

Lecture Notes in Nuclear Structure Physics ¹

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Chapter 1

The Woods-Saxon potential

1.1 General form

The Woods-Saxon potential is a convenient phenomenological choice for the one-body potential. It provides a model for the properties of bound-state and continuum single-particle wavefunctions. The Woods-Saxon potential (or any other one-body potential) cannot be used for the total binding energy since it is not based upon a specific two-body interaction. The parameters of the Woods-Saxon are chosen for a best fit of nuclear single-particle energies and nuclear radii. The Woods-Saxon potential is based upon the sum of a spin-independent central potential, a spin-orbit potential, and the Coulomb potential:

$$V(r) = V_o(r) + V_{so}(r) \vec{\ell} \cdot \vec{s} + V_c(r), \quad (1.1)$$

where $V_o(r)$ is the spin-independent central potential:

$$V_o(r) = V_o f_o(r), \quad (1.2)$$

with a fermi shape

$$f_o(r) = \frac{1}{1 + [\exp(r - R_o)/a_o]}, \quad (1.3)$$

$V_{so}(r)$ is the spin-orbit potential:

$$V_{so}(r) = V_{so} \frac{1}{r} \frac{df_{so}(r)}{dr}, \quad (1.4)$$

with

$$f_{so}(r) = \frac{1}{1 + [\exp(r - R_{so})/a_{so}]}, \quad (1.5)$$

and $V_c(r)$ is the Coulomb potential for protons based upon the Coulomb potential for a sphere of radius R_c :

$$V_c(r) = \frac{Ze^2}{r} \text{ for } r \geq R_c$$

and

$$V_c(r) = \frac{Ze^2}{R_c} \left[\frac{3}{2} - \frac{r^2}{2R_c^2} \right] \text{ for } r \leq R_c. \quad (1.6)$$

The radii R_o , R_{so} and R_c are usually expressed as:

$$R_i = r_i A^{1/3}. \quad (1.7)$$

For nuclei with a neutron excess the protons will feel a stronger potential than the neutrons, since the average proton-neutron potential is stronger than the average neutron-neutron (or proton-proton) potential. Thus we take:

$$V_{op} = V_0 + \frac{(N - Z)}{A} V_1 \text{ for protons} \quad (1.8)$$

and

$$V_{on} = V_0 - \frac{(N - Z)}{A} V_1 \text{ for neutrons} \quad (1.9)$$

In principle, r_o and a_o could also be a little different for proton and neutrons in a nucleus with $N \neq Z$. Thus the spin-independent potential could have six parameters (and even more if any of them are allowed to take some additional mass dependence). The values of these parameters are chosen to give an overall accounting of the observed single-particle energies, the rms charge radii, and the electron scattering form factors.

The form of the spin-orbit interaction was originally taken from the form of the spin-orbit interaction used for electrons in atoms which was derived from the Dirac theory by Thomas and Frenkel [1]. For nuclear physics the spin-orbit interaction ultimately comes from the nucleon-nucleon interaction, but there is still some debate over whether or not relativistic effects are very important. The spin-orbit interaction for nucleons in the nucleus has the opposite sign and is much larger than for electrons in the atom. In the nuclear interior a nucleon sees an equal number of spin-up and spin-down nucleons on both sides and the spin-orbit interaction must vanish. Thus, the spin-orbit potential is peaked at the nuclear surface as in Eq. (1.4), and the particular form of this equation gives a good overall description of the ℓ and mass dependence observed experimentally. In analogy to the spin-independent potential, one could introduce up to six parameters for the spin-orbit interaction. The strength of the spin-orbit interaction is chosen to reproduce spin-orbit splittings (especially for large ℓ). For $N \neq Z$ the strength V_{so} could be different for protons and neutrons, but in practice they are nearly the same. For the geometry one usually takes $r_{so} = r_o$ and $a_{so} = a_o$. Thus, the introduction of one parameter in the spin-orbit interaction V_{so} gives a good overall accounting of the data.

A typical set of parameters for the Woods-Saxon potential is $V_0 = -53$ MeV, $V_1 = -30$ MeV and $V_{so} = 22$ MeV for the strengths, and $r_o = r_{so} = 1.25$ fm and $a_o = a_{so} = 0.65$ fm for the geometry. For the Coulomb term the radius is a little smaller with $r_c = 1.20$ fm. One can find in the literature many other sets of parameters which are better for specific nuclei or mass regions.

The Woods-Saxon potential is written in terms of a potential for a core nucleus specified by A and Z . The solutions for the energy and wavefunctions for a nucleon “in this core” will present those for both the filled and empty orbitals and relationship of these quantities to experimental observables will be assumed to be the same as those obtained in the Hartree-Fock approximation.

Fig. (1.1) from page 239 of Bohr and Mottelson Vol I [2] shows the neutron single-particle energies for nuclei near the valley of stability obtained with a set of Woods-Saxon parameters similar to those given above (note that the radial quantum numbers in this figure start with one rather than with our convention of zero). One observed that as A increases more single-particle states become bound so that the energy of the most loosely bound filled orbit is always around -8 MeV.

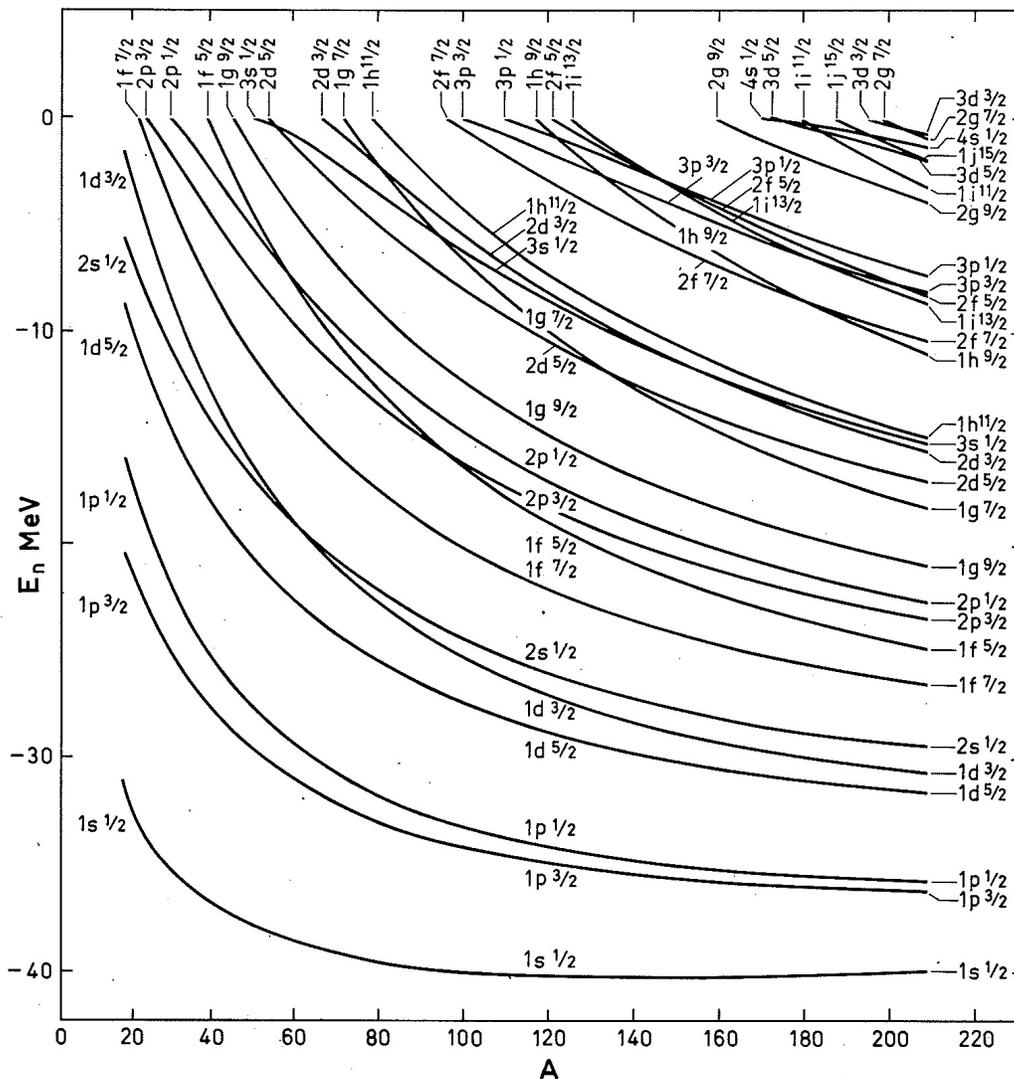


Figure 1.1: Neutron single-particle energies for nuclei near the valley of stability as a function of mass (from page 239 of Bohr and Mottelson Vol I [2]).

1.2 Computer program for the Woods-Saxon potential

The program WSPOT.FOR can be used to obtain the single-particle energies and single-particle radial wavefunctions for the bound states of the Woods-Saxon potential with quantum numbers n_r , ℓ and j . (It could also be used for other potentials if the program is modified.) It can also be used to calculate the nucleon scattering cross sections for a given ℓ and j . values. The program is initialized with a typical set of Woods-Saxon parameters:

$$\begin{aligned}V_0 &= -53 \text{ MeV}, \\V_1 &= -30 \text{ MeV}, \\V_{so} &= 22 \text{ MeV}, \\r_o = r_{so} &= 1.25 \text{ fm}, \\a_o = a_{so} &= 0.65 \text{ fm and} \\r_c &= 1.20 \text{ fm}.\end{aligned}$$

Some parameters can be modified in the input files. Other parameters can be changed internally to the program and then program can be recompiled.

The input file is called x.dai. To run the wspot program with this input file type

```
wspot x
```

The output in the file x.dao.

The following provides some sample inputs for various type of calculations.

1.2.1 Example for bound states

To obtain results for protons in ^{16}O the input is, o16.dai,

16 8 1 1

0 0 1

0 1 3

0 1 1

1 0 1

0 2 5

0 2 3

The first line is (A, Z) of the nucleus and (a, z) of the nucleon. A proton would be $(a, z) = (1, 1)$ and a neutron is $(a, z) = (1, 0)$. After the blank line the next lines are a list of $n_r, \ell, 2j$ values.

The output will give $(n_r, \ell, 2j, \text{spe}, k, \text{kt}, \text{rms}, \text{rmst})$

for each orbit, where spe is the single-particle energy, k is the expectation value of the kinetic energy $\langle T \rangle$, and rms is $\sqrt{\langle r^2 \rangle}$. The program will indicate if the orbit is unbound. Kt means

$$\sum_i (2j_i + 1) \langle T \rangle_i$$

where the sum runs over all orbits up to the current one, with the assumption that there are $(2j_i + 1)$ nucleons in orbit j_i . At the end this will tell you the total kinetic energy for neutrons (if the complete set is included in the input). Rmst is obtained from:

$$x_1 = \sum_i (2j_i + 1) \langle r^2 \rangle_i$$

$$x_2 = \sum_i (2j_i + 1)$$

where the sum runs over all orbits up to the current one. $\text{Rmst} = \sqrt{(x_1/x_2)}$ is the total rms radius for all orbits up to the current one.

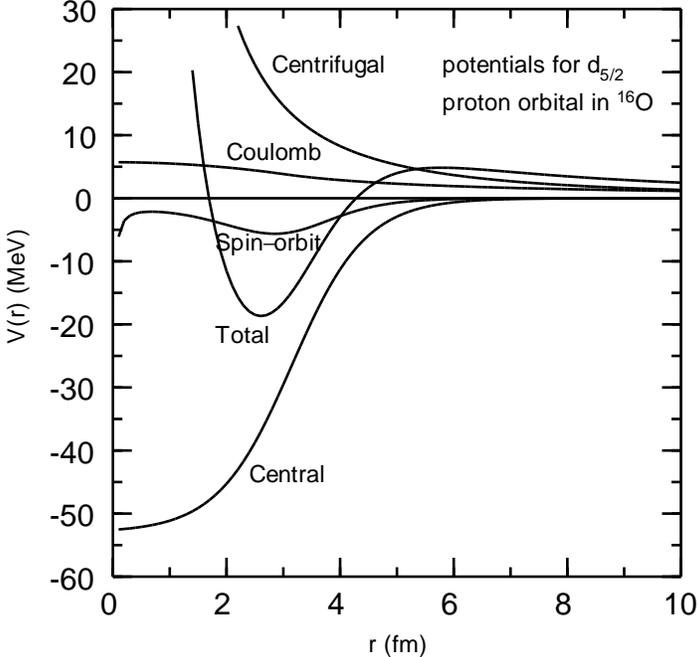


Figure 1.2: Woods-Saxon potential for a $0d_{5/2}$ proton with an ^{16}O core. The total and parts of the potential are shown.

1.2.2 Changing the potential parameters

In order to change the parameters of the central spin-independent potential the input can be modified as shown below for the example of a neutron in ^{40}Ca , ca40.dai,

```
40 20 1 0

0 0 1 0.9 0.7 1.3
0 1 3
0 1 1
1 0 1
0 2 5
0 2 3
```

where $V_N = 0.9$ modifies the central potential strength to $V'_o = V_N V_o$, and a_o and r_o are changed to $a'_o = 0.7$ and $r'_o = 1.3$. These values are used for this and all following orbits. Other parameters can be changed by modifying the program.

One can loop over both protons and neutrons in the following way, ca40b.dai,

```
40 20 1 1

0 0 1 0.9 0.7 1.3
0 1 3
0 1 1
1 0 1
0 2 5
0 2 3
-1
40 20 1 0

0 0 1 0.9 0.7 1.3
0 1 3
0 1 1
1 0 1
0 2 5
0 2 3
```

1.2.3 Width of an unbound state resonance

For a given (ℓ, j) value the program calculates the phase shift $\delta(E)$ and the scattering cross section $\sigma(E)$ as a function of (positive) energy, where

$$\sigma(E) = \frac{4\pi(2\ell + 1)}{k^2} [\sin(\delta)]^2$$

where $E = \hbar^2 k^2 / 2m$ (m is the mass of the nucleon).

As an example, the input for the $0d_{5/2}$ proton resonance in ^{13}N , n13.dai,

```
12 6 1 1
0.1 10.0
0 2 5
```

The first line is (A, Z) of the “core” nucleus and (a, z) of the nucleon. The next line contains e_{\min} , e_{\max} for the energy interval over which to look for a resonance. The next line contains $n_r, \ell, 2j$.

If a resonance peak is found in that interval, the program iterates over a series of steps in which this energy interval is narrowed down until the full resonance curve is found. It outputs $e_i = E_r$ and $G = \Gamma$ where E_r is the resonance energy peak and Γ is the full width at half maximum of the resonance. This particular input gives $E_r = 0.99$ MeV and $\Gamma = 0.010$ MeV. The output for the cross section vs E is given in the output file x.dao. Also a x.top file is made. Typing “cps x” will produce an x.eps file that shows a picture of the resonance.

The width is obtained by finding the values of sigma above and below the peak energy where the value is a factor of two smaller than that at the peak. No attempt is made to subtract a possible background due to non-resonant scattering. Making the appropriate modifications for subtracting this background would make an addition to this code.

For the scattering n_r is not a quantum number and the value in the input has no effect on the output. The value given in the x.dao file indicates how often the wavefunction crosses zero. Sometimes the resonance is a quasi-bound state corresponding to a particular the n_r value of interest. One should check in the output that the correct n_r value is given. If not, the correct value may correspond to a different energy range (or to a different value for V_N discussed in the next section).

A resonance width is useful if the resonance is reasonably sharp. Broad resonances will be asymmetric and the width itself is not a full measure of its nature. Rather one must look at the full function of σ vs E . Narrow resonances (if they exist) are at an energy below the Coulomb plus centrifugal barrier. For these light nuclei this is not more than 10 MeV which is what determines e_{\max} in this input.

1.2.4 Width of an unbound state resonance at a fixed energy

Experimentally one observes a $5/2^+$ resonance at 3.547 MeV in ^{13}N with a one-proton resonance energy of $E_r=1.60$ MeV. This is below the energy (0.99 MeV) obtained from the standard potential. We could ask - if the potential is slightly adjusted so that the resonance is exactly at 1.60 MeV then what is the width? We need to slightly reduce the potential in order to increase the resonance energy. We could do this by hand, for example, n13b.dai,

```
12 6 1 1
0.1 10.
0 2 5 0.96
```

where the potential depth has been multiplied by 0.96 to give the resonance energy at 1.68 MeV (closer but not exactly the required value). We would then change V_N by trial and error until the resonance is at the correct position. However, the program will automatically search for the correct V_N by the following input, n13c.dai,

```
12 6 1 1
1.60 1.60 0.8 1.2
0 2 5
```

The input 1.60,1.60,0.8,1.2 will search on V_N over the interval 0.8 to 1.2 to put the $0d_{5/2}$ resonance at exactly 1.6 MeV. Then the program will scan from $E_r/2$ to $2E_r$ to find the width. The result is 64 keV which is not far from the observed width of 47(7) keV. The ratio of experiment over theory provides a measure of the spectroscopic factor, $S = 0.73(9)$, in this case.

Bibliography

- [1] Thomas, *Nature* **107**, 514 (1926); Frenkel, *Zeits. fur Phys.* **37**, 243 (1926).
- [2] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. I, (W. A. Benjamin, 1969), *Nuclear Structure* Vol. II, (W. A. Benjamin, 1975).