Coupled Channels Program

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1. General Description of CHUCK3

The program CHUCK3 calculates nuclear reaction cross sections by numerically solving an appropriate set of coupled equations. This treatment differs from that of the distorted wave Born approximation (DWBA) program DWUCK4 which solves the scattering problem to first order only in the interaction potential. The solution to the set of coupled equations, however, solves the problem to all orders in the interaction potential. However, the program CHUCK3 can be run with a one way coupling from the initial state channels to the final state channels (no back coupling) to give the same result as the the DWBA program DWUCK4. An effort has been made in this program to keep the input data for CHUCK3 in a form as similar as feasible to the DWBA code DWUCK4. It is recommended that refernce be made to the DWUCK4 instructions for details (*e.g.* the form factor, non-locality correction, *etc.*) which are only sketched in this description.

The are necessary differences between the two programs in the mode of running the coupled channels program. The principal one is that, since the coupled channels program is necessarily non-linear in the interaction potentials, it is mandatory to include the exact coupling strengths and angular momentum dependences explicitly in the input parameters instead of scaling the final output as is done in the DWBA programs. Hence, the output of CHUCK3 will be absolutely normalized in terms of the model used to describe the reaction.

The wave function for a channel c is expanded in a representation which couples the orbital angular momentum of the relative motion l_c to the intrinsic spin of the projectile s_c to give j_c. Then, j_c is coupled to the target spin I_c to give a total angular momentum J and projection M. The wave function for all channels is

$$\Psi^{J\,M} = \sum_{c\,j_c\,l_c} \chi^{c\,J}_{j_c\,l_c}(k_c, r_c)(\mathcal{Y}^c_{l_c j_c}\Phi^c_{I_c})^M_J\,. \tag{1.1}$$

The channel index c denotes the quantum numbers (l_c, s_c, j_c, I_c) plus any other quantities needed to completely define the channel. The function $\Phi^c_{I_c}$ is the intrinsic wave function for the target and $\mathcal{Y}^c_{l_c j_c}$ is the coupling of the relative angular momentum and projectile intrinsic spin function. The $\chi^{c\,J}_{j_c l_c}(k_c; r_c)$ are the radial wave functions for the relative motion in the channel c and is the generalization of the distorted radial wave functions in DWUCK4. The coupled equations for the radial functions are

$$[\frac{d^2}{dr_c^2} - \frac{l_c(l_c+1)}{r_c^2} + (k_c^2 - U_{cc})\chi^J_{j_c l_c}(k_c; r_c) = \sum_{c'} \frac{2\gamma_c}{\hbar^2} V_{cc'}\chi^J_{j_{c'} l_{c'}}(k_{c'}; r_{c'}). \tag{1.2}$$

The diagonal term U_{cc} is the usual distorting potential

$$U_{cc} = \frac{2\gamma_c}{\hbar^2}(V_{cc} + V_c) \tag{1.3}$$

where γ_c is the reduced mass for the channel c, V_{cc} may be an optical potential and V_c is a Coulomb potential. The off diagonal couplings $V_{cc'}$ are

$$V_{cc'} = \sum_{m_c m_{c'} \nu_c \nu_{c'} M_c M_{c'}} \mathcal{J}(l_c\, s_c\, m_c\, \nu_c | j_c\, \mu_c)(l_{c'}\, s_{c'}\, m_{c'}\, \nu_{c'} | j_{c'}\, \mu_{c'}) \\ (j_c\, I_c\, \mu_c\, M_c | J\, M)(j_{c'}\, I_{c'}\, \mu_{c'}\, M_{c'} | J\, M) \left\langle i^{l_c} Y_{l_c}^{m_c} \phi_{s_c}^{c} \Phi_{J_c M_c}^{c} | V | i^{l_{c'}} Y_{l_{c'}}^{m_{c'}} \phi_{s_{c'}}^{c'} \Phi_{J_{c'} M_{c'}}^{c'} \right\rangle \tag{1.4}$$

where $\mathcal{J}$ is the usual Jacobian of transformation to relative coordinates. It is convenient to expand the matrix element in (4) into tensorial components

$$\left\langle i^{l_c} Y_{l_c}^{m_c} \phi_{s_c}^{c} \Phi_{J_c M_c}^{c} | V | i^{l_{c'}} Y_{l_{c'}}^{m_{c'}} \phi_{s_{c'}}^{c'} \Phi_{J_{c'} M_{c'}}^{c'} \right\rangle = \sum_{lsj} (I_{c'}\, j\, M_{c'}\, M_c - M_{c'} | I_c\, M_c)(s_c\, s\, \nu_c\, \nu_{c'} - \nu_c | s_{c'}\, \nu_{c'}) \\ (l\, s\, m\, \nu_{c'} - \nu_c | j\, M_c - M_{c'}) i^{l_{c'} - l - l_c} \mathcal{G}_{lsj}^{c c'} \tag{1.5}$$

where $m = M_c - M_{c'} - \nu_c + \nu_{c'}$ and $\mathcal{G}_{lsj}^{c c'}$ can be obtained by inverting the above expression and can be expressed as a sum of products of form factors f_{lsj} which can be calculated as an option in the program and strengths B_{lsj} which must be supplied as input.

The form factors f_{lsj} calculated in CHUCK3 assume the zero range approximation as in DWUCK4. With this assumption the distorted waves and the form factors depend upon only a single coordinate $r_{c''}$ which differs from r_c or $r_{c'}$ only by a scaling factor. The coupling matrix element is then

$$V_{cc'}(r_{c'}) = \sum_{lsj} (-1)^{J - j_{c'} - I_{c'}} \frac{i^{l_{c'} - l - l_c}}{\sqrt{4\pi}} \hat{l}_c \hat{l} \hat{j}_c \hat{j}_{c'} \hat{j} \hat{s}_{c'} \hat{I}_c (l_c\, l\, 0\, 0 | l_{c'}\, 0) W(j_c I_c j_{c'} I_{c'}; J j) \\ \begin{Bmatrix} l_c & s_c & j_c \\ l & s & j \\ l_{c'} & s_{c'} & j_{c'} \end{Bmatrix} B_{lsj}^{c c'} f_{lsj}^{c c'}(r_{c''}), \tag{1.6}$$

where we use the notation $\hat{k} = \sqrt{2k+1}$.

The coupled equations for the channels are solved for the channel radial wave functions $\chi_{j_c l_c}^{J}(k_c; r_c)$ satisfying the boundary conditions $\chi_{j_c l_c}^{J}(k; 0) = 0$ at the origin and for large r_c (where V_{cc} can be neglected),

$$\chi_{j_c l_c}^{J}(k_c; r_c) \longrightarrow e^{i\sigma_{l_c}^{c}} [F_{l_{c1}}^{c1} \delta_{c c_1} \delta_{l_c l_{c1}} \delta_{j_c j_{c1}} + \sqrt{\frac{v_{c1}}{v_c}} T_{c c_1}^{J} H_{l_c}^{+}(k_c r_c)]. \tag{1.7}$$

Here, $F_{l_c}^{c}(k_c r_c)$ is the Coulomb function regular at the origin and $H_{l_c}^{+}(k_c r_c) = G_{l_c}(k_c r_c) + i F_{l_c}(k_c r_c)$ is the outgoing Coulomb wave, $T_{c c1}^{J}$ is the outgoing wave amplitude and $\sigma_{l_c}^{c}$ is the Coulomb phase shift. The label c_1 refers to the initial (or elastic) channel. If we define the scattering amplitude as

$$D_{M_c \nu_c, M_{c1} \nu_{c1}}^{l_c m_c} = \sum_{j_c J M} \frac{e^{2i(\sigma_{l_c}^{c} - \sigma_1^{c})}}{k_c} (2 l_c + 1) \left[\frac{(l_c + m_c)!}{l_c - m_c)!}\right]^{\frac{1}{2}} \\ (l_{c_1}\, s_{c_1}\, 0\, \nu_{c_1} | j_{c_1}\, \nu c_1)(j_{c_1}\, I_{c_1}\, \nu_{c_1}\, M_{c_1} | J\, M) \\ (l_c\, s_c\, m_c\, \nu_c | j_c\, M - M_c)(j_c\, I_c\, M - M_c\, M_c | J\, M)\, T_{c c_1}^{J}(j_c l_c; j_{c_1} l_{c_1}), \tag{1.8}$$

then the resulting cross section is

$$\frac{d\sigma_c}{d\Omega} = \frac{1}{2 I_{c_1} + 1} \frac{1}{2 s_{c_1} + 1} \sum_{M_{c_1} M_c \nu_{c_1} \nu_c} |f_{coulomb}(\theta) + \sum_{l_c m_c} D_{M_c \nu_c M_{c_1} \nu_{c_1}}^{l_c m_c} P_{l_c}^{m_c}(\theta)|^2. \tag{1.9}$$

CARD SET 1 (one card)

ICON(12) ALPHA
FORMAT (12I1,8X,15A4)

i	ICON(i)	FUNCTION
1	≠9	Read additional cards for this run.
	9	Call EXIT, do not read in additional cards.
2	-	Not used.
3	-	Not used.
4	-	Not used.
5	≠0	Write scattering amplitudes onto scratch file "TAPE4" for external use.
6	≠0	Print out elastic T matrix elements ($S^{\ell j}_{c_o c_o} = 1+2i\, T^{\ell j}_{c_o c_o}$)
7	≠0	Print out all T matrix elements.
8	≠0	Print out scattering amplitudes, D.
9	0	Suppress plot of cross-sections.
	N	Plots differential cross-sections with N-decade log scale.
10	≠0	Uses relativistic kinematics.
11	≠0	Prints out diagonal $k^2_{cc}(r_c)$ of channel functions where $k^2_{cc} = k^2_c - U_{cc}$.
12	≠0	Prints out off-diagonal $k^2_{cc'}(r_{c'}) = -\frac{2}{\hbar^2} f^{cc'}_{\ell sj}$.

ALPHA — Any 60 character identification starting in column 21.

CARD SET 2 (one card)

THETAN, THETA1, DTHETA
FORMAT (3F8.4)

THETAN	No. of angles at which to calculate angular distributions.
THETA1	First angle.
DTHETA	Angle increment.

CARD SET 3 (one card)

LMAX, NCHAN, $J_1, J_2, \ldots, J_{NCHAN}, K_1, K_2, \ldots, K_{NCHAN}$
FORMAT (18I3)

LMAX	Number of partial waves (LMAX ≤ 50).
NCHAN	Number of intrinsic channels, including elastic channel (NCHAN ≤ 8).
$J_c = 2*\pi_c*I_c$	π_c=parity of target in channel c. I_c=spin of target in channel c. (For a 1^- state, $J_c = -2$.) One number for each channel.
$K_c = 2*\kappa_c$	κ_c=spin projection for the bandhead of a collective rotational band. One number for each channel where applicable.

CARD SET 4 (one card)

DR, RMAX
FORMAT (2F8.4)

DR	Integration step size (Fm).
±RMAX	Max radius (Fm). Positive sign allows program to override this number with criteria depending upon the problem. Negative sign forces program to integrate to RMAX. (RMAX/DR ≤ 200.)

CARD SET 5 (One set for each channel declared in card set 3. Elastic channel must be first channel described. Set defines charges, masses, etc., as well as optical potentials for the channels. Minimum of 3 cards per card set.)

Cards for Some Channel NZ

Card 1 E,MP,ZP,MT,ZT,R_{oc},AC,PNLOC,2*FS
FORMAT (9F8.4)

E	=Lab energy (MeV) of elastic channel projectile (if NZ=1). =Q value relative to elastic channel (for NZ≠1).
MP	Projectile mass (AMU).
ZP	Projectile charge.
MT	Target mass (AMU).
ZT	Target charge.
R_{oc}	Coulomb charge radius (Fm). ($R_c=R_{oc}\ MT^{1/3}$).
AC	Coulomb charge diffuseness.
PNLOC	Nonlocal range parameter.
2*FS	Twice spin of projectile intrinsic spin.

Card 2 NZ,NZ,LDFRM,BETA, BETA2,BETA4
FORMAT (2I3,12X,I3,3X,3F8.4)

NZ	Defines channnel.
LDFRM	Order of deformation for optical potentials, if deformed.
BETA	Deformation parameter of order LDFRM. Used only if options 11 and/or 12 are used for optical potentials. Program extracts monopole part of potential.
BETA2	Deformation parameter of order LDFRM+2.
BETA4	Deformation parameter of order LDFRM+4.

Cards 3-M OPT,VR,R_{OR},AR,VSOR,VI,R_{OI},AI,POWR
FORMAT (8F8.4,8X,F8.4)

OPT	Potential option.
VR	Real well depth.
R_{OR}	Real well radius ($R_R=R_{OR}\ MT^{1/3}$).
AR	Real well diffuseness.
VSOR	Thomas spin-orbit factor.
VI	Imaginary well depth.
R_{OI}	Imaginary well radius ($R_I=R_{OI}\ MT^{1/3}$).
AI	Imaginary well diffuseness.
POWR	Extra variable for use in some options.

These cards have substantially the same meaning as in DWUCK and are detailed later in this write-up. Any number of potential cards may be used and the resulting potential will be the sum of potentials defined on the cards. The card with a negative OPT option is the last for this card set.

CARD SET 6 (One set for each coupling between channels. A blank card will terminate the scan of coupling card set and start the numerical integration.)

Cards for Coupling Between Channels NI and NJ

Card 1 NJ,NI,LTR,2*STR,2*JTR,ICODE,LDFRM,ICOUEX,BETA, BETA2,BETA4,R_{OCX},FNRNG
FORMAT (8I3,5F8.4)

NJ,NI — Channels which are coupled (these channels described on card sets 3 and 5). If NJ is negative, coupling is one way from NI → NJ. If NJ is positive, coupling is both ways, NI ⇆ NJ.

LTR — Transferred orbital angular momentum.

2*STR — Twice transferred projectile spin.

2*JTR — Twice transferred $J(\vec{LTR} + \vec{STR} = \vec{JTR})$.

ICODE=0 — Collective form factor.

=1 — Single-particle transfer form factor.

=2 — Two-particle transfer form factor.

LDFRM — Order of deformation for collective form factor (ICODE=0).

ICOUEX≠0 — If Coulomb excitation is to be calculated.

BETA,BETA2, BETA4 — BETA=$B_{\ell sj}$, coupling strength.

ICODE=0. $B_{\ell sj}=\beta_{LDFRM}$, deformation for collective excitations. BETA2=$\beta_{LDFRM-2}$. BETA4=$\beta_{LDFRM-4}$.

ICODE=1. $B_{\ell sj}$=product of light and heavy particle spectroscopic factors and D_o, the zero range strength, i.e.

$$BETA=S^{\frac{1}{2}}_{\ell j}(\text{target})* S^{\frac{1}{2}}(\text{projectile})*D_o.$$

Caution: D_o is the true overlap and is not the same as for DWUCK. ($D^2_{o\,DWUCK}=D^2_{o\,CHUCK}/10^4$). BETA2 and BETA4 are not used in this case.

ICODE=2.

$$BETA=B_{\ell sj}=\sqrt{9.7}\ \ S^{\frac{1}{2}}_{\ell j}\ (\text{target})\ S^{\frac{1}{2}}\ (\text{projectile})\ D_o.$$

BETA2 and BETA4 are not used.

R_{OCX} — Radius for Coulomb excitation (Fm). ($R_{CX}=R_{OCX}\ MT(NJ)^{1/3}$).

FNRNG — Finite range parameter.

Cards 2-N

ICODE=0

Cards 2-N

OPT,VR,R_{OR},AR,VSOR,VI,R_{OI},AI,POWR
FORMAT (8F8.4,8X,F8.4)

Macroscopic form factor cards using the optical potential options.

ICODE=1

Card 2

E,MP,ZP,MT,ZT,R_{oc}
FORMAT (6F8.4)

E	Binding energy of transferred particle to core.
MP	Mass of transferred particle.
ZP	Charge of transferred particle.
MT	Mass of core which binds particle.
ZT	Charge of binding core (less that of transferred particle).
R_{oc}	Coulomb radius ($R_c=R_{oc}\ MT^{1/3}$)

Cards 3-M

Cards which describe binding well. Same as cards 3-M of card set 5.

Card M+1

FN,FL,2*F2,2*FS,VTRIAL,FISW,DAMP
FORMAT (7F8.4)

FN	Number of nodes of orbital (excluding origin and infinity).
FL	Orbital angular momentum.
2*FJ	Twice total angular momentum of particle.
2*FS	Twice intrinsic spin of particle.
VTRIAL	Scaling factor for bound state potential.
FISW	Search control for bound state. =0 Search on well depth. =1 Search on binding energy for fixed potential. =2 No search.
DAMP≠0	Multiplies "bound" state function by exp(-DAMP*r) and renormalize to 1.0.

ICODE=2

Card 2

CNTRL,QCODE,FLMU,VZERO,FJ2,FJ1,FJF
FORMAT (7F8.4)

CNTRL	=0.0 Read zero sets of orbital cards and EXIT from form factor calculation. =1.0 Read one set of orbital cards. =2.0 Read two sets of orbital cards.
QCODE	=0.0 No option. 1.0 Yukawa potential microscopic form factor. 2.0 Coulomb potential microscopic form factor. 3.0 Tensor potential microscopic form factor. 4.0 Not used 5.0 Two nucleon transfer microscopic form factor. 6.0 Zero range knockout microscopic form factor.

ICODE=2, Card 2 (continued)

FLMU = (Range)$^{-1}$ of potentials for options 1 and 3 above.
= RMS radius of Gaussian wave function for "triton" in option 5 above (default for FLMU=0 is 1.7).

VZERO = Strength of potential for options 1, 2, and 3.
= Spectroscopic amplitude for transfer reaction in option 5.
= Volume integral of two-body potential in option 6. Here volume integral = $\int V(r)\exp(i\vec{K}\cdot\vec{r})d\vec{r}$

FJ2 = 2* spin of core (j2) to which single particle is coupled in options 1, 2, 3, and 6.

FJI = 2* spin of initial nucleus $\vec{j1} + \vec{j2} = \vec{JI}$ in options 1, 2, 3, and 6.

FJF = 2* spin of final nucleus $\vec{j1}' + \vec{j2} = \vec{JF}$ in options 1, 2, 3, and 6.

Cards (3-M)

Following card 1, place one or two sets of cards defining the single-particle orbitals, (as used for ICODE=1 for example). If the second orbital is identical to the first you may use CNTRL=1.0 and the program will use the results of the first orbital calculation as the second orbital wave function and will not read in the second set. The program will attempt to read in a new card 1 of this data set and coherently add the form factor contributions until a CNTRL=0.0 is detected and the program will terminate the calculation of this form factor. A negative CNTRL will also give a coherent contribution to the form factor but the calculation will terminate similarly to CNTRL=0.0.

A number of variations to the standard options may be made. The list described for QCODE ≤ 10.0 will treat the form factor as a purely real function. If you increase the QCODE by 10.0, i.e., 1.0 becomes 11.0, etc., the program will treat that part of the form factor as purely imaginary. Thus by use of a QCODE number less than 10.0 and another one greater than 10.0 one can generate both real and imaginary parts of the form factor. Further, the use of a negative QCODE will use the single-particle orbitals generated by the previous QCODE to calculate the contribution to the form factor. Hence the single-particle orbital cards can be omitted and in this way one can calculate multiple contributions such as a Yukawa interaction plus a Coulomb term without repunching the orbital cards for the second potential option.

Potential options available for card sets 5 and 6.

OPT = 1.0 Volume Wood-Saxon potential

$$V(r) = VR*f(X_R) + iVI*f(X_I)$$

$$f(X_i) = [1 + \exp(\frac{r-R_{oi}MT^{1/3}}{A_i})]^{-1}$$

Note that for attractive real and absorptive imaginary potentials VR and VI must be negative.

OPT = 2.0 Surface Wood-Saxon potential

$$V(r) = VR*df(X_R)/dX_R + iVI*df(X_I)/dX_I$$

Note that for attractive real and absorptive imaginary potentials VR and VI must be positive. Further, this form has no factors of 4 so that VI=4 W_D where W_D is the other convention of expressing the strength of the surface shape potential.

OPT = 3.0 Second derivative Wood-Saxon potential

$$V(r) = VR*d^2f(X_R)/dX^2_R + iVI*d^2f(X_I)dX_I^2$$

OPT = 4.0 L·S potential from a volume Wood-Saxon potential

$$V_{LS}(r) = (-VR*\frac{1}{r}\,df(X_R)/dr - iVI*\frac{1}{r}\,df(X_I)/dr)\ L\cdot S$$

Note that the potential is defined in terms of L·S and contains no $(\hbar/m_\pi c)^2 \approx 2$ factor. The strength VR is 4 times the strength used when the $(\hbar/m_\pi c)^2$ factor and L·σ conventions are used for protons and neutrons, but only a factor of two is needed for deuterons since the V_{LS} then is usually given in terms of L·S.

The VSOR parameter used in option 1 will give a spin orbit potential the same as option 4 with the same geometry as used in option 1. The VSOR parameter is then the usual λ factor used in multiplying the Thomas term and has a value of about 25 for protons and neutrons. In this case

$$V_{LS}(r) = -VR*\frac{VSOR}{45.2}*\frac{1}{r}\frac{df(X_R)}{dr}\ L\cdot S$$

OPT = 5.0 L·S potential from a surface Wood-Saxon potential

$$V_{LS}(r) = (-VR*\frac{1}{r}\frac{d}{dr}\,df(X_R)/dX_R - iVI*\frac{d}{dr}\,df(X_R)/dX_R)\ L\cdot S$$

OPT = 6.0 Volume Wood-Saxon potential *r**POWR

$$V(r) = [VR*f(X_R) + iVI*f(X_I)]*r^{**}POWR$$

OPT = 7.0 Surface Wood-Saxon potential *r**POWR

$$V(r) = [VR*df(X_R)/dX_R + iVI*df(X_I)/dX_I] \quad *r**POWR$$

OPT = 8.0 External Form Factor

This option reads in a form factor from the input file. In addition to the option card you must add a separate card to specify the number of points to be read in and whether the form factor is real or imaginary.

F1,F2
FORMAT (2F8.4)

F1 Number of points to be read in
F2 =0.0 places points in real part of form factor
=1.0 places points in imaginary part of form factor

This card is followed by the necessary cards containing the form factor, 5 points per card.

FF(I) Form factor points
FORMAT (5E16.7)

The form factor FF(I) is scaled by VR or VI depending upon whether F2 is 0.0 or 1.0. If VR and VI are zero or blank, the form factor is scaled by 1.0.

OPT = 9.0 Normalized Harmonic Oscillator

$$V(r) = VR*N\ L(r/R_{OR})\ \exp\ (-\tfrac{1}{2}(r/R_{OR})^{**}2)$$

where N=normalizing constant for the harmonic oscillator functions such that

$$\int_0^\infty (V(r))^2 r^2 dr = (VR)^2$$

$L_n^{\ell+\frac{1}{2}}(r/R_{OR}^2)$ is the Laguerre polynomial

Note that the radius parameter is R_{OR}, the quantity input on the potential card and <u>not</u> R_R, which is R_{OR}MT**(1/3).

OPT = 10.0 GAUSSIAN *r**POWR

$$V(r) = VR*\ \exp\ (-(r)/R_{OR})^{**}2)*\ r\ ^{**}POWR$$
$$+iVI*\ \exp(-(r/R_{OR})^{**}2)*\ r\ ^{**}POWR$$

OPT = 11.0 Legendre expansion of volume Wood-Saxon potential

$$V(r)_{LTR} = \int [VR * f(X_R(r,\theta)) + iVIf(X_I(r,\theta))]\ Y^\circ_{LTR}(\theta)\ d\ \Omega_r$$

where $f(X) = 1.0/[1.0 + \exp(X)]$

$$X_i = [r - R_i(1.0 + \beta_{LDFRM} Y^\circ_{LDFRM})]/A_i$$

LTR is the L transfer specified for this form factor.

OPT = 12.0 Legendre expansion of surface Wood-Saxon potential

$$V(r)_{LTR} = \int [VR * \frac{df}{dX_R}(X_R) - iVI * \frac{df}{dX_I}(X_I)]\, Y^{\circ}_{LTR}(\theta)\, d\Omega_r$$

where f(X) and X have the same meanings as in OPTION=11.0.

In options 11 and 12, VR and VI have the same sign conventions as are used in options 1 and 2. Further CHUCK treats LTR=0 differently from other values of LTR when used as a form factor as a result of options 1,2,3,11, and 12. In these cases the V(r) for LTR=0 is considered to be the coefficient of $P_0(0)$, the zeroth Legendre polynomial, whereas V(r) for LTR > 0 is considered to be the coefficient of Y°_{LTR}, the usual assumption of the angular momentum algebra. Hence the LTR=0 strength needs to be multiplied by an extra $\sqrt{4\pi}$ factor. This ambiguity does not apply to the single particle, microscopic, or two nucleon transfer form factors.

OPTIONS 11 and 12 may also be used as potential for the distorted waves. In this case, LTR=0 is set by the program.

Form Factors for ICODE=0.

These options have the basic form as denoted in the potential option description except for the following cases.

OPT=1.0

$$F(r) = VR*f(X_R) * (R_R/A_R)^{**}POWR + VI*F(X_I)*(R_I/A_I)^{**}POWR$$

Note that this is different from DWUCK.

OPT=2.0 and OPT=3.0 The real and imaginary parts of the form factor are multiplied by $(R_R/A_R)^{**}POWR$ and $(R_R/A_R)^{**}POWR$ respectively, as in the case of OPT=1.0.

OPT=4.0 and OPT=5.0 These options are not available as form factors.

OPT=6.0 - 12.0 These options give the same form and normalization as in the options for the potential functions.

Light particle spectroscopic factors in CHUCK

Particles	Isospin factor	$\sqrt{n}$	Amplitude	$S^{\frac{1}{2}}$	$S^{\frac{1}{2}}D_0$
p(n)d	$(\frac{1}{2}\frac{1}{2}-\frac{1}{2}\frac{1}{2}\|00) = -1/\sqrt{2}$	$\sqrt{2}$	1	-1	122.5
n(p)d	$(\frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2}\|00) = 1/\sqrt{2}$	$\sqrt{2}$	1	+1	-122.5
d(n)t	$(0\frac{1}{2}0\frac{1}{2}\|\frac{1}{2}\frac{1}{2}) = 1$	$\sqrt{3}$	$1/\sqrt{2}$	$\sqrt{3/2}$	-225
d(p)^{3}He	$(0\frac{1}{2}0-\frac{1}{2}\|\frac{1}{2}-\frac{1}{2}) = 1$	$\sqrt{3}$	$1/\sqrt{2}$	$\sqrt{3/2}$	-225
p(2n)t	$(\frac{1}{2}1-\frac{1}{2}1\|\frac{1}{2}\frac{1}{2}) = -\sqrt{2/3}$	$\sqrt{3}$	$-1/\sqrt{2}$	1	-1560*
p(np)^{3}He $(^{S=0}_{T=1})$	$(\frac{1}{2}1-\frac{1}{2}0\|\frac{1}{2}-\frac{1}{2}) = -1/\sqrt{3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$1/\sqrt{2}$	-1100
p(np)^{3}He $(^{S=1}_{T=0})$	$(\frac{1}{2}0-\frac{1}{2}0\|\frac{1}{2}-\frac{1}{2}) = 1$	$\sqrt{3}$	$1/\sqrt{2}$	$\sqrt{3/2}$	-1920
n(2p)^{3}He	$(\frac{1}{2}1\frac{1}{2}-1\|\frac{1}{2}-\frac{1}{2}) = \sqrt{2/3}$	$\sqrt{3}$	$-1/\sqrt{2}$	-1	1560
n(np)t $(^{S=0}_{T=1})$	$(\frac{1}{2}1\frac{1}{2}0\|\frac{1}{2}\frac{1}{2}) = \sqrt{1/3}$	$\sqrt{3}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1100
n(np)t $(^{S=1}_{T=0})$	$(\frac{1}{2}0\frac{1}{2}0\|\frac{1}{2}\frac{1}{2}) = 1$	$\sqrt{3}$	$1/\sqrt{2}$	$\sqrt{3/2}$	-1920
^{3}He(n)α	$(\frac{1}{2}\frac{1}{2}-\frac{1}{2}\frac{1}{2}\|00) = -1/\sqrt{2}$	$\sqrt{4}$	1	$-\sqrt{2}$	678
t(p)α	$(\frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2}\|00) = 1/\sqrt{2}$	$\sqrt{4}$	1	$\sqrt{2}$	-678

Triton or ^{3}He function $= 1/\sqrt{2}\,[(S=1+\frac{1}{2})(T=0+\frac{1}{2}) - (S=0+\frac{1}{2})(T=1+\frac{1}{2})]$

*This value includes the factor $\sqrt{9.7}$ to normalize the two-nucleon transfer.

2. Specific Cases of Reactions.

In CHUCK3 the off diagonal terms which need to be calculated are of the form

$$V^J_{c'c} = \mathcal{J}\left\langle i^{l_{c'}}(l_{c'}s_{c'})j_{c'}I_{c'};JM\right|\sqrt{2j+1}(T(j)U(j))^0\left|i^{l_c}(l_cs_c)j_cI_c;JM\right\rangle \tag{2.1}$$

where $I_{c'}$ and I_c are the target spins,
$s_{c'}$ and s_c are the projectile spins,
$l_{c'}$ and l_c are their relative angular momentum,
$\vec{j}_c = \vec{l}_c + \vec{s}_c$ are the coupled angular momentum,
$\vec{J} = \vec{j}_c + \vec{I}_c$ is the total angular momentum and
T(j) is the tensor operator for the relative motion with coupling $(ls)j$.

Applying equations (5.4.1) and (7.1.5) of Edmonds [Angular Momentum in Quantum Mechanics, Princeton Univ. Press 1957] we obtain

$$\begin{aligned} V^J_{c'c} =& (-1)^{j_{c'}+I_c+j-J}\hat{j}_c\hat{j}_{c'}\hat{l}_c\hat{l}_{c'}\hat{j}W(j_cj_{c'}I_cI_{c'};jJ)(l_c\,l\,0\,0|l_{c'}\,0) \\ & \left\{\begin{array}{ccc} l_c & l & l_{c'} \\ s_c & s & s_{c'} \\ j_c & j & j_{c'} \end{array}\right\} B^j_{I_cs_cI_{c'}s_{c'}} f^{c'c}_{lj}(r_c) \end{aligned} \tag{2.1}$$

and

$$B^j_{I_cs_cI_{c'}s_{c'}} = \left\langle s_{c'}||s||s_c\right\rangle\left\langle I_{c'}||U(j)||I_c\right\rangle. \tag{2.2}$$

The coefficents B^j contain the nuclear structure information and for coherent processes the phasing is very important. The reduced matrix elements in B^j satify the symmetry property

$$\left\langle\gamma' j_{c'}||U(j)||\gamma j_c\right\rangle = \left\langle\gamma j_c||U(j)||\gamma' j_{c'}\right\rangle(-1)^{j+j_{c'}-j_c} \tag{2.3}$$

and the hermitean adjoint is defined as

$$U^\dagger(k\,q) = (-1)^{k+q}U(k\,-q). \tag{2.4}$$

Thus all of the spherical harmonics Y^m_l are assumed to carry a factor i^l with them. In particular for the rotational model of the nucleus the wave functions should be defined as in Bohr and Mottleson [Nuclear Structure Vol. II, W.A. Benjamin Inc., eq(4.39)].

a) ICODE = 0 Inelastic excitation using the strong coupling collective nuclear model.

The off-diagonal interaction potential for this model is given by

Application to various models

ICODE = 0 Collective Strong Coupling Model

$$V = \beta_\ell Y^0_\ell(\theta'_p) \left(-\frac{R_0}{a}\frac{dV(x)}{dx}\right) \quad , \quad x = \frac{r - R_0(\beta,\theta)}{a} \quad ,$$

where θ'_p is defined as the angle between the projectile coordinate and the body fixed frame of the nucleus. In terms of the formalism the potential is rearranged as

$$V = (-)^\ell \beta_\ell \hat{\ell}^{-1}\sqrt{4\pi}\left[\hat{\ell}\,(Y_\ell(\hat{\theta}_p)\,Y_\ell(\vec{\xi}_c))_0\right]\left(-\frac{R_0}{a}\frac{dV}{dx}\right).$$

Using the Bohr and Mottelson phase conventions for the nuclear wave functions the reduced matrix element is

$$B^{J\ell} = (-)^\ell \sqrt{2J_c+1}\sqrt{2I_c+1}\;(I_c\,\ell\,K\,0\,|\,I_{c'}K)\;i^{\ell+[c]-[c']} \quad ,$$

where [c] or [c′] = 0 for a positive symmetry state and

[c] or [c′] = 1 for a negative symmetry state.

The subroutine INTEG calculates

$$(-)^\ell\sqrt{(2J_{c'}+1)(2I_c+1)}\;(I_c\,\ell\,K\,0\,|\,I_{c'}K)\;i^{\ell+[c]-[c']} \quad ,$$

so that β_ℓ must be entered via the channel coupling cards.

ICODE = 1 Particle Transfer

1.1 Stripping

The interaction needed for stripping is

$$V = D_0\,(Q_j(r)\,a_j^\dagger)_0\sqrt{2j+1} \quad ,$$

hence

$$B^{J\ell} = D_0\,\langle J_{c'}\|A\|J_c\rangle\langle I_{c'}\|a_j^\dagger\|I_c\rangle$$

$$= (-)^{J+J_c-J_{c'}}\sqrt{2J_c+1}\sqrt{2I_{c'}+1}\;S_\beta^{\frac{1}{2}}\,S_\alpha^{\frac{1}{2}}\,D_0 \quad ,$$

where the definitions of the spectroscopic factor ,

$$S_\beta^{\frac{1}{2}} = (2I_{c'}+1)^{-\frac{1}{2}}\,\langle I_{c'}\|a_j^\dagger\|I_c\rangle \quad ,$$

is that of Bohr and Mottelson Eq(3E-8), and the $\sqrt{S_\alpha}$ is the projectile spectroscopic amplitude. The quantity calculated in INTEG is

$(-)^{\Delta+\Delta_c'-\Delta_c}\sqrt{(2\Delta_c+1)(2I_c'+1)}$ and the input on

the coupling card is

$$\beta_b^{1/2}\beta_a^{1/2}D_0 \ .$$

1.1a Strong Coupling Model

The spectroscopic amplitude

$$\beta^{\frac{1}{2}}(K_c\Lambda_c \to K_c'\Lambda_c')$$
$$= g\sqrt{\frac{2I_c+1}{2I_c'+1}}\ (I_c j \mp K_c \Omega_c' \pm \Omega_c | I_c' K)$$
$$\times \langle\alpha_c'|\alpha_c\rangle C_{jc}(\Omega_c' \pm \Omega_c) \ ,$$

where $g=\sqrt{2}$ if $K_1 = \Omega_1 = 0$ or $K_2 = \Omega_2 = 0$ and unity otherwise. The $\mp$ is taken to be whether the captured particle transfers to an orbit $\Omega=|\Omega_c\pm\Omega_c|$ and the factor $\langle\varphi_c'|\varphi_c\rangle$ is the core overlap.

1.1b Weak Coupling Model

Here the final state is represented usually as $\Psi_{j'}=(\varphi_j Y_{I_c})j'$. The spectroscopic amplitude is

1.2 Pickup

This reaction is related to the stripping reaction by time reversal

$$B^{1\ell}_{p.u.} = B^{j\ell}_{str.}\cdot(-)^{I_c+j-I_c'+\Delta_c'+\Delta-\Delta_c}\cdot\frac{\hat{I}_c\,\hat{\Delta}_c'}{\hat{I}_c'\,\hat{\Delta}_c} \ .$$

The phase and $\hat{I}_c\hat{\Delta}_c'/\hat{I}_c'\hat{\Delta}_c$ factor are computed in INTEG hence only $D_0\beta_A^{\frac{1}{2}}\beta_b^{\frac{1}{2}}$ need be entered by the channel coupling cards

ICODE = 3 Weak Coupling Model

The interaction is the same as for the strong coupling model

$$V = \beta_2\, Y_2^0(\theta_{R'})\left(-\frac{R}{a}\frac{dV}{dR}\right) \ .$$

The wave function is given by

$$\psi_{I_c} = (\alpha_j\, \Psi(J_c K_c))_{I_c}$$

where $\Psi(J_c K_c)$ is the rotational model of Bohr and Mottelson (4-39). The

matrix element

$$B^{1}_{I_c I_{c'} \lambda_c \lambda_{c'}} = (-)^{\ell} \sqrt{2\lambda_{c'}+1}\; i^{\ell} \langle j\, J_c, I_{c'} \| Y_\ell \| j\, J_c\, I_c \rangle$$

$$= (-)^{\ell} \sqrt{2\lambda_{c'}+1} \quad i^{\ell + [c] - [c']} (-)^{\ell + j - I_c - J_{c'}} \sqrt{2J_c+1}$$

$$\times \sqrt{(2J_c+1)(2I_{c'}+1)} \quad W(J_{c'} I_{c'} J_c I_c ; j \ell) \langle J_c\, \ell\, K 0 \mid J_{c'} K \rangle .$$

For the present only a very simple case of this expression can be used. The states are limited to

$$J_c = 0$$

$$I_c = j \ .$$

In this case

$$B^{1}_{I_c I_{c'} \lambda_c \lambda_{c'}} = (-)^{\ell} \sqrt{2\lambda_{c'}+1} \quad i^{\ell + [c] - [c']} \sqrt{(2I_{c'}+1)/(2\ell+1)} \ .$$

The states must be entered in the program such that states with $J'_c \neq 0$ must come after the state $J_c = 0$ in the ordering of the channels.

Special cases for strong coupling model spectroscopic factors

1.1a1 Stripping on an even nucleus with K=0

$$K_c = \Omega_c = 0 \qquad K_{c'} = \Omega_{c'} = K$$

$$S^{\frac{1}{2}}_{\alpha} = \sqrt{\frac{2}{2I_{c'}+1}} \quad \langle I_c\, j\, 0\, K \mid I_{c'} K \rangle \; C_{j\ell}(K) \; \langle \varphi_{c'} \mid \varphi_c \rangle \ .$$

1.1a2 Stripping on an odd nucleus going to an even nucleus with $K'_c = 0$

$$K_c = \Omega_c = K \qquad K_{c'} = \Omega_{c'} = 0$$

$$S^{\frac{1}{2}}_{\alpha} = \sqrt{2} \sqrt{\frac{2J_c+1}{2I_{c'}+1}} \quad \langle I_c\, j\, +K\, -K \mid I_{c'} 0 \rangle \; C_{j\ell}(-K) \; \langle \varphi_{c'} \mid \varphi_c \rangle$$

$$= \sqrt{2} \quad \langle I_{c'}\, j\, 0\, K \mid I_c K \rangle \; (-)^{I_c - I_{c'} + j} \; C_{j\ell}(K) \; \langle \varphi_{c'} \mid \varphi_c \rangle$$

where use has been made of

$$C_{j\ell}(-K) = C_{j\ell}(K) (-)^{j+K} \ .$$

5) Microscopic Interaction Model for Inelastic Scattering

a) Central interactions

The potential between the projectile and one of the target nucleons may be expanded in a Legendre polynomial series

$$V(\underset{\sim}{r}_0 - \underset{\sim}{r}_1) = V_0 \sum_\ell (2\ell+1)\, v_\ell(r_0, r_1)\; P_\ell(\hat{r}_0 \cdot \hat{r}_1)$$

$$= 4\pi V_0 \sum_\ell v_\ell(r_0, r_1) \sum_m Y_\ell^m(\hat{r}_0)^*\, Y_\ell^m(\hat{r}_1) .$$

Using this form we find for the matrix element of V

$$\langle J_B M_B s_b m_b | V(r_0 - r_1) | J_A M_A s_a m_a \rangle =$$

$$4\pi V_0 \sum_\ell \langle J_A \ell M_A M_B - M_A | J_B M_B \rangle \langle J_B || v_\ell(r_0, r_1)\, i^\ell Y_\ell || J_A \rangle$$

$$\times \langle s_b s m_b m_a - m_b | s_a m_a \rangle (-i)^\ell\; Y_\ell^m(\hat{r}_0)^* \frac{1}{\sqrt{2J_B+1}} .$$

The program DWUCK calculates for the form factor the expression

$$FF = 4\pi V_0 \sqrt{2J_A+1}\; \langle j_1' j_2 J_B || Y_\ell(\hat{r}_1) || j_1 j_2 J_A \rangle$$

$$\times \int R_{\ell' j_1'}(r_1)\, v_\ell(r_0, r_1)\, R_{\ell j_1}(r_1)\, r_1^2\, dr_1 ,$$

where the $R_{\ell j}(r_1)$ are the normalized radial functions for the initial and final nucleon states, and the reduced matrix element $\langle j_1' j_2 J_B || Y_\ell || j_1 j_2 J_A \rangle$ is defined by Edmonds.[1] The quantity j_2 is the spin for the coupling of the A-1 nucleons. The reduced matrix element $\sqrt{4\pi}\, \langle j_1' j_2 J_B || Y_\ell || j_1 j_2 J_A \rangle$ is printed out as RME in the output.

If there is more than one particle in the shell then the strength VZERO in the microcopic form factor option for each configuration is to be multiplied by

the factor accounting for the number of identical particles and amplitudes for the configurations used. Thus

$$B_{\ell_0 \ell} = a_{j_1 j_2 J_A} a_{j_1' j_2 J_B} \sum_{\ell} \langle j_1 j_2 J_B \| Y_\ell(i) \| j_1 j_2 J_A \rangle / \langle j_1' j_2 J_B \| Y_\ell(i) \| j_1' j_2 J_B \rangle$$

If VZERO includes $B_{\ell_0 \ell}$, then

$$\frac{d\sigma^\ell}{d\Omega} = \sigma_{0\omega}^{\ell_0 \ell}(\theta)$$

b) $\sigma_i \cdot \sigma_j$ interaction

A central interaction $V(r_0 - r_i)$ with a $\vec{\sigma}_0 \cdot \vec{\sigma}_1$ dependence can be written in the following form

$$V(\underset{\sim}{r}_0 - \underset{\sim}{r}_i)\, \underset{\sim}{\sigma}_0 \cdot \underset{\sim}{\sigma}_i = 4\pi V_0 \sum_{\ell j \mu} v_\ell(r_0, r_i)(-)^{\ell - 1 + \mu + 1} \mathcal{Y}_{\ell 1 j - \mu}(\hat{r}_0)\, \mathcal{Y}_{\ell 1 j \mu}(\hat{r}_i)$$

where

$$\mathcal{Y}_{\ell 1 j \mu} = \sum_m (\ell\, 1\, \mu\, \mu - m \mid j \mu)\, i^\ell\, Y_\ell^m(\theta \omega)\, \sigma^{\mu - m}$$

The matrix element for V in the transition amplitude is

$$\langle J_B M_B s_b m_b | V | J_A M_A s_a m_a \rangle$$

$$= 4\pi V_0 \sum_{\ell j \mu} \langle \ell\, s\, m\, \mu - m \mid j \mu \rangle (-i)^\ell\, Y_\ell^m(\hat{r}_0)^*$$

$$\times \langle s_b\, s\, m_b\, m_a - m_b \mid s_a m_a \rangle \sqrt{2 s_a (s_a + 1)}\, (-)^{2j}$$

$$\times \langle j_1' j_2 J_B \| v_\ell(r_0, r_i)\, i^\ell\, \mathcal{Y}_{\ell 1 j}(\hat{r}_i, \sigma_i) \| j_1 j_2 J_A \rangle$$

$$\times \langle J_A\, j\, M_A\, M_B - M_A \mid J_B M_B \rangle \frac{1}{\sqrt{2 J_B + 1}},$$

where s=1 and $s_a=s_b$.

The form factor computed by DWUCK4 is

$$f_{\ell s j}(r_0) = 2\sqrt{S_a(S_a+1)}\; i^{\ell} \langle j_1' J_1' \| V_\ell(r_0 r_1)\, Y_{\ell i}(\hat{r}_1 \vec{s}_1) \| j_1 J_1 J_A \rangle \times \sqrt{\frac{(2\ell+1)}{(2j+1)(2J_A+1)}}$$

In the above expressions for the form factor we have used

$$\langle S_b \| \sigma_0 \| S_a \rangle = 2\sqrt{S_a(S_a+1)}$$

which is the reduced matrix element appropriate to inelastic scattering $s_a=s_b$. If the spin of the projectile changes such as in (^{6}Li, ^{6}He) inelastic scattering then the appropriate matrix element ratio

$$\langle S_a \| \sigma_0 \| S_b \rangle / \left(2\sqrt{S_a(S_a+1)(2S_b+1)}\right)$$

must multiply the form factor.

The quantity is

$B_{\ell ij} = -1$ when $s_a=s_b$.

We obtain the cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{DW}^{\ell ij}.$$

Note: It should be pointed out again that the cross section for the microscopic interaction is done in terms of a single particle transition $j_1 \to j_1'$. If there is more than one equivalent particle then the matrix element ratio

$$\sum_i \langle J_B \| Y_{\ell i} \| J_A \rangle / \langle j_1' j_2 J_B \| Y_{\ell i}(r_1) \| j_1 j_2 J_A \rangle$$

will have to multiply the $B_{\ell sj}$ factors which are outlined here.

6) Two-nucleon transfer option

The description for the (p,t) reaction is given in the article, H. Baer et al., Annals of Physics (NY) 76, 437 (1973), Appendix.

The usual normalization condition gives the cross section for (p,t) as

$$\frac{d\sigma}{d\Omega} = D_0^2 \left(\pi\Delta^2/2\right)^{3/2} \left(T_B \| N_B \| T_A N_A\right)^2 \sigma_{DW}^{lsj} / (2j+1),$$

where $(\pi\Delta^2/2)^{3/2} = 9.72$ fm^3 for $\Delta = \Delta' = 1.7$ fm.

For the (t,p) reaction, we have

$$\frac{d\sigma}{d\Omega} = D_0^2 \left(\pi\Delta^2/2\right)^{3/2} \left(T_A \| N_A \| T_B N_B\right)^2 \frac{2T_B+1}{2T_A+1} \sigma_{DW}^{lsj} / (2j+1).$$

The value of D_0^2 ranges from 15 - 30 MeV^2-Fm^3.

August 8, 1977

Users of DWUCK and CHUCK:

There has been a major change in the codes in regard to the phasing of the microscopic inelastic and two-nucleon transfer form factor. This change also carries over in CHUCK to the sequential transfer case. The new two-particle wave functions should carry an extra phase in the amplitude of $(i)^{\ell_1 + \ell_2 - LTR}$ over the old amplitude convention. This will cause a ground state to ground state L=0 transition to have all positive coefficients, e.g. ^{90}Zr

$$\psi = +.8(p_{1/2})^2 + .6(g_{9/2})^2.$$

Another change has been made in CHUCK for the collective form factor (ICODE=0). When using options 1, 2, or 3, the power of the (R/a) factor starts with <u>one</u>, not zero, so that the usage is now consistent with DWUCK,

Form factor $\propto$ (R/a)**(POWR + 1.0).