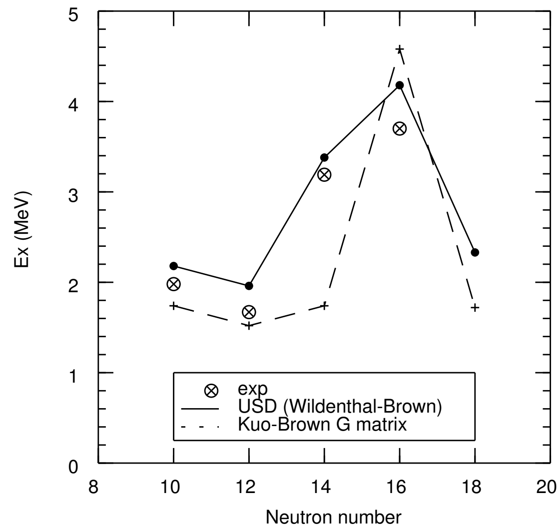

Figure 3: Excitation energy of 2^+ states in the oxygen isotopes. The experimental value for $N = 16$ (^{24}O) is a lower limit.

30 TBME. The values of the TBME were obtained from a least squares fit to 447 binding energy data in the mass region A=16-40 as they were known up to year 1983. The USD energy levels for all of the sd-shell nuclei are given in [6] (the levels in this database labeled by an asterisk are those used for the least-squares fit). Of particular importance for this work are the constraints placed on the $T = 1$ matrix elements by the oxygen data known in 1983 - the ground state binding energies of $^{18-21}$O, the excitation energy of the 2^+, 4^+ and 3^+ states in ^{18}O, seven excited states in ^{19}O, and three excited states in ^{20}O. The results I discuss for $^{21-24}$O are predictions of this USD interaction. Experimental and calculated excitation energies for the 2^+ states in the oxygen isotopes are shown in Fig. 3.

In order to understand the results it is useful to use the effective single-particle energies (ESPE) for the three neutron orbits. These are the bare single-particle energies (those for one neutron plus ^{16}O as observed in ^{17}O) plus the addition of the monopole part of the diagonal TBME. The monopole interaction contribution is the $(2J + 1)$ weighted average of the diagonal TBME. The ESPE for the configurations ^{16}O, ^{22}O [$d^6_{5/2}$], ^{24}O [$d^6_{5/2}s^2_{1/2}$] and ^{28}O [$d^6_{5/2}s^2_{1/2}d^4_{3/2}$] with the G-matrix and USD interactions are shown in Fig. 4. For USD the $d_{5/2}$-$s_{1/2}$ gap starts out at 0.8 MeV for ^{16}O and increases to 4.3 MeV in ^{22}O. This gap is large enough to make ^{22}O a doubly-magic nucleus (e.g. the shell gap is much larger than the pairing gap). In addition there is a 4.5 MeV gap between the $s_{1/2}$ and $d_{3/2}$ orbits that also makes ^{24}O a doubly-magic nucleus. With the G-matrix interaction the gap $d_{5/2}$-$s_{1/2}$ gap does not appear and ^{22}O is not doubly magic.

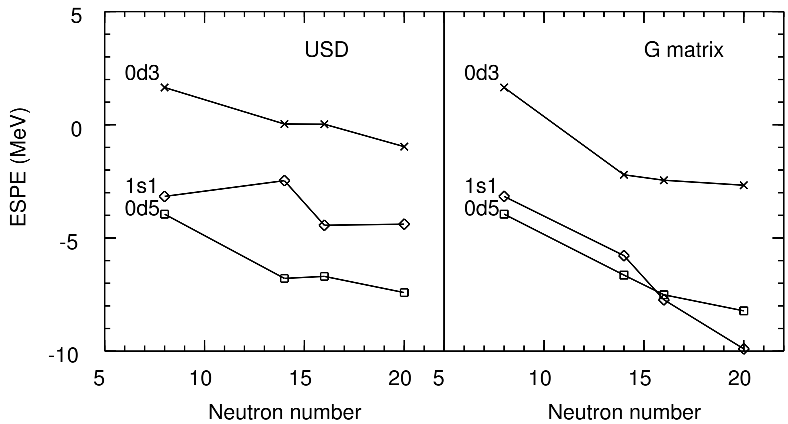

Figure 4: Effective single-particle energies for the oxygen isotopes.

The closed-shell nature of ^{22}O and ^{24}O means that the structure of $^{21-25}$O can be interpreted in terms of their dominate components relative to the "closed-shell" $0p-0h$ structure for the ground states of ^{22}O (with a filled $d_{5/2}$ orbit) and ^{24}O (with filled $d_{5/2}$ and $s_{1/2}$ orbits). There is a dramatic difference between the G-matrix and USD in the properties of ^{22}O. With USD the sd-shell wave function for the ^{22}O ground state is 77% $0p-0h$, 22% $2p-2h$ and 1% $4p-4h$ (typical of a quickly converging sequence for a doubly-magic nucleus) and the 2^+ energy is 3.38 MeV. The G-matrix ground state is much more complicated: 29% $0p-0h$, 61% $2p-2h$, 3% $3p-3h$ and 7% $4p-4h$, and with a 2^+ energy of 1.71 MeV.

The doubly-magic nature of ^{22}O predicted by USD was first confirmed by a radioactive beam Coulomb excitation experiment at the NSCL [7] where a 2^+ state was observed at 3.17 MeV with a E2 transition strength are in agreement with USD [7]. Relative to the dominant $0p-0h$ ground state, the excited states in ^{22}O are $1p-1h$ with one hole in the $d_{5/2}$ orbit. The lowest of these are the 2^+ and 3^+ states which are dominated by the one-particle in the $s_{1/2}$ orbit. Recently more detail has been found for the spectrum of ^{22}O from the prompt gamma-ray spectrum measured at GANIL [8]. In particular, another excited state at 4.6 MeV was observed whose energy and decay properties are in excellent agreement with the predicted 3^+. The energies of the 2^+ and 3^+ are split by the residual $1p-1h$ interaction. Relative to the ESPE gap of 4.3 MeV the 2^+ state is pushed down to 3.4 Mev and the 3^+ state is pushed up to 4.8 MeV. The $(2J+1)$ weighted average of 4.2 MeV is close to the ESPE gap. Higher energy states were also observed in the gamma decay which are consistent with $2p-2h$ states predicted at 6.5-6.9 MeV. Thus the spectrum of ^{22}O turns out to be perhaps the simplest example which exists in nuclei of a doubly-closed shell and excited states of just one kind of nucleon (neutrons in this case).

The first information on excited states ^{21}O was found in 1989 with the ^{18}O(^{18}O,^{15}O) reaction [9]. A much more complete level scheme and its gamma decay properties has recently been observed [8]. Relative to the dominant $0p-0h$ configuration for the ^{22}O ground state, the structure of ^{21}O can be simply interpreted in terms of the $0p-1h$ $5/2^+$ ground state and the $1p-2h$ excited states based upon coupling of the $s_{1/2}$ (particle) orbit to the $(d_{5/2})^{-2}$ $2h$ state 0^+, 2^+ and 4^+ to give the ^{21}O excited states $1/2^+$, $(3/2^+,5/2^+_2)$ and $(7/2^+,9/2^+)$, respectively. This accounts for all of the shell-model levels (in the full space calculation) up to 4.7 MeV and all of the levels observed in the recent experiment [8]. The predicted gamma decay properties of $^{20-22}$O given in [10] are in good overall agreement with the new experiments [8]. The decay of the $9/2^+$ state is particularly interesting. It is unbound to $\ell = 4$ neutron emission to the ^{20}O ground state by 1.1 MeV, but its gamma decay is observed. In the sd-shell this $\ell = 4$ transition is forbidden, and the gamma decay puts a limit on the spectroscopic factor which may arise from mixing from the *sdg* major shell. The calculated gamma-decay lifetime is 57 fs [10]. The single-particle width for an $\ell = 4$ decay obtained from a typical Woods-Saxon potential is 0.31 keV or a lifetime of 0.0020 fs. Thus the spectroscopic factor for the $\ell = 4$ decay of the $9/2^+$ state must be less than about 10^{-4}. I have used a much larger $2\hbar\omega$ model space with the WBP interaction [11] to include the *sdg* major shell in the $9/2^+$ wave function (the J-scheme matrix dimension is about 33,000). The $g_{9/2}$ occupancy comes out to only 0.0002 in this calculation.

Relative to the $0p-0h$ model for ^{22}O, the ^{23}O levels are $1p-0h$ levels and $2p-1h$. Of these only the lowest $1p-0h$ $s_{1/2}$ $(1/2^+)$ is predicted to be bound and this agrees with the present experiment. The first excited state $5/2^+$ state at 2.72 MeV is the lowest state which is dominated by $2p-1h$. This is predicted to be very near the one-neutron decay threshold. No gamma transitions are found experimentally in ^{23}O [8] indicating that there are no excited bound states, consistent with the USD theory. The properties of this unbound state could be studies in a ^{24}O one-neutron knockout experiment as a sharp low-energy peak in the neutron decay spectrum. The cross section and momentum distribution observed for one-neutron knockout from the ^{23}O ground state [12] are in agreement with theory [13].

The $1p-0h$ state which is dominated by $d_{3/2}$ is the $3/2^+$ predicted to be at 3.28 MeV and thus also unbound to neutron decay. The importance of the high (unbound) energy for the $d_{3/2}$ orbit is that all of the nuclei beyond ^{24}O where one or more nucleons goes into the $d_{3/2}$ orbit are unbound - ^{24}O is at the drip line. The strong proton-neutron interaction [4], [14] between the $d_{5/2}$ proton and the $d_{3/2}$ neutron lowers the ESPE of the $d_{3/2}$ by about 1.2 to 2.0 MeV (depending on the J coupling) for the fluorine isotopes which gives enough extra binding to make fluorine bound out to ^{29}F. The fact that fluorine is known to be bound at least out to ^{31}F is an indication that at least one of the pf-shell orbits also becomes effectively bound by the interaction with the $d_{5/2}$ proton.

The $s_{1/2}$ state which is relatively strongly bound in ^{23}O fills to make another doubly-magic nucleus for ^{24}O. Relative to a dominant $0p-0h$ wave function for the

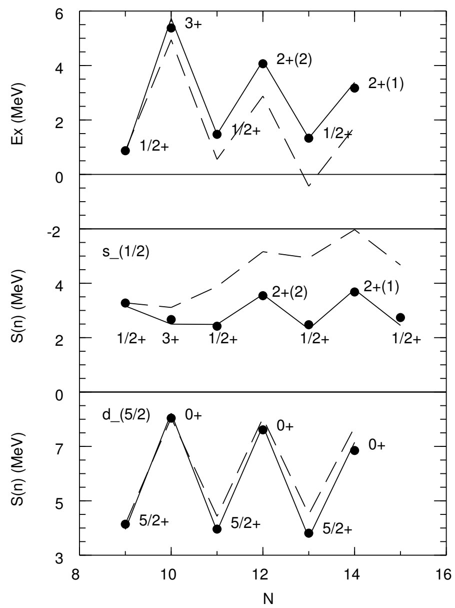

Figure 5: Experimental energies (filled circles) compared with G-matrix (dashed lines) and USD (solid lines) calculations.

^{24}O ground state $[d^6_{5/2}s^2_{1/2}]$, the excited states are $1p-1h$; $d_{3/2}$-$s^{-1}_{1/2}$ with $J=1^+$ and 2^+ and $d_{3/2}$-$d^{-1}_{5/2}$ with $J=1^+$ to 4^+. Of these the lowest is the 2^+ predicted to be at 4.18 MeV and is close to the neutron-decay threshold of 3.7(3) MeV. The nonobservation of excited states in ^{24}O indicates that the 2^+ state is above 3.4 MeV and consistent with theory. Beyond ^{4}He, ^{24}O is the most tightly bound nucleus with no bound excited states.

Energies of excited states in $^{18-20}$O which have a large component with one neutron in the $s_{1/2}$ orbit were important in the determination of the USD matrix elements which are essential for the theoretical extrapolations to $^{21-23}$O [15]. There are key early experiments which determined the ℓ=0 properties of these states such as ^{16}O(d,p)^{17}O [16], ^{17}O(d,p)^{18}O and ^{18}O(d,p)^{19}O [17], ^{17}O(t,p)^{19}O [18], and ^{18}O(t,p)^{20}O [19]. The properties of the key excited states are shown in Fig. 5. The top panel shows the excitation energy of states whose dominant shell-model configuration is $[d^{(n-1)}_{5/2}s_{1/2}]$ with $n=A-16$. The middle panel shows the one-neutron separation energy $S(n)$ for these excited states, while the bottom panel shows $S(n)$ for the ground state. For the $s_{1/2}$ states one observes a linear divergence of the G-matrix from experiment so that already by $N=12$ (^{20}O) with data known up to 1983 the failure of the G-matrix was evident and its correction with the USD interaction was made. Thus we find that the successful shell-model extrapolations to the drip lines are determined by the correct description of excited states in nuclei near stability that are related to orbitals which will become filled further from stability.

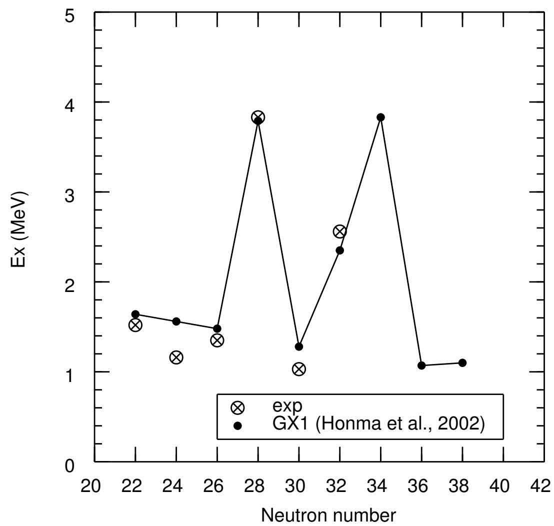

Figure 6: Excitation energy of 2^+ states in the calcium nuclei.

In fact there are only two specific matrix elements which are important for difference in Fig. 5; $< (d_{5/2}, s_{1/2}); J, T = 1 \mid V \mid (d_{5/2}, s_{1/2}); J, T = 1 >$ with $J = 2$ and 3. The Kuo-Brown G-matrix values for these matrix elements are -1.29 and 0.17 MeV, respectively, and the USD values are -0.82 and 0.76 MeV, respectively. Although the values of the TBME differ only by about 0.5 MeV, the consequence is the dramatic differences shown in Figs. 3, 4 and 5. This difference between the G-matrix and USD is not understood. The G-matrix values are insensitive to the NN interaction. The original Kuo-Brown values are based on the Hamanda-Johnson potential; results with the more recent Bonn-A potential (Table 20 of [20]) are -1.23 MeV ($J = 2$) and 0.28 MeV ($J = 3$). Hartree-Fock calculations of the change in single-particle energy between ^{16}O and ^{22}O [15] cannot account for the increase in splitting between the $d_{5/2}$ and $s_{1/2}$ orbits shown on the bottom of Fig. 4. A theoretical explanation is needed. One should explore the dependence of the ^{16}O core breaking on neutron number and the role of effective (and real) three-body interactions on the effective TBME.

Results for the calcium isotopes are also very interesting. The experimental 2^+ energies are compared with the results from the recently obtained effective interaction GXPF1 [21] in Fig. 6. The earliest shell-model calculations with the G matrix [22] showed a problem with the ESPE from ^{40}Ca to ^{48}Ca for the $f_{7/2}$-$p_{3/2}$ splitting which is completely analogous to the $d_{5/2}$-$s_{1/2}$ splitting in the oxygen isotopes. With the G matrix ^{48}Ca is not a doubly-magic nucleus [22], [20]. Only the monopole corrected TBME which were derived by McGrory [22] and employed in all subsequent effective pf-shell interactions are able to describe the correct properties of ^{48}Ca. As observed in Fig. 6, ^{52}Ca is also doubly-magic (although not as strongly as ^{48}Ca). Furthermore ^{54}Ca is predicted to be doubly magic with the GXPF1 interaction. The doubly-magic property of ^{54}Ca depends upon the $p_{1/2}$-$f_{5/2}$ shell gap shown in Fig. 1 of [21]. Recent experiment data on high-spin states in ^{54}Ti [23] is sensitive to the position of the $f_{5/2}$ neutron orbit and indicates that GXPF1 value is 0.5-1.0 MeV too high. This means that the 2^+ state in ^{54}Ca is about 0.5-1.0 MeV lower than shown in Fig. 6. Hopefully ^{54}Ca will be experimentally accessible within the next few years.

Table 1: Doubly-magic nuclei. The nuclei in brackets have not yet been observed. The pairs of nuclei are mirrors.

$n\ell j$	N=2	N=8	N=20	N=40
$0s_{1/2}$	^{4}He			
$0p_{3/2}$	^{8}He	^{14}O ^{14}C		
$0p_{1/2}$		^{16}O		
$0d_{5/2}$		^{22}O ^{22}Si	^{34}Ca ^{34}Si	
$1s_{1/2}$		^{24}O	^{36}Ca ^{36}S	
$0d_{3/2}$			^{40}Ca	
$0f_{7/2}$			^{48}Ca ^{48}Ni	^{68}Ni
$1p_{3/2}$			^{52}Ca	
$1p_{1/2}$			[^{54}Ca]	
$0f_{5/2}$			[^{60}Ca]	
$0g_{9/2}$			[^{70}Ca]	^{90}Zr
$1d_{5/2}$				^{96}Zr

In summary we have discussed two new doubly-magic nuclei, ^{22}O and ^{24}O, which were predicted to be magic with the empirical USD interaction derived in the early 1980's and have only recently been confirmed by experiment. In addition, we predict a new doubly-magic nucleus in the calcium isotopes, ^{54}Ca. These new magic nuclei together with others that have been known for many years as summarized in Table 1 point to a new rule for magic numbers.

If there is an oscillator magic number (2, 8, 20 or 40) for one kind of nucleon, then the other kind of nucleon has a magic number for the filling of every possible (n,ℓ,j) value. This rule accounts for all of the nuclei in Table 1. There are no exceptions to the rule. The oscillator magic number becomes weak for $N = 40$. Since $N = 20$ is expected to be a magic number for protons out to the neutron drip line, the new rule predicts that ^{60}Ca and ^{70}Ca will be doubly magic, but we will have to wait for an advanced radioactive-beam facility such as RIA for experimental verification. It would be interesting to understand the physical principle behind this rule. If the first (oscillator) condition is not met, one finds very few doubly-magic nuclei, only ^{12}C, ^{56}Ni, ^{132}Sn and ^{208}Pb with the expectation that ^{78}Ni and ^{100}Sn will also be doubly magic (there is no data on the excited states for these two). Otsuka et al. [14] have discussed the part of the interaction, $V_{\sigma\tau}$, which is responsible for changing the magic numbers, for example how the $N = 16$ neutron gap at $Z = 8$ disappears as protons are added. But we are left with the puzzle as to why every possible nucleus is magic when one kind of nucleon has an oscillator shell magic number.

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