Cluster Models for Light Nuclei

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Purpose of the present study

Cluster model
explore exotic structure and intruder states, but the interactions used are simple

Just in between, we would like to make a model for light neutron-rich nuclei, which has the model space to express the exotic states, and the interaction used is an effective one but the nature of the original nuclear force is well-reflected.

GFMC, NCSM
Ab initio and honest calculations
Outline of the talk

• Apply the cluster model to neutron-rich side. Using the Molecular-orbit (MO) approach, what kind of physics can be discussed? (4 examples)

• Improve the cluster model (model space and interactions)
  * cluster-shell competition can be discussed
  * direct treatment of the tensor contribution
The MO approach in nuclear physics

“Molecular Viewpoints in Nuclear Structure”
J.A. Wheeler, Physical Review 52 (1937)

Applied to $^9$Be in 1973
Y. Abe, J. Hiura, and H. Tanaka, P.T.P. 49
(application of LCAO-SCF method
to nuclear systems)

Systematic Analyses of the Be isotopes
M. Seya, M. Kohno, and S. Nagata, P.T.P. 65 (1981)

\( \pi \)- and \( \sigma \)-orbits are described
as linear combination of p-orbits
(Gaussian around each \( \pi \)-cluster with zero distance)
Molecular-Orbit is one bridge for the Cluster model to neutron-rich nuclei.

(a) $\pi$-orbit One node

(b) $\sigma$-orbit Two nodes
Our Extension of the Model Space

• p-orbits around each $\Xi$-cluster is described as the linear combination of shifted Gaussians $\rightarrow$ Spatial distribution of each neutron depends on the orbit.

• The coefficients for the linear combination is optimized $\rightarrow$ Deviation from pure MO

• Spin-triplet contribution to the $(\Xi)^2$ is included $\rightarrow$ Lowering of the intruder $(\Xi)^2$ configuration by several MeV $\rightarrow$ Breaking of N=8 magic number in $^{12}$Be
Hamiltonian

\[ \hat{H} = \sum_{i=1} \hat{t}_i - \hat{T}_{c.m.} + \sum_{i>j} \hat{v}_{ij}, \]

2-body effective interaction

\[ V(r) = (W - M P^\sigma P^\tau + B P^\sigma - H P^\tau) \]
\[ (V_1 \exp(-r^2/c_1^2) + V_2 \exp(-r^2/c_2^2)), \]

where \( W = 1 - M, M = 0.60 \) and \( B = H = 0.125 \). For the spin-orbit term, we introduce the G3RS potential\[38]\] as

\[ V_{ls} = V_0 \{e^{-d_1 r^2} - e^{-d_2 r^2}\} P(3O) \vec{L} \cdot \vec{S}, \]

where \( d_1 = 5.0 \text{ fm}^{-2}, d_2 = 2.778 \text{ fm}^{-2}, V_0 = 2000 \text{ MeV}, \)

\[ \Box + \Box + \text{n+n model for }^{10}\text{Be} \]

$\Box + \Box + 4N$ model for $^{12}\text{Be}$
Triaxial deformation of $^{10}\text{Be}$

( $\bigcirc + \bigcirc + n + n$ )

If the two valence neutrons perform independent particle motions
$\rightarrow$ charge distribution is still axial symmetric

If di-neutron correlation becomes important
$\rightarrow$ Triaxial deformation $\rightarrow$ inter-band transition

Davydov-Filiipov model
FIG. 2. $B(E2)$ ratios $B(E2: 2^+_2 \rightarrow 2^+_1)/B(E2: 2^+_1 \rightarrow 0^+_1)$ (solid line) and $B(E2: 2^+_2 \rightarrow 0^+_1)/B(E2: 2^+_1 \rightarrow 0^+_1)$ (dotted line), as a function of $\gamma$ (degree). Our results of 0.34 and 0.059 cross with the Davydov-Filippov model around $15^\circ \sim 20^\circ$. 
TABLE II. The electromagnetic transition probability \( B(E2) \) from the first \( 2^+ \) state to second \( 2^+ \) state calculated by changing the strength of the spin-orbit interaction (\( V_0^{\text{ls}} \)). The Majorana parameter (\( M \)) for the central interaction is also changed to keep the calculated ground \( 0^+ \) state energy constant. The original interaction is \( V_0^{\text{ls}} = 2000 \) MeV and \( M = 0.6 \). The magnetic dipole moment is also predicted. \( \mu(pI) \) and \( \mu(ns) \) represent proton-orbital part and neutron-spin part, respectively.

<table>
<thead>
<tr>
<th>( V_0^{\text{ls}} ) (MeV)</th>
<th>( 1000 )</th>
<th>( 1500 )</th>
<th>( 2000 )</th>
<th>( 2500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>0.58</td>
<td>0.59</td>
<td>0.60</td>
<td>0.61</td>
</tr>
<tr>
<td>( 0^+_1 ) (MeV)</td>
<td>-60.48</td>
<td>-60.38</td>
<td>-60.51</td>
<td>-60.75</td>
</tr>
<tr>
<td>( 2^+_1 ) (MeV)</td>
<td>-57.69</td>
<td>-57.30</td>
<td>-57.26</td>
<td>-57.33</td>
</tr>
<tr>
<td>( 2^+_2 ) (MeV)</td>
<td>-56.71</td>
<td>-55.78</td>
<td>-54.88</td>
<td>-53.93</td>
</tr>
<tr>
<td>( B(E2: 2^+_1 \rightarrow 2^+_2) (e^2 \text{ fm}^4) )</td>
<td>17.53</td>
<td>9.53</td>
<td>3.99</td>
<td>2.20</td>
</tr>
<tr>
<td>( \mu(2^+_1) ) (( \mu_N ))</td>
<td>0.57</td>
<td>0.73</td>
<td>0.72</td>
<td>0.69</td>
</tr>
<tr>
<td>( \mu(pI), \mu(ns) )</td>
<td>(0.70, -0.13)</td>
<td>(1.04, -0.30)</td>
<td>(1.13, -0.41)</td>
<td>(1.16, -0.48)</td>
</tr>
<tr>
<td>( \mu(2^+_2) ) (( \mu_N ))</td>
<td>0.91</td>
<td>0.59</td>
<td>0.48</td>
<td>0.43</td>
</tr>
<tr>
<td>( \mu(pI), \mu(ns) )</td>
<td>(1.03, -0.12)</td>
<td>(0.65, -0.06)</td>
<td>(0.51, -0.04)</td>
<td>(0.46, -0.03)</td>
</tr>
</tbody>
</table>
Equilateral triangular shape of $^3\Xi$

$D_{3h}$ symmetry
$K^{\Xi}=0^+, 3^-$ rotational bands are possible

$^{12}\text{C} \ 3^- \text{ at } 9.6 \text{ MeV}$
$4^-, 5^- \text{ have not been observed (above the } \Xi^- \text{-threshold) }$
Energy level of $^{14}$C

“Phase” of nuclei

Excitation energy

mean-field-like (liquid)

Cluster crystallization (solid)

threshold

Alpha-condensation (gas)
Now we extend the cluster model and include the cluster-shell competition due to the spin-orbit interaction.
Antisymmetrized Molecular Dynamics (AMD)  
(H. Horiuchi et al.)

Single-particle wave function

\[ \psi_i = \left( \frac{2\nu}{\pi} \right)^{\frac{3}{4}} \exp\left[ -\nu (\vec{r} - \vec{z}_i/\sqrt{\nu})^2 + \frac{\vec{z}_i^2}{2} \right], \]

\[ \langle \vec{r} \rangle = \frac{1}{\sqrt{\nu}} \text{Re}[\vec{z}_i], \]

\[ \langle \vec{p} \rangle = 2\sqrt{\nu} \hbar \text{Im}[\vec{z}_i]. \]

Slater determinant

\[ \begin{bmatrix} \end{bmatrix} = A \left[ \begin{bmatrix} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix} \cdots \right] \]

Projection of parity and angular momentum

\[ \begin{bmatrix} \end{bmatrix} = P^J P \begin{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix} \]
\[
\frac{d\bar{z}_i}{d\tau} = -\frac{\partial E}{\partial \bar{z}_i^*}, \quad \frac{d\bar{z}_i^*}{d\tau} = -\frac{\partial E}{\partial \bar{z}_i^*}.
\]

**AMD cooling equation**

\[
\frac{dE}{d\tau} = \sum_i^A \frac{\partial E}{\partial \bar{z}_i} \cdot \frac{d\bar{z}_i}{d\tau} + \sum_i^A \frac{\partial E}{\partial \bar{z}_i^*} \cdot \frac{d\bar{z}_i^*}{d\tau},
\]

\[
= -2 \sum_i^A \frac{d\bar{z}_i}{d\tau} \cdot \frac{d\bar{z}_i^*}{d\tau} < 0.
\]

\[
\frac{d\bar{z}_i}{d\tau} = -Im\left[\frac{\partial E}{\partial \bar{z}_i^*}\right]_i, \quad \frac{d\bar{z}_i^*}{d\tau} = \cdot Im\left[\frac{\partial E}{\partial \bar{z}_i}\right]_i.
\]

**AMD triple-S cooling equation**
Antisymmetrized Molecular Dynamics (AMD) has been proposed to express this effect, but the single Slater determinant is not enough to express the quantum mechanical mixing of shell-like structure and cluster-like structure.

- We superpose many AMD wave function
- We use the idea of Stochastic Variation Method (Suzuki, Varga …) for the selection of the basis
Superposition of Selected Snapshots (AMD SSS)
N. Itagaki and S. Aoyama, PRC68 (2003) 054302
Density distribution of $^6$He (□+n+n)

N. Itagaki and S. Aoyama, PRC68 (2003) 054302
We prepare $\square - \square$ (3, 4, 5 fm) and add $\square + p + p + n + n$ model space. The squared overlap between the first 3 basis (cluster) and the final state is 0.91. $\Rightarrow$ Cluster structure survives. 

$^8\text{Be}$ $0^+$ state
$^{12}\text{C}$ $0^+$ energy convergence

- Energy (MeV)
- Number of the trial basis state

- $3\alpha$
- $2\alpha + 2p + 2n$
- $\alpha + 4p + 4n$
Formation of $^{12}$C in starts

$0^+_2 \xrightarrow{\square \text{decay}} 2^+$

$2^+ \xrightarrow{\square \text{decay}} 0^+$

$B(E2) \quad \text{present} \quad \text{exp.} \quad 3 \quad \square \text{model}$

$0^+_2 \rightarrow 2^+_1 \quad 14.1 \quad 13 \quad 4 \quad 5.6$

$2^+_1 \rightarrow 0^+_1 \quad 7.1 \quad 7.8 \quad 0.4 \quad 9.3$

All units are $e^2fm^4$
Nuclear chart of cluster-shell competition

\[ Z \]

\[ ^{12}\text{C} \]
\[ \Delta = 5.2, \quad \Delta (2) = 6.2 \]

\[ ^{10}\text{B} \]
\[ \Delta = 1.9 \]

\[ ^{12}\text{B} \]
\[ \Delta = 1.7 \]

\[ ^{8}\text{Be} \]
\[ \Delta = 2.0 \]

\[ ^{10}\text{Be} \]
\[ \Delta = 1.9 \]

\[ ^{12}\text{Be} \]
\[ \Delta = 1.4 \]

\[ N \]
$^{14}$C $0^+$ energy convergence

- Shell-like
- $3\alpha + 2n$

Energy (MeV) vs. number of trial AMD basis states
$^{16}$C $0^+$ energy convergence

- shell-like ($\alpha + 4p + 8n$)
- $3\alpha + 4n$

energy (MeV)

number of trial AMD basis states
Anomalously Hindered $E2$ Strength $B(E2; 2^+_1 \rightarrow 0^+)$ in $^{16}\text{C}$

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The electric quadrupole transition from the first $2^+$ state to the ground $0^+$ state in $^{16}\text{C}$ is studied through measurement of the lifetime by a recoil shadow method applied to inelastically scattered radioactive $^{36}\text{C}$ nuclei. The measured mean lifetime is $77 \pm 14(\text{stat}) \pm 19(\text{syst})$ ps. The central value of mean lifetime corresponds to a $B(E2; 2^+_1 \rightarrow 0^+)$ value of $0.63 \text{e}^2\text{fm}^4$, or 0.26 Weisskopf units. The transition strength is found to be anomalously small compared to the empirically predicted value.

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Preliminary result of the present calculation

$B(E2; 2^+ \rightarrow 0^+) = 0.1 \text{e}^2\text{fm}^4$ (protons)

$B(E2; 2^+ \rightarrow 0^+) = 5.4 \text{e}^2\text{fm}^4$ (neutrons)

$2^+$ state has mainly neutron-excitation configuration?

$<I|E2|F> = <p|E2|p><n|n>$

No conclusion for the deformation of the protons
Conclusion

Cluster model

- explore exotic structure and intruder states, but the interactions used are simple
- MO model is one way to extend the cluster models to neutron-rich side.
- Using AMD SSS, we can discuss the cluster-shell competition.
- Refinement of the interaction has just started and Tensor can be directly treated.

GFMC, NCSM

Ab initio and honest calculations