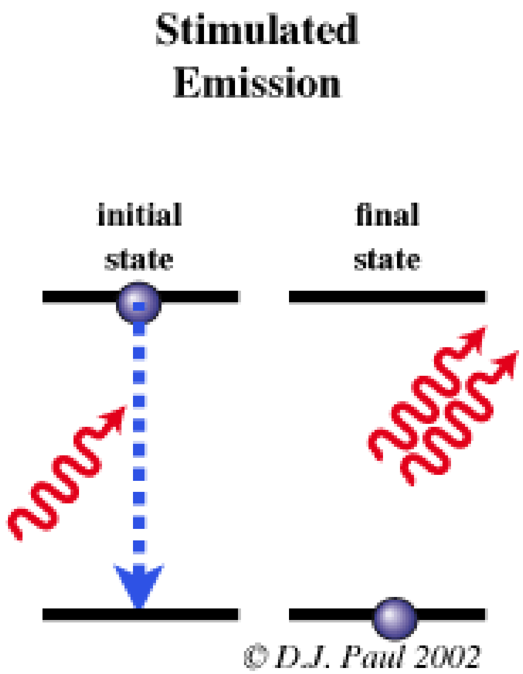
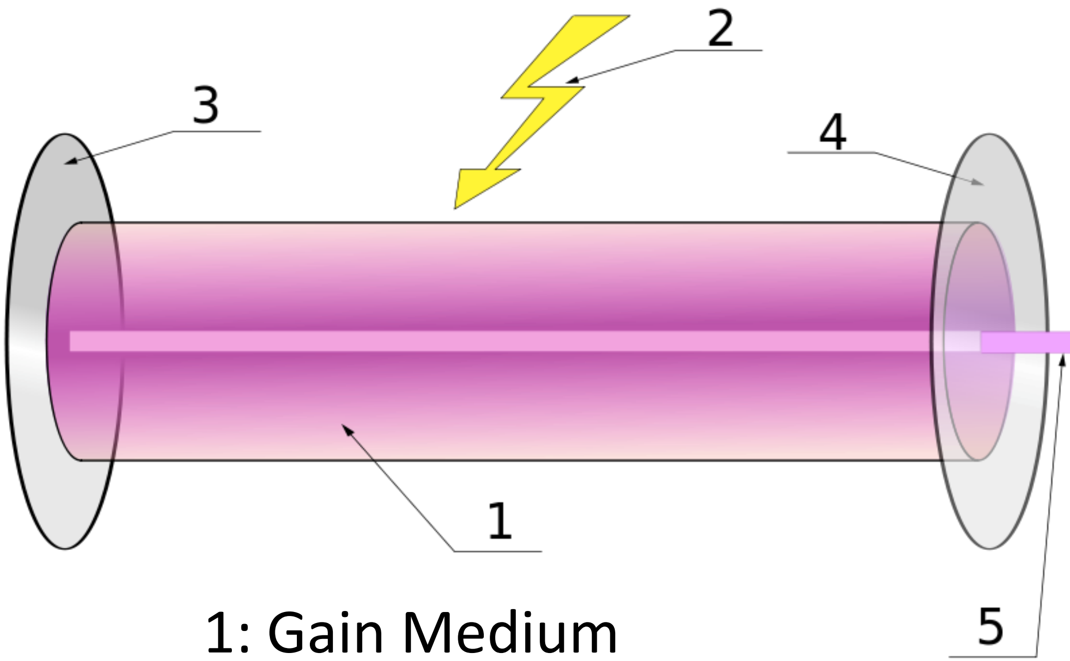
Free Electron Laser

Tradition Laser





Materials	Wave length (nm)
CO2	10600
TI: Sapphire	800 (650-1100)
Yb: YAG	1030
Dye lasers	390-1000

1: Gain Medium
2. Pumping energy
3. Reflctor
4. Output coupler
5. Laser output.

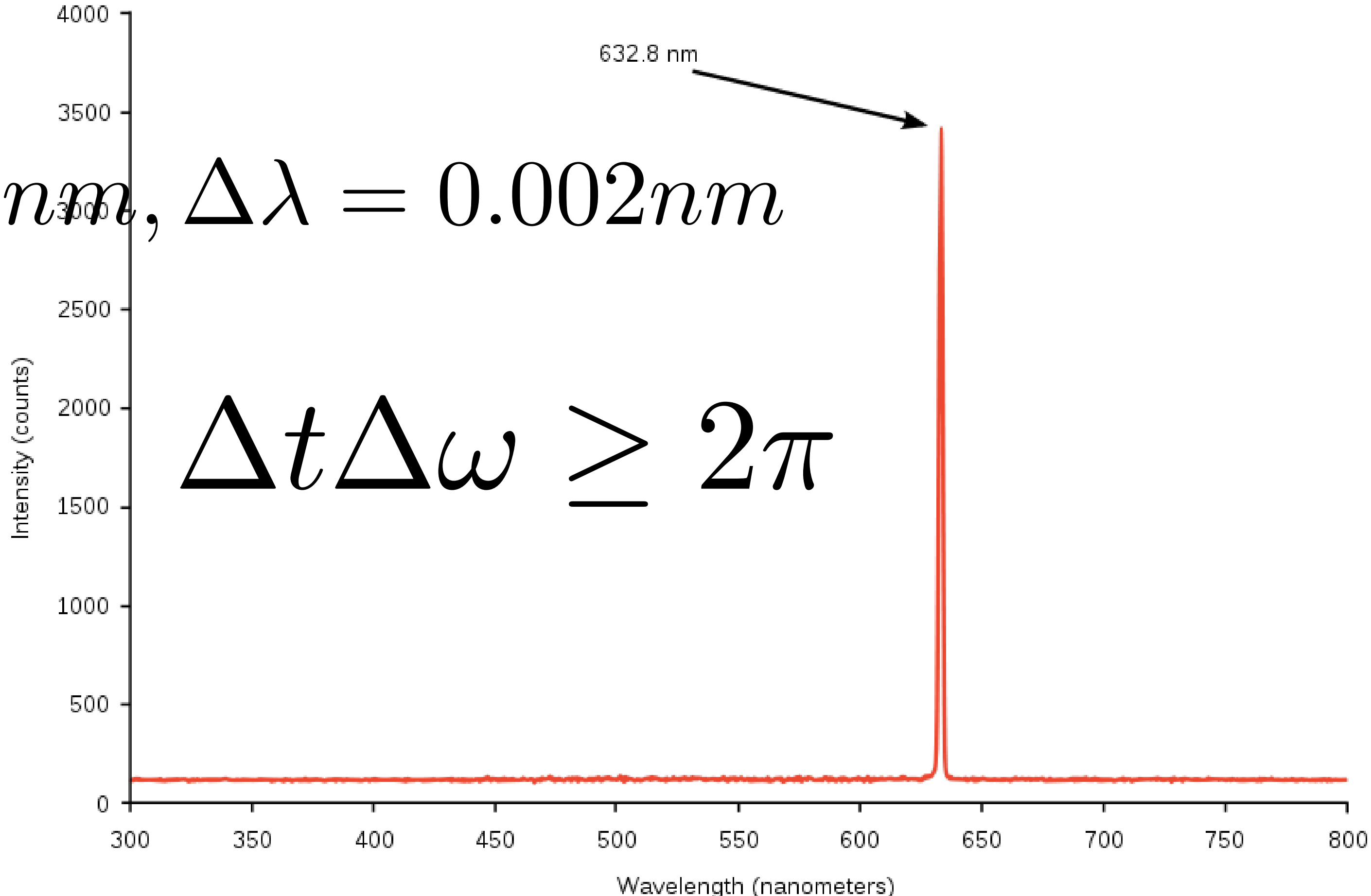
@Wikipedia

Properties of Laser I

- Almost single frequency

$$\lambda = 632nm, \Delta\lambda = 0.002nm$$

$$\Delta t \Delta\omega \geq 2\pi$$

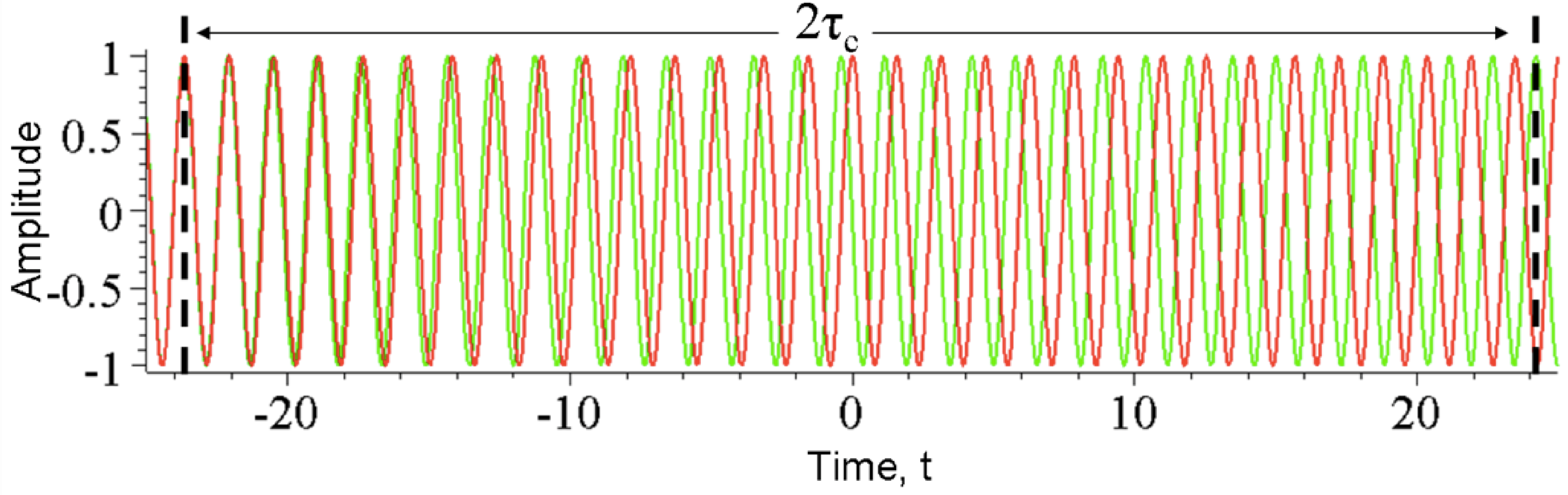


Properties of Laser II

- Monochromatic light source leads to 'temporal coherence'.

$$\xi(\tau) = \langle E(t)E(t+\tau)\rangle_t$$

- Beyond certain time, the wave becomes irrelevant



Properties of Laser III

- The coherence time:

$$\tau_c \sim 1/\Delta_f$$

- And the temporal coherence length

$$L_T = c\tau_c \sim \lambda^2/\Delta\lambda$$

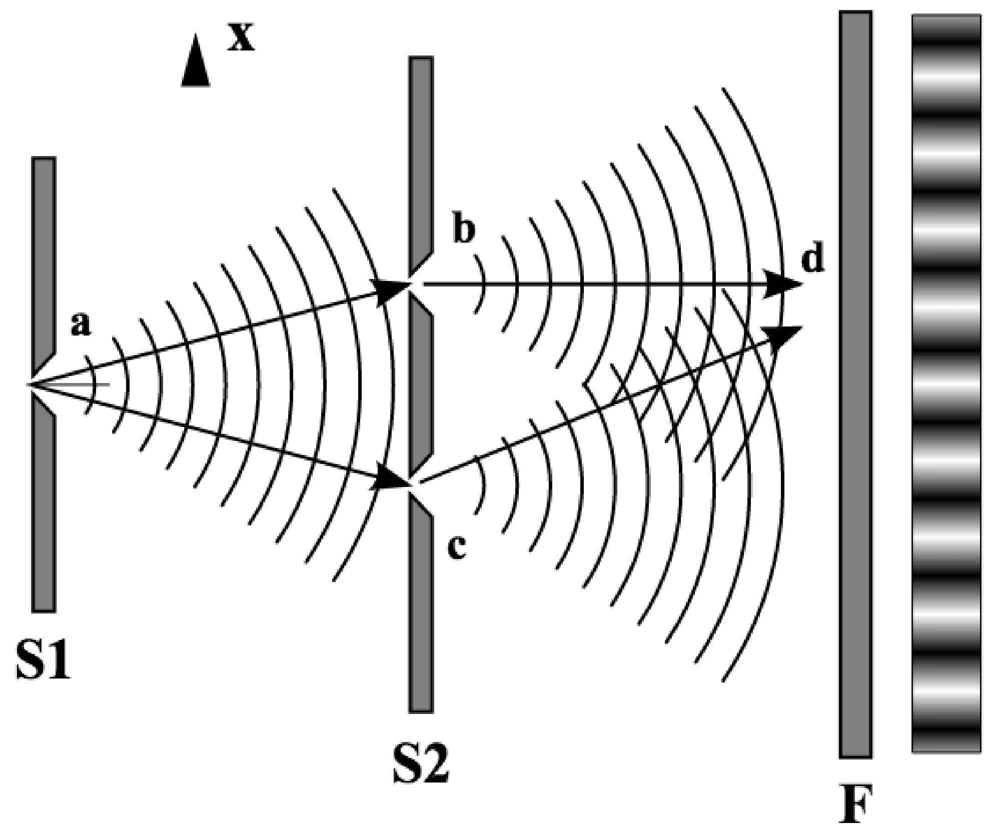
Properties of Laser IV

- Spatial coherence
- In Young's experiement the wave at b and c must be coherent

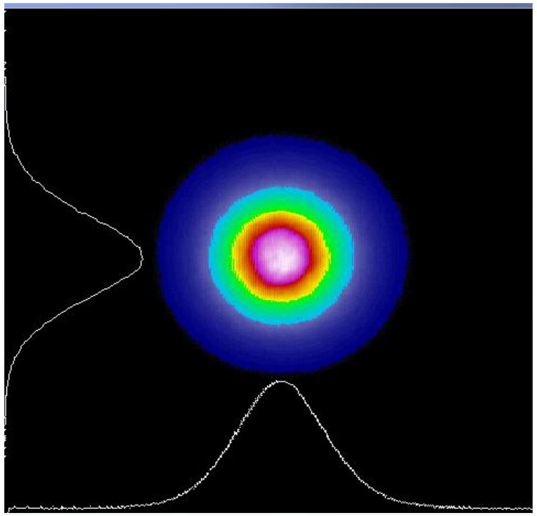
$$d_a\theta \leq \lambda$$

Spatial coherence length:

$$L_s \sim \frac{\lambda R}{d}$$



Gaussian Wave Packet



Emittance:

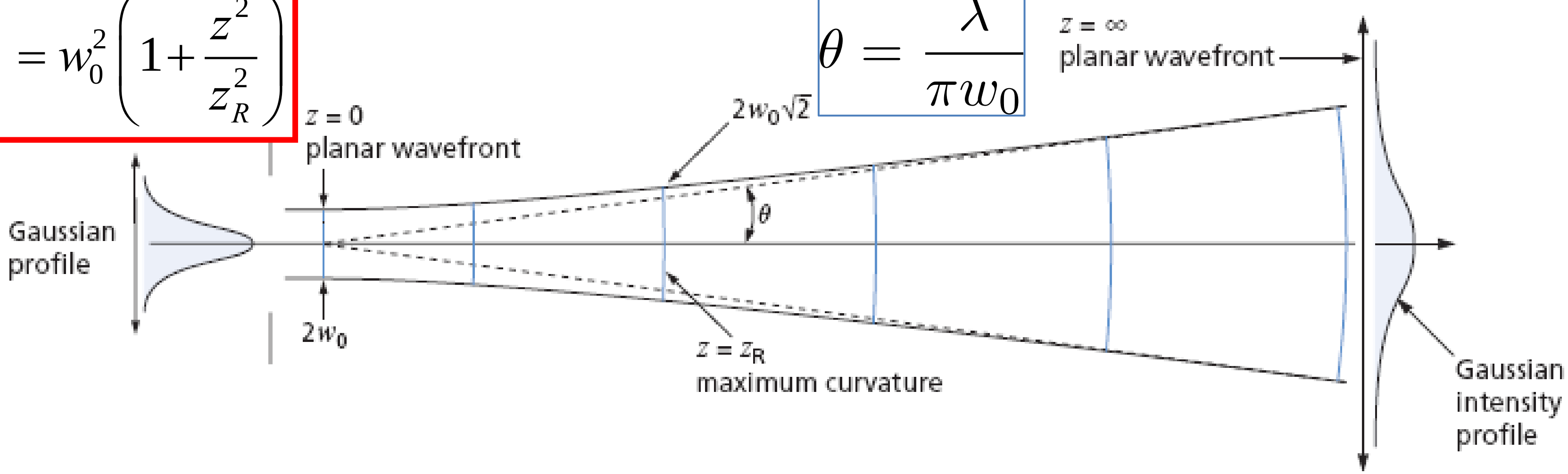
$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

Rayleigh length:

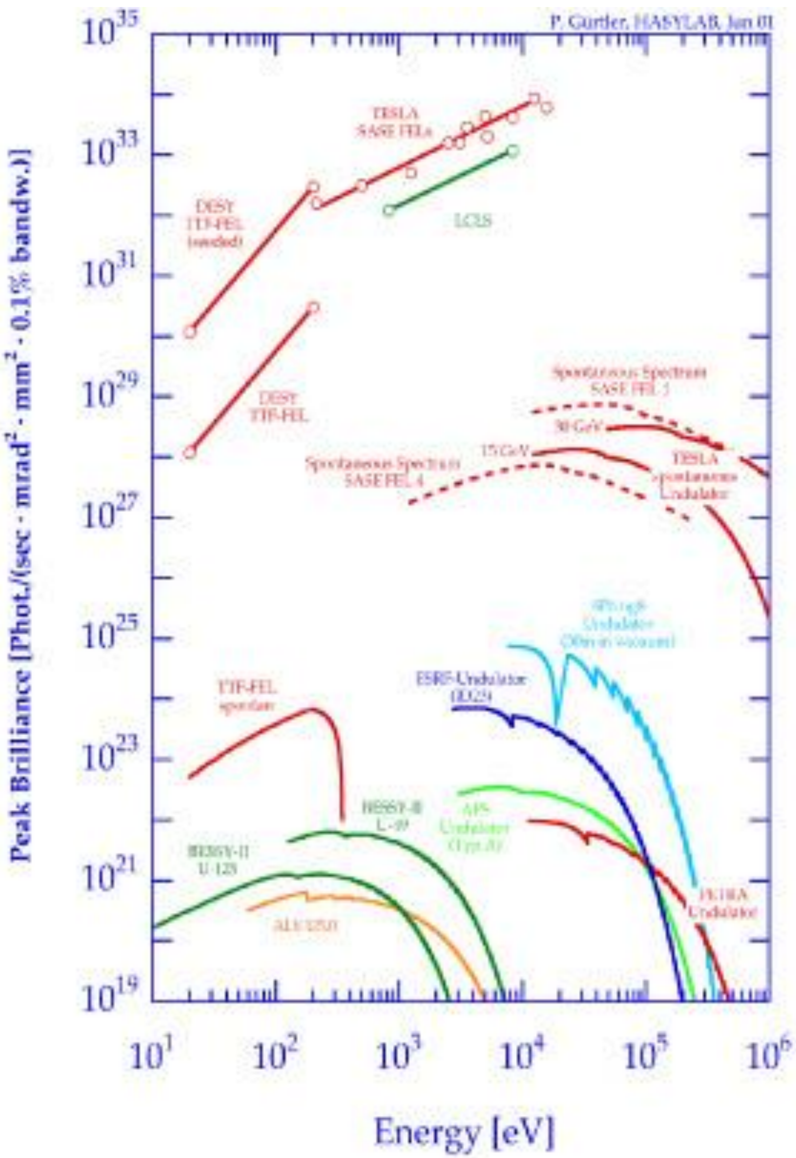
$$Z_R = \frac{\pi w_0^2}{\lambda}$$

$$w^2 = w_0^2\left(1 + \frac{z^2}{z_R^2}\right)$$

$$\theta = \frac{\lambda}{\pi w_0}$$



From SR Light Source to FEL



- Transversely SR is not coherent.
  - Emittance in x is ~nm, too large for angstrom x-ray
- Longitudinal not coherent
  - Micro bunching trick

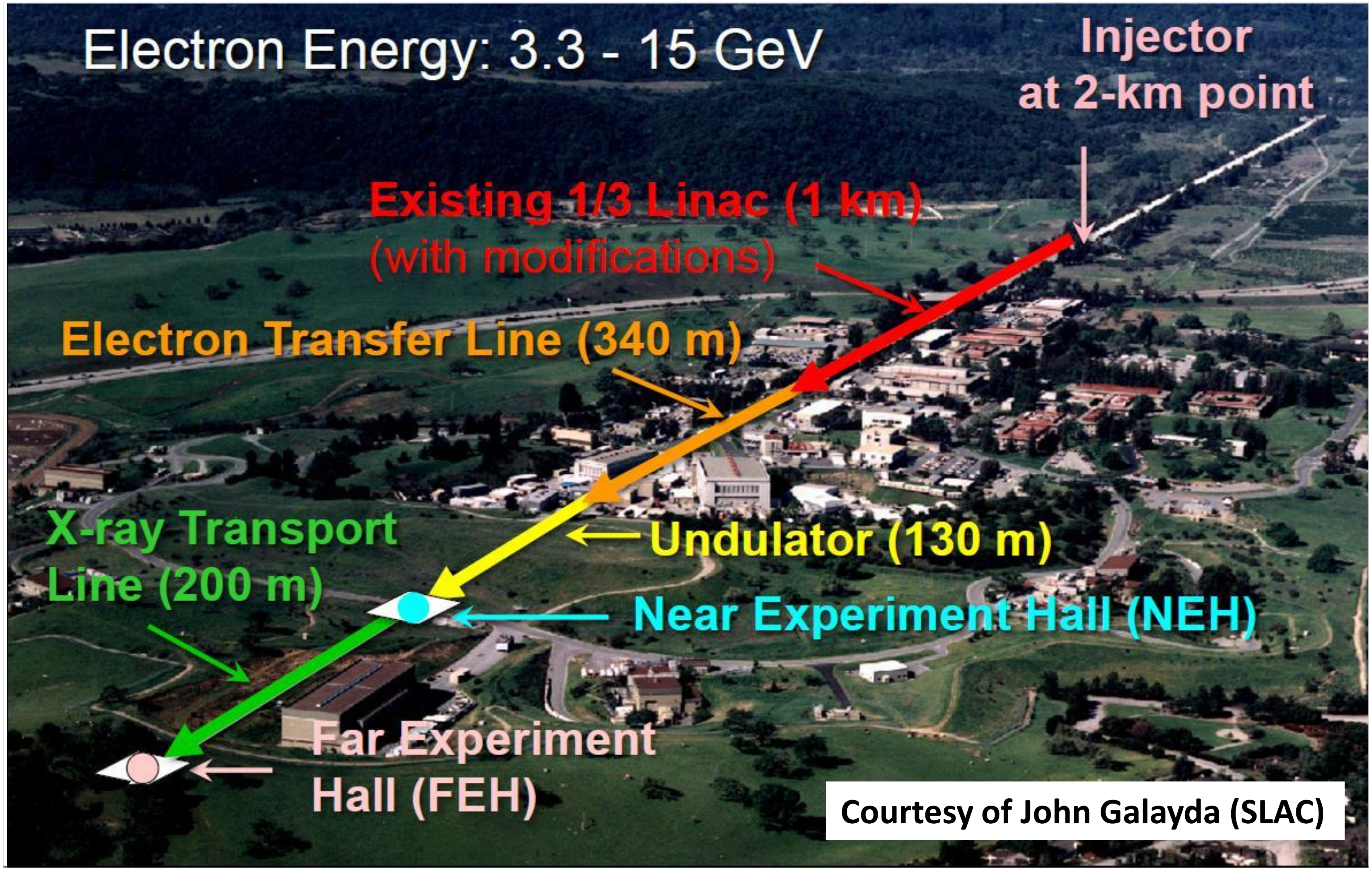
Promising New Lasing Method

- Tuning ability, wave length adjustable by undulator strength, or energy of the e-beam
- Can reach wavelength region that is lack of traditional laser. (x-ray!)

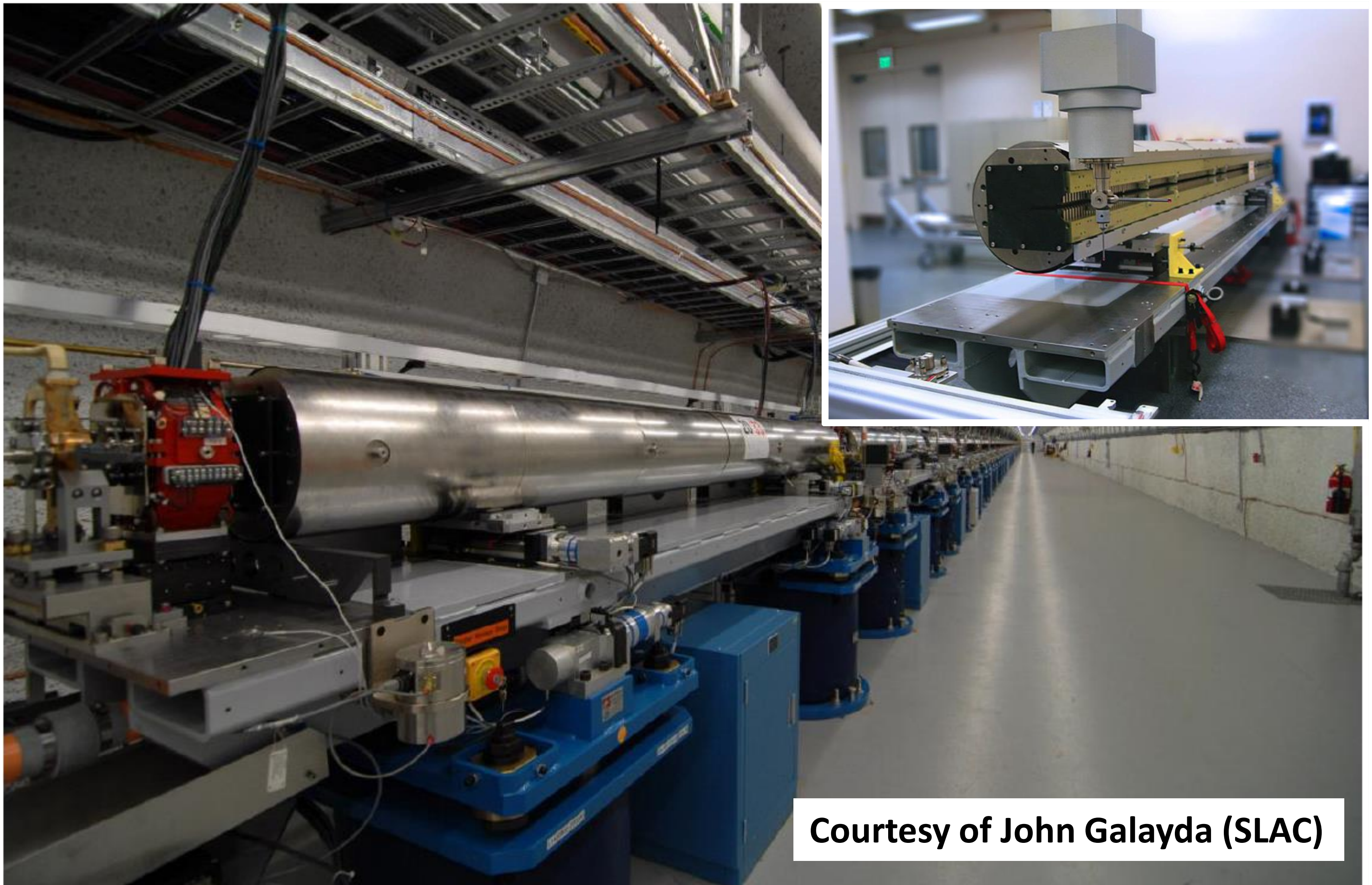
A Brief History

- 1951 – Motz showed an electron beam propagating through an undulator magnet could be used to amplify radiation
- 1960 – Philips developed a microwave tube which is very similar to FEL
- 1971 – Madey theoretically and experimentally (10μm) proved the possibility of exchanging energy between free electrons and EM radiation
- 1980 – Infrared radiation FEL (Oscillator)
- 1984 – Bonifacio et al. brought up the first theory of a SASE FEL in 1-D case
- 1985 – X-ray FEL
- 1992 – Proved 0.1nm to 1nm SASE FEL to be possible
- 2000 – First demonstration of seeded lasing (HGHG)
- 2009 – 4th generation X-ray light source LCLS successfully commissioned.
- On-going – X-ray FEL with high rep. rate; seeded lasing in x-ray range; etc.

Layout: LCLS

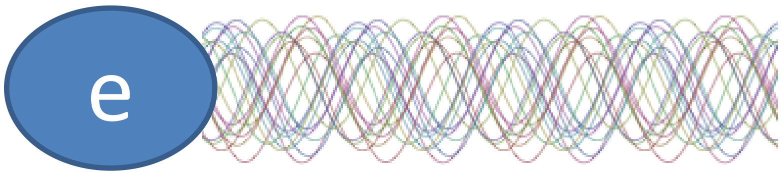

Electron Energy: 3.3 - 15 GeV
Injector
at 2-km point
Existing 1/3 Linac (1 km)
(with modifications)
Electron Transfer Line (340 m)
X-ray Transport
Line (200 m)
Undulator (130 m)
Near Experiment Hall (NEH)
Far Experiment
Hall (FEH)
Courtesy of John Galayda (SLAC)

LCLS Undulator



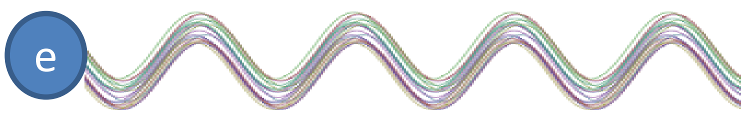
Courtesy of John Galayda (SLAC)

Naïve Idea



Incoherent Radiation

$$A = \left| \sum_i E_0 \exp\left(kz - \omega t + \phi_i\right) \right|^2 \sim N E_0^2$$



Coherent Radiation

$$A = \left| \sum_i E_0 \exp\left(kz - \omega t + \phi_i\right) \right|^2 \sim N^2 E_0^2$$

It is hard to produce such short bunch, 1nm wavelength requires 3e-18 second bunch.

FEL I: The resonance

The energy exchange has its phase as:

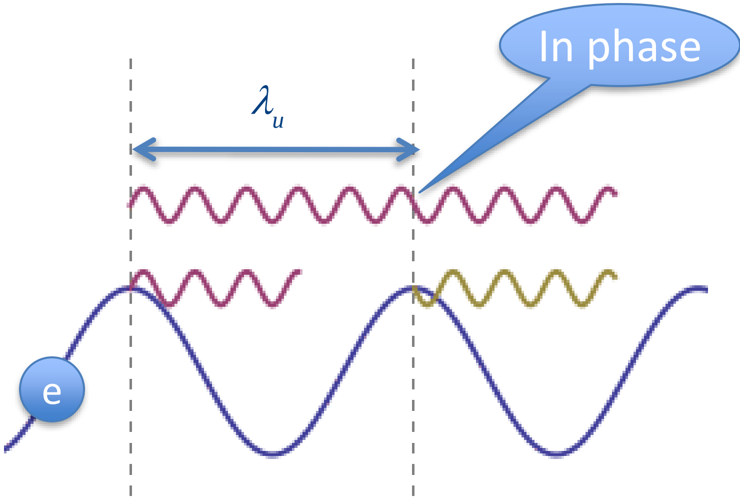
$$\phi = (k_u + k_r)z - \omega t + \theta_0$$

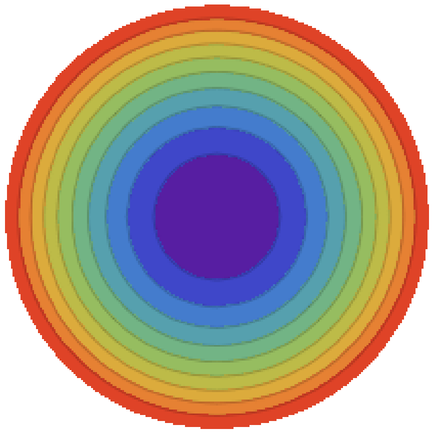
The resonance occurs when:

$$\frac{d\phi}{dt} = (k_u + k_r)v_z - \omega = 0$$

The resonance radiation wavelength at off axis angle ψ

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}(1 + K^2/2 + \gamma^2\psi^2)$$

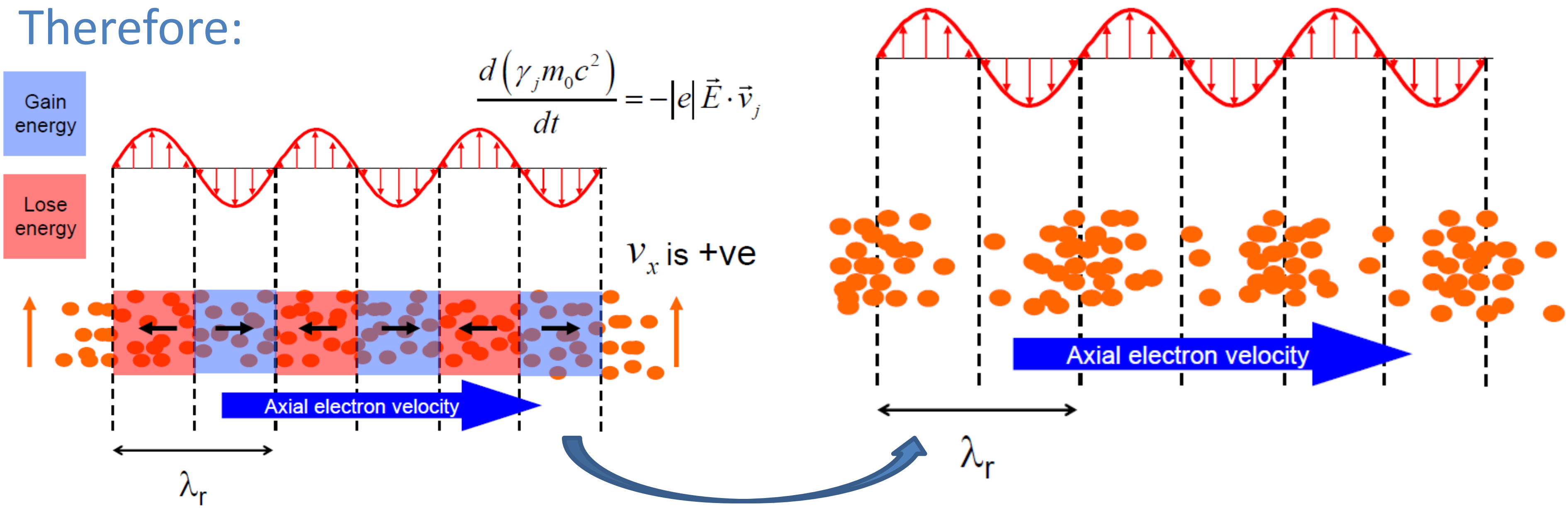




FEL II: Micro-Bunching

The energy change has a sinusoidal form,
Therefore:

$$\frac{d\left(\gamma_j m_0 c^2\right)}{dt} = -|e|\vec{E}\cdot\vec{v}_j$$



Or we can write down differential equation for the energy exchange phase and the energy deviation. It is a pendulum equation which defines the separatrix.

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r}, \theta = (k + k_u)z - \omega t + \theta_0$$

$$\frac{d\theta}{dz} = 2k_u\eta$$

$$\frac{d\eta}{dz} = \frac{eE_0K[JJ]}{2\gamma_r^2 mc^2}\sin\theta$$

$$[JJ] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)$$

Low Gain Regime

We identify a pendulum equation of the phase space variable pair:

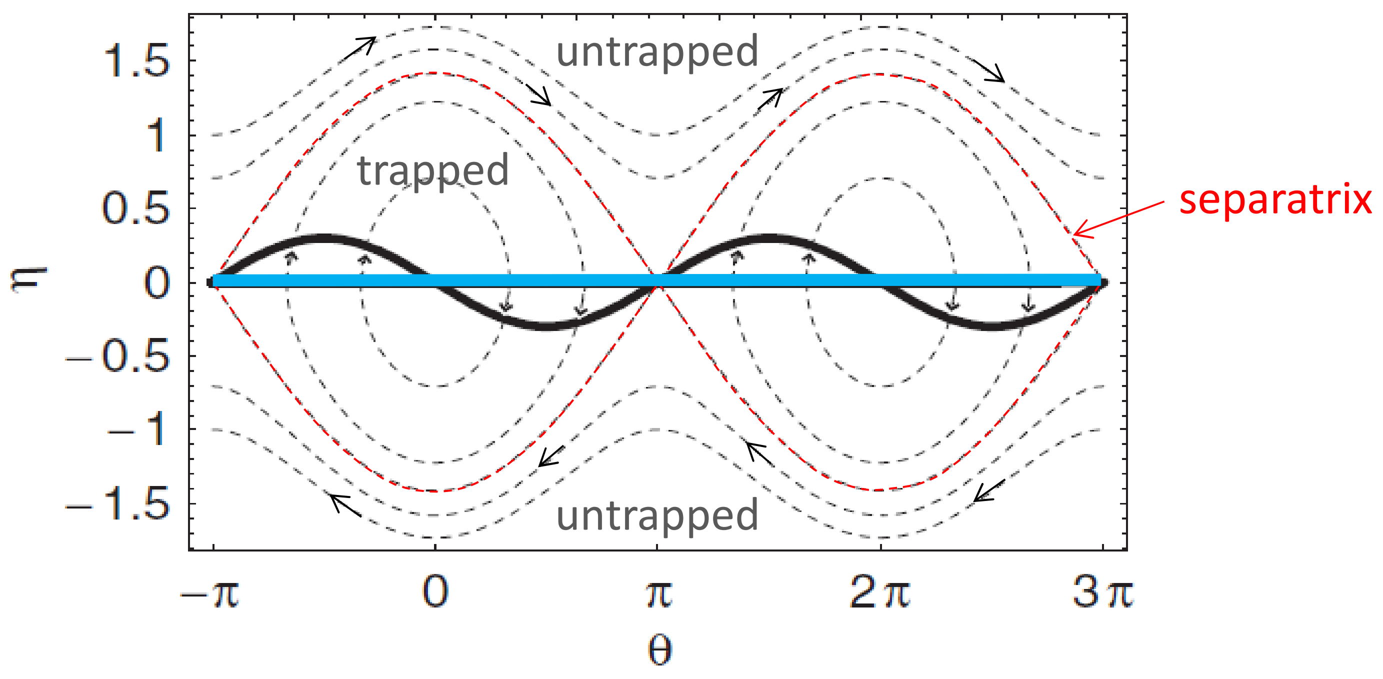
- Ponderomotive phase
- Energy deviation

It is similar to the longitudinal motion.

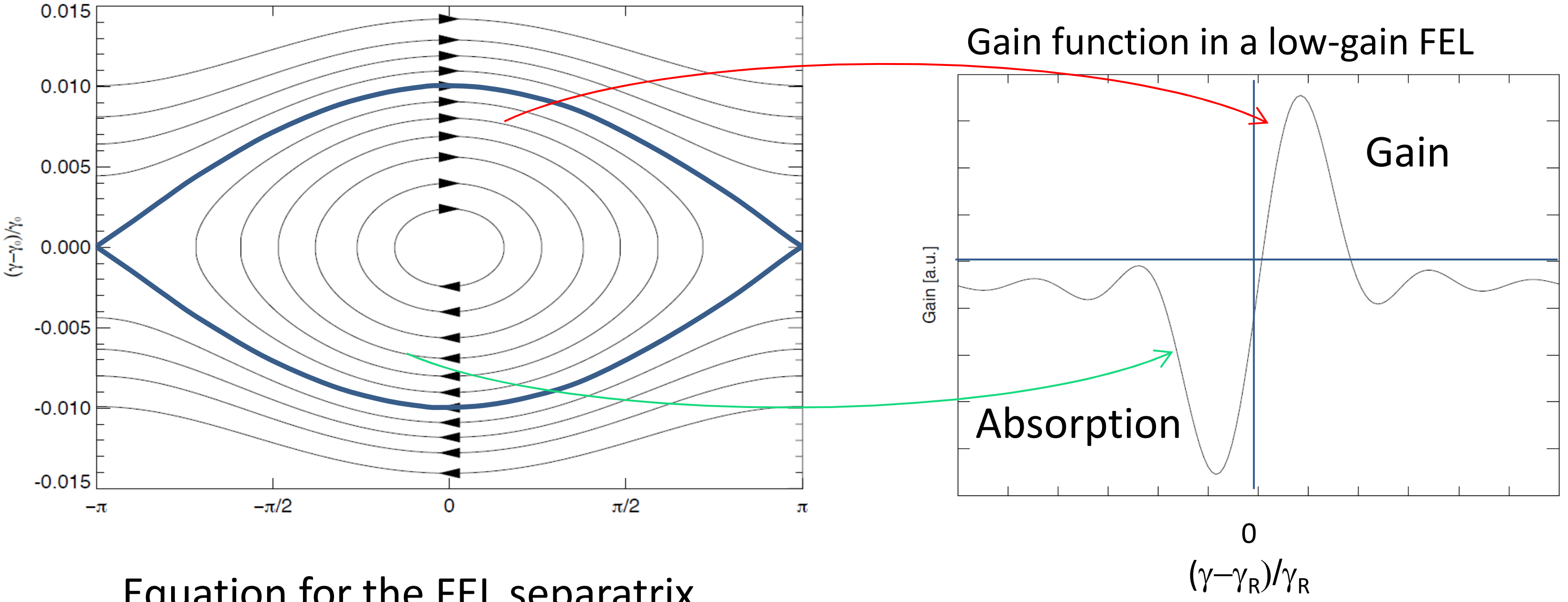
$$\frac{d\theta}{dz} = 2k_u\eta$$

$$\frac{d\eta}{dz} = \frac{eE_0K[JJ]}{2\gamma_r^2 mc^2}\sin\theta$$

$$[JJ] = J_0\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)$$



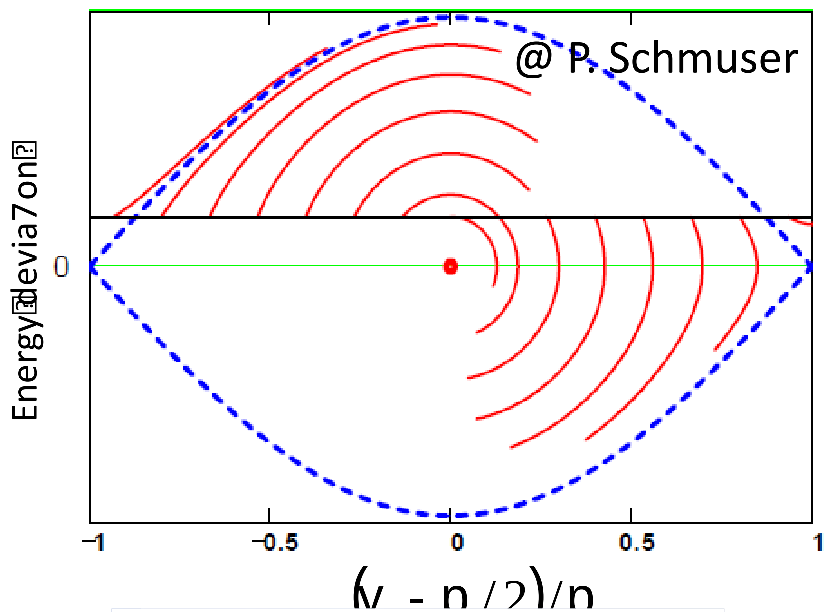
Low Gain Regime II



Equation for the FEL separatrix

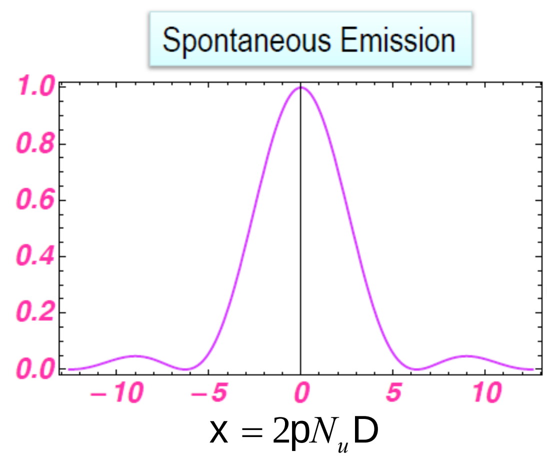
No net gain if the average beam energy equals the reference energy, i.e. the beam is not detuned

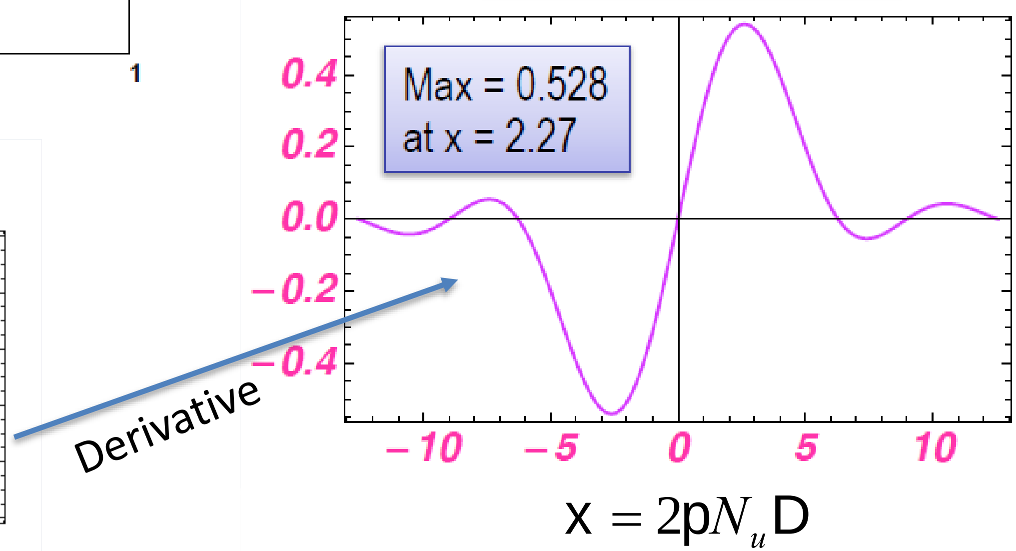
Low Gain Regime III



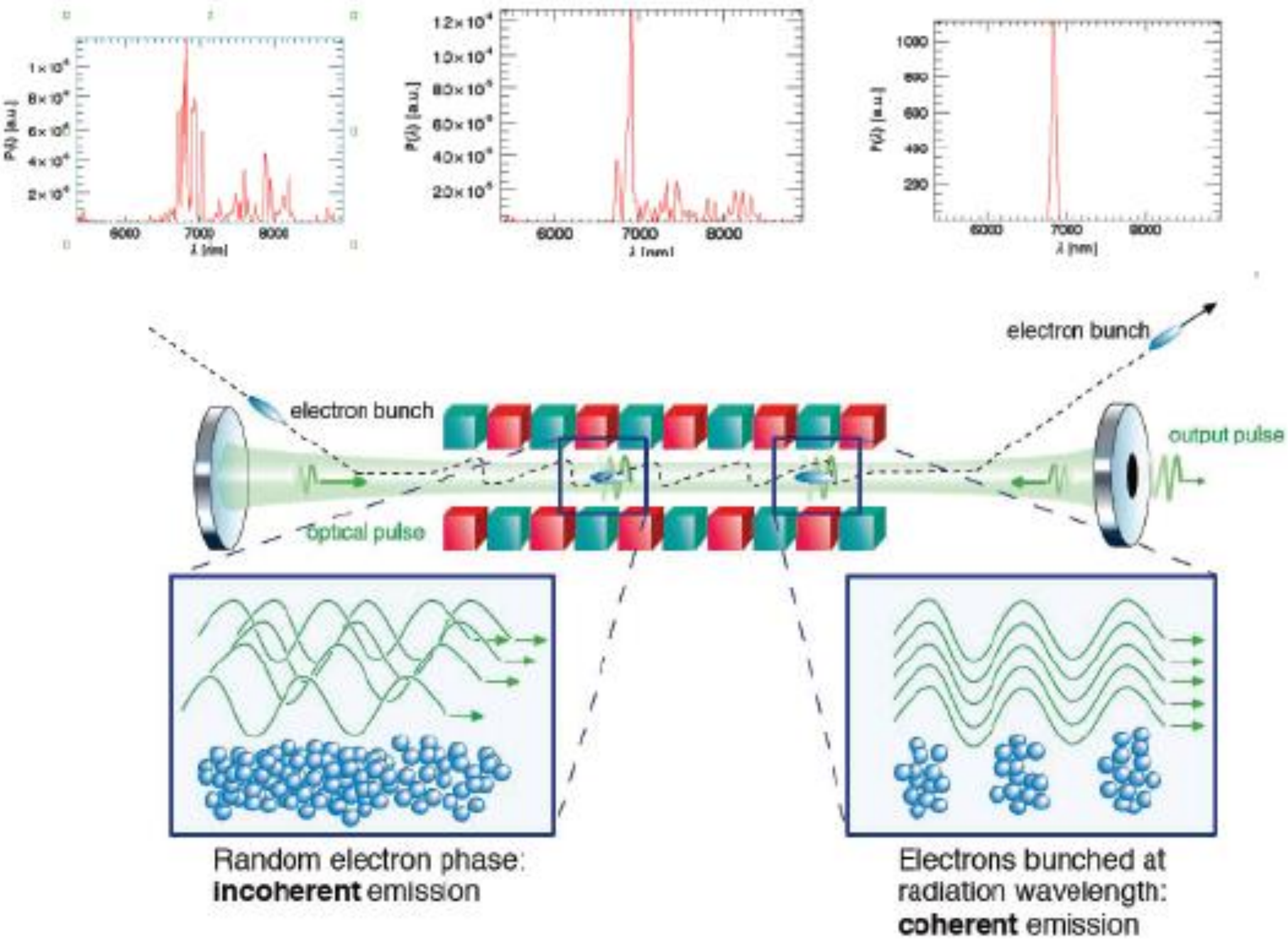
$$G(\xi) = -\frac{\Omega^4 c k_u T^3}{4}\frac{d}{d(\xi)}\left(\frac{\sin^2(\xi)}{\xi^2}\right)$$

Madey Theorem





FEL Oscillator



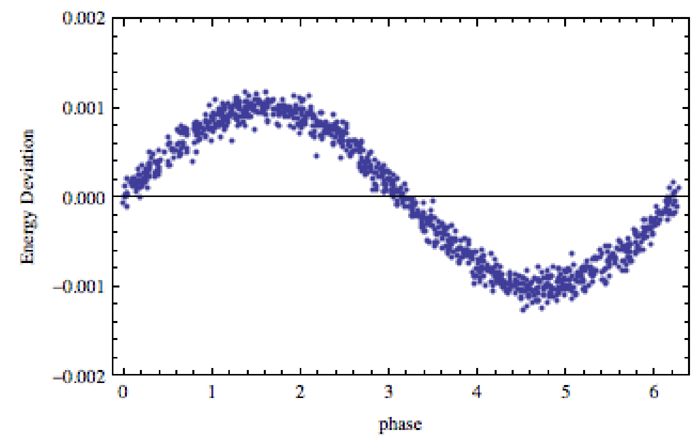
FEL III: Instability Exponential Growth

- Jump to the answer directly by assuming
  - Steady state, no energy spread, ignore transverse detail

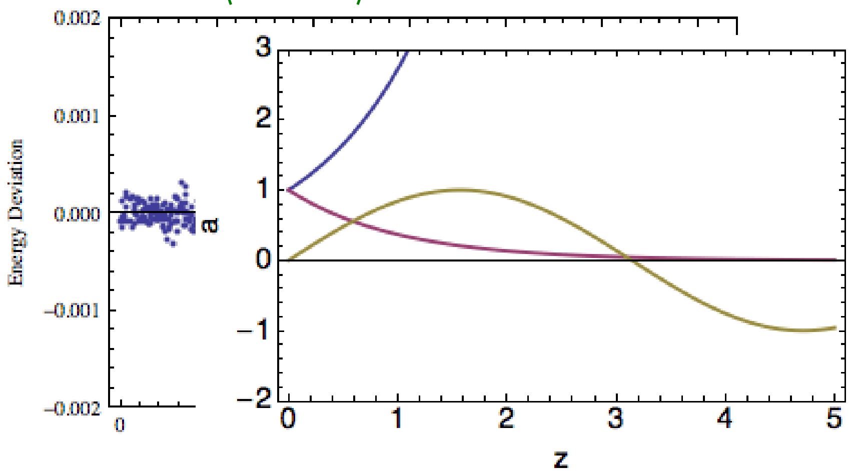
$$\frac{da}{d\bar{z}} = -b \qquad a\text{: normalized radiation field}$$

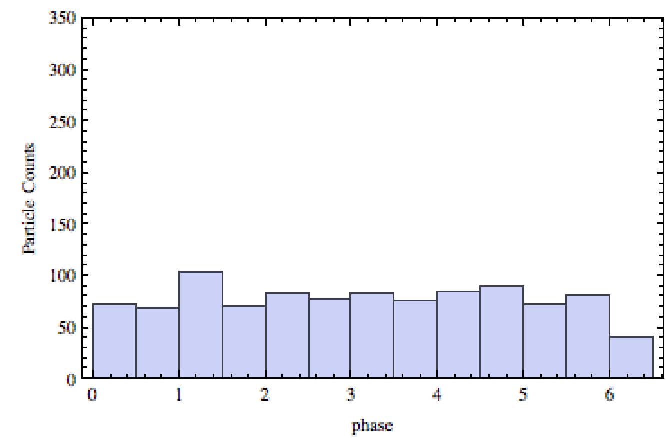
$$\frac{db}{d\bar{z}} = -iP \qquad b = \left\langle e^{iq_j} \right\rangle\text{: bunching factor}$$

$$\frac{dP}{d\bar{z}} = a \qquad P = \left\langle g_j e^{iq_j} \right\rangle\text{: energy modulation}$$



$$\frac{d^3 a}{d\bar{z}^3} = ia$$





High-Gain Regime

- The field amplitude is not constant!
- Pendulum equation + Maxwell-Vlasov Equation is required.

$$\left[\left(\frac{1}{c}\frac{\partial}{\partial t}\right)^2 - \left(\frac{\partial}{\partial z}\right)^2 - \boldsymbol{\nabla}_\perp^2\right] E_x(\boldsymbol{x}, t; z) = -\frac{1}{\epsilon_0 c^2}\left[\frac{\partial j_x}{\partial t} + c^2 \frac{\partial (e n_e)}{\partial x}\right],$$

$$j_x = e K_0 \cos(k_u z) \sum_{j=1}^{N_e} \frac{1}{\gamma_j} \delta[\boldsymbol{x} - \boldsymbol{x}_j(z)] \delta[t - t_j(z)]$$

- We use slowly varying phase and amplitude approximation, slow means constant within wavelength:

$$E_x = E(z,t)\cos(kz - \omega t + \phi(z,t))$$

- In 1-D theory, transverse derivative is ignored.

Dimensionless FEL Parameter

The Maxwell equation under this slow approximation:

$$\frac{d\theta}{dz} = 2\eta k_u$$

$$\frac{d\eta}{dz} = \Xi_1 \left(\tilde{E} e^{i\theta} + \tilde{E}^* e^{-i\theta} \right)$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t} \right) \tilde{E} = -\Xi_2 \left\langle e^{-i\theta} \right\rangle_{\Delta}$$

$$\tilde{E} = \frac{E}{2} e^{i\phi}$$

$$\Xi_1 = \frac{eK[JJ]}{2\gamma_0^2 mc^2}$$

$$\Xi_2 = \frac{eK[JJ]n_e}{4\epsilon_0 \gamma}$$

Choose the FEL scaling parameter, so that the differential equation has constant 1.

$$\bar{z} = 2k_u \rho z$$

$$\rho = \left[\frac{\Xi_1 \Xi_2}{4k_u^2} \right]^{1/3}$$

Solving the Equation

$$\frac{da}{d\bar{z}} = -b$$ a: normalized radiation field

$$\frac{db}{d\bar{z}} = -iP$$ $b = \left\langle e^{iq_j} \right\rangle$: bunching factor

$$\frac{dP}{d\bar{z}} = a$$ $P = \left\langle g_j e^{iq_j} \right\rangle$: energy modulation

$$\frac{d^3a}{d\bar{z}^3} = ia$$

$$a(\bar{z}) = \sum_{j=1}^{3} C_j e^{-i\mu_j \bar{z}}$$

$$\mu_1 = 1, \mu_{2/3} = \frac{-1 \pm \sqrt{3}i}{2}$$

The evolution dominated by the growth mode:

$$aa^* \sim e^{\sqrt{3}\bar{z}}$$

Gain Length and FEL parameter

The Power Gain length under 1-D Assumption:

The FEL parameter ρ
A dimensionless parameter

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

$$P = P_0 \exp\left(z / L_G\right)$$

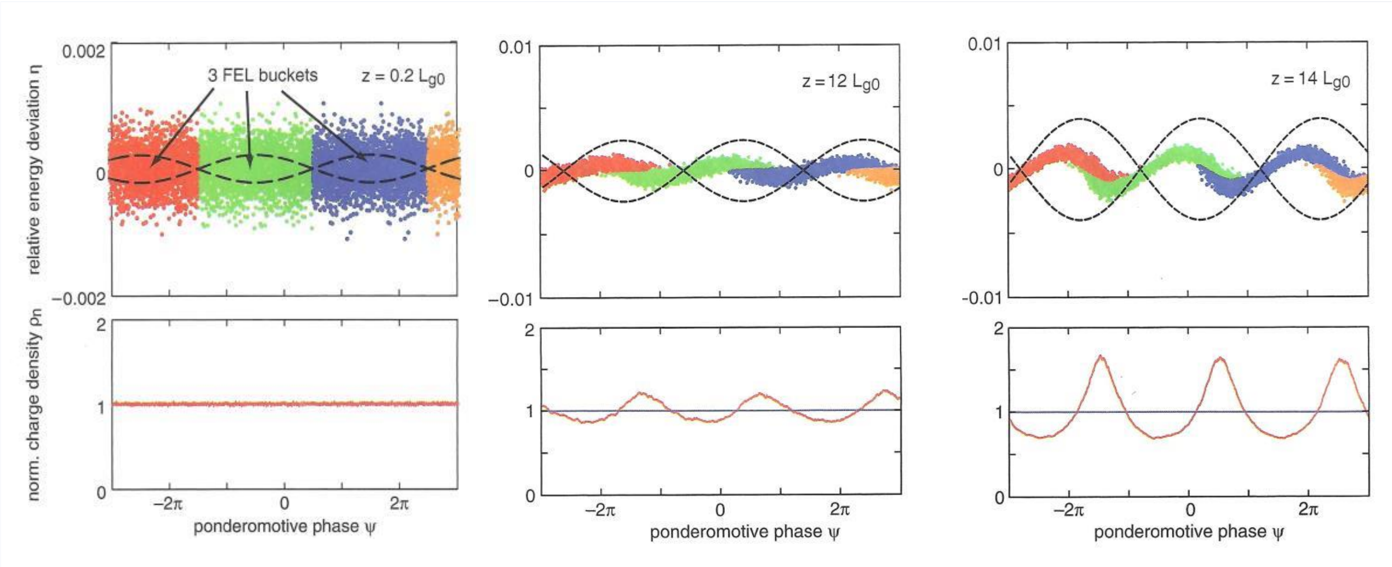
$$\rho = \left(\frac{e^2 K^2 [JJ]^2 n_e}{32\varepsilon_0 \gamma_0^3 m c^2 k_u^2} \right)^{1/3}$$

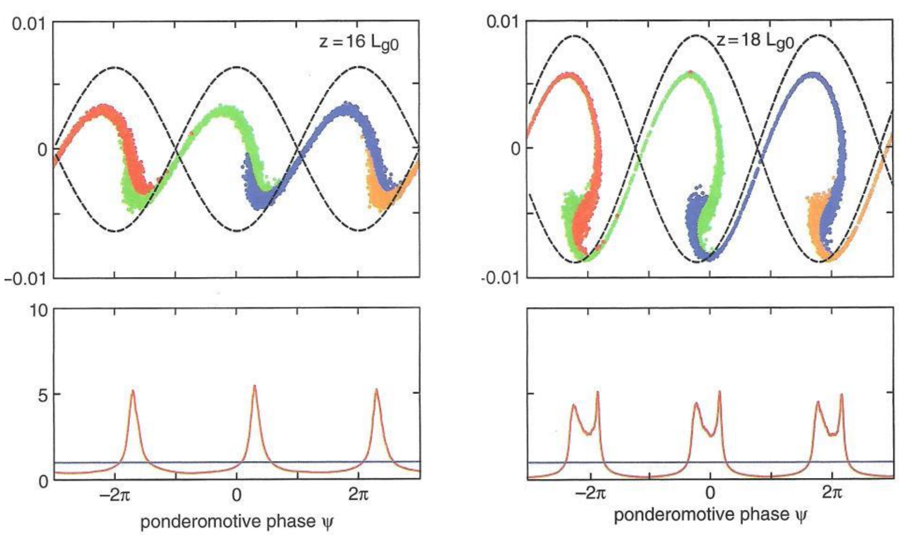
$$\approx 1\times10^{-3}$$

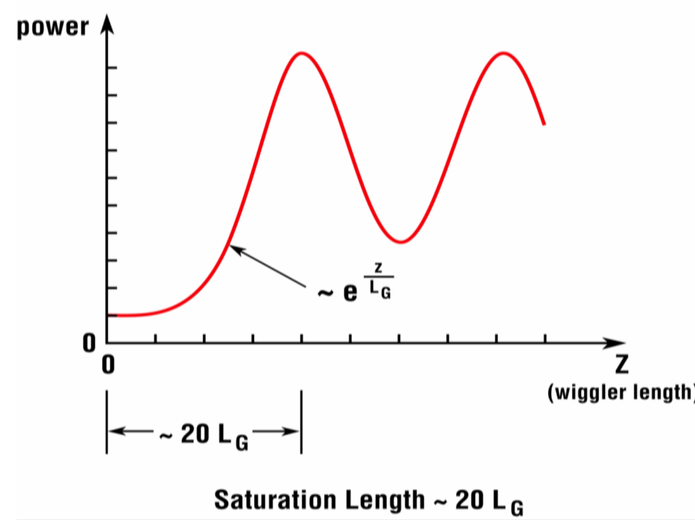
Given without proof:

$$\rho = \frac{\text{Radiation energy at saturation}}{\text{Electron beam energy}} = \text{Saturation Efficiency}$$

FEL Evolution







Beam Quality Requirement

- Energy spread: $$\Delta\gamma/\gamma < \rho$$
- Beam emittance: $$\epsilon_x \leq \lambda/(4\pi)$$
- Beam size $$\sigma_x \sim \sqrt{\frac{\lambda\lambda_u}{16\pi^2\rho}}$$
- Beam current:
  - Large enough to get reasonable FEL parameter

3-D Effects

- 3-D effect include both EM wave and particle:

Electron Emittance Parameter

$$X_{e} = \frac{L_{1D}}{b_{ave}} \frac{4pe_{u}}{\lambda_{r}}$$

Electron Energy Spread Parameter

$$X_{\gamma} = \frac{4pL_{1D}}{\lambda_{u}} \frac{\sigma_{\gamma}}{\gamma}$$

Diffraction of Radiation

$$X_{d} = \frac{L_{1D}}{z_{R}}$$

Ming-Xie 3D Parameterization

$$L_{G,3D} = L_{G,1D} F(X_d, X_\epsilon, X_\gamma)$$

F function is retrieved by fitting:

$$F(X_d, X_e, X_\gamma) = 1 + a_1 X_d^{a_2} + a_3 X_e^{a_4} + a_5 X_\gamma^{a_6}$$ e and energy spread

$F(\eta_d, \eta_\varepsilon, \eta_\gamma)$

$$+ a_7 X_e^{a_8} X_\gamma^{a_9} + a_{10} X_d^{a_{11}} X_\gamma^{a_{12}} + a_{13} X_d^{a_{14}} X_e^{a_{15}} + a_{16} X_d^{a_{17}} X_e^{a_{18}} X_\gamma^{a_{19}}$$

The fitting coefficients are:

$a_1 = 0.45$	$a_2 = 0.57$	$a_3 = 0.55$	$a_4 = 1.6$
$a_5 = 3$	$a_6 = 2$	$a_7 = 0.35$	$a_8 = 2.9$
$a_9 = 2.4$	$a_{10} = 51$	$a_{11} = 0.95$	$a_{12} = 3$
$a_{13} = 5.4$	$a_{14} = 0.7$	$a_{15} = 1.9$	$a_{16} = 1140$
$a_{17} = 2.2$	$a_{18} = 2.9$	$a_{19} = 3.2$	

Initial Conditions and FEL Schemes

- The coupled differential equation can start from different initial condition
  - Initial EM wave (Case 1 and 2)
  - Initial bunching factor (Case 3)

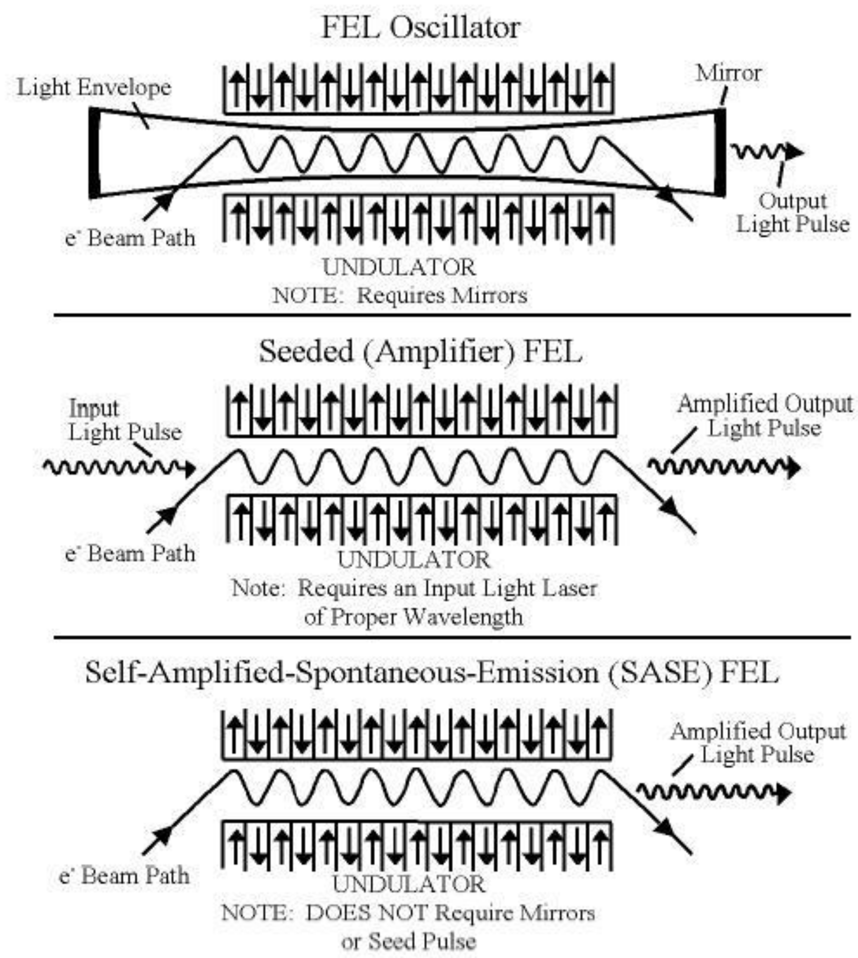
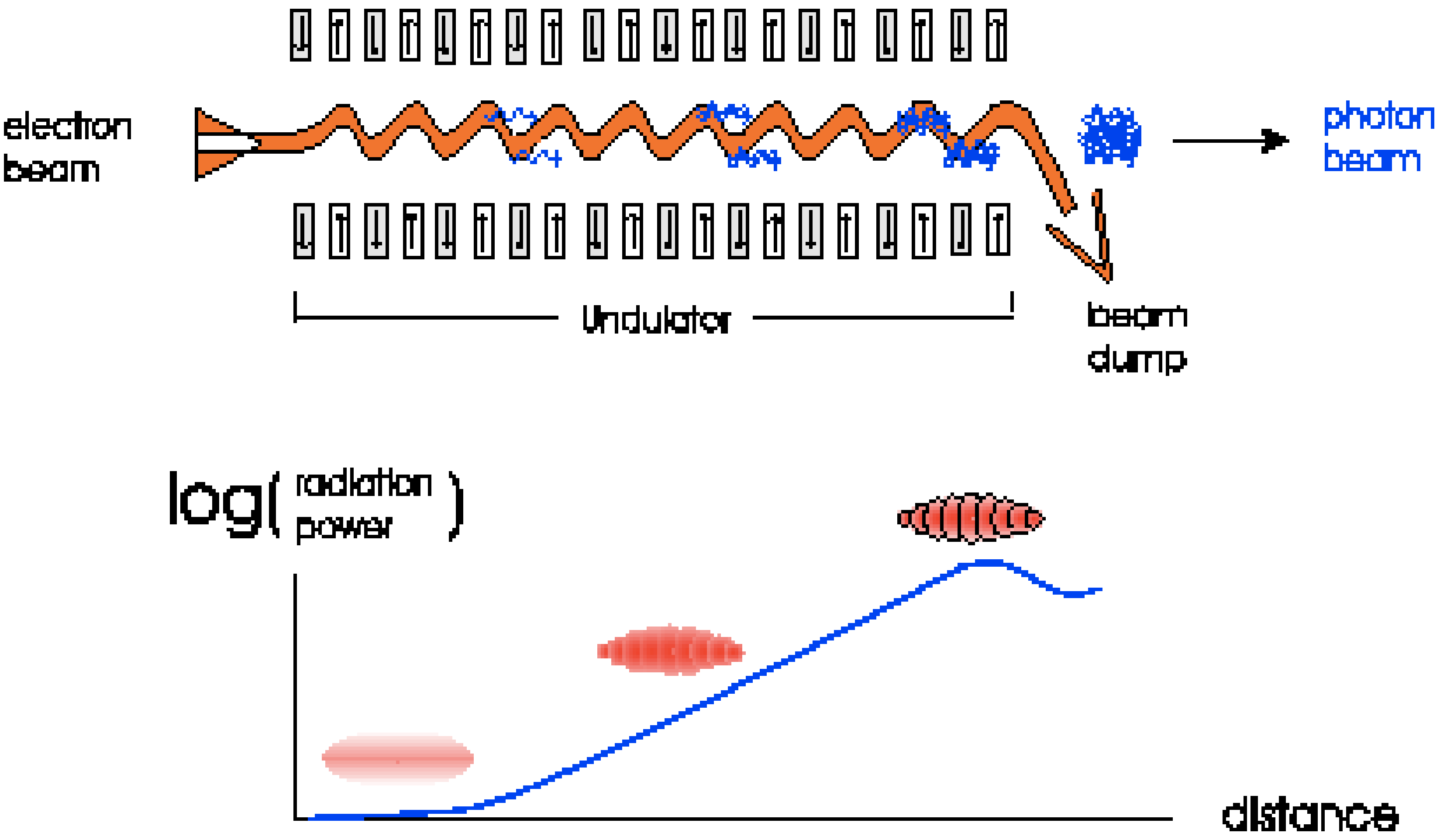


Figure 4.3: Three FEL operating modes.

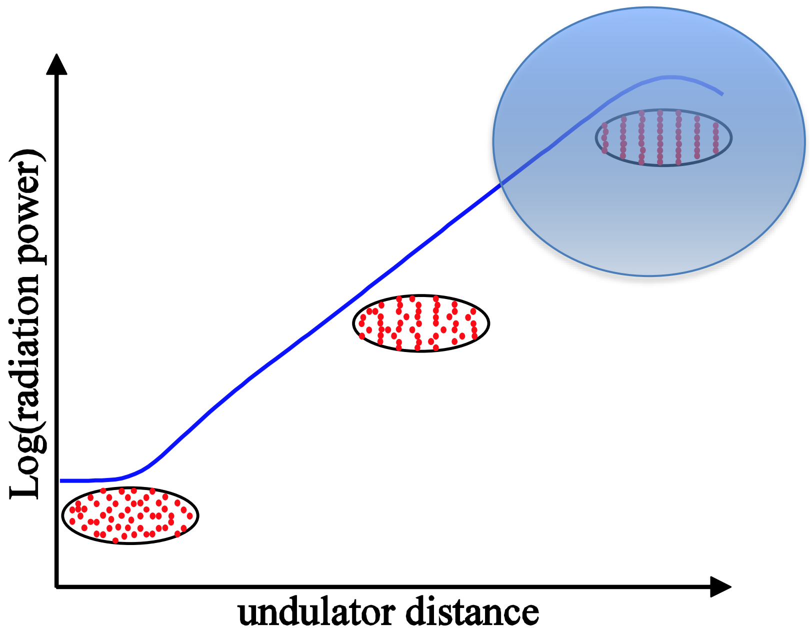
RoadMap to X-ray Laser

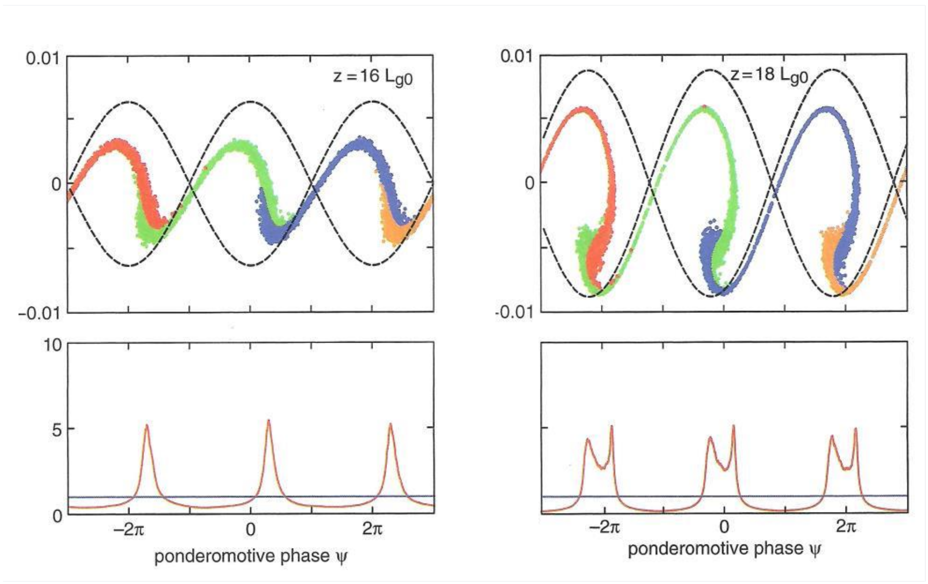
- SASE, do not need x-ray seed. Generate X-ray pulses from nose, poor longitudinal coherence
- Higher harmonic generation, start from large wavelength, obtain its higher harmonic as seed.
- SASE self-seeding, apply monochromator in the part of SASE radiation, and use it as seed for the second part.
- X-ray FEL oscillator

SASE: Born from Noise



FEL Saturation



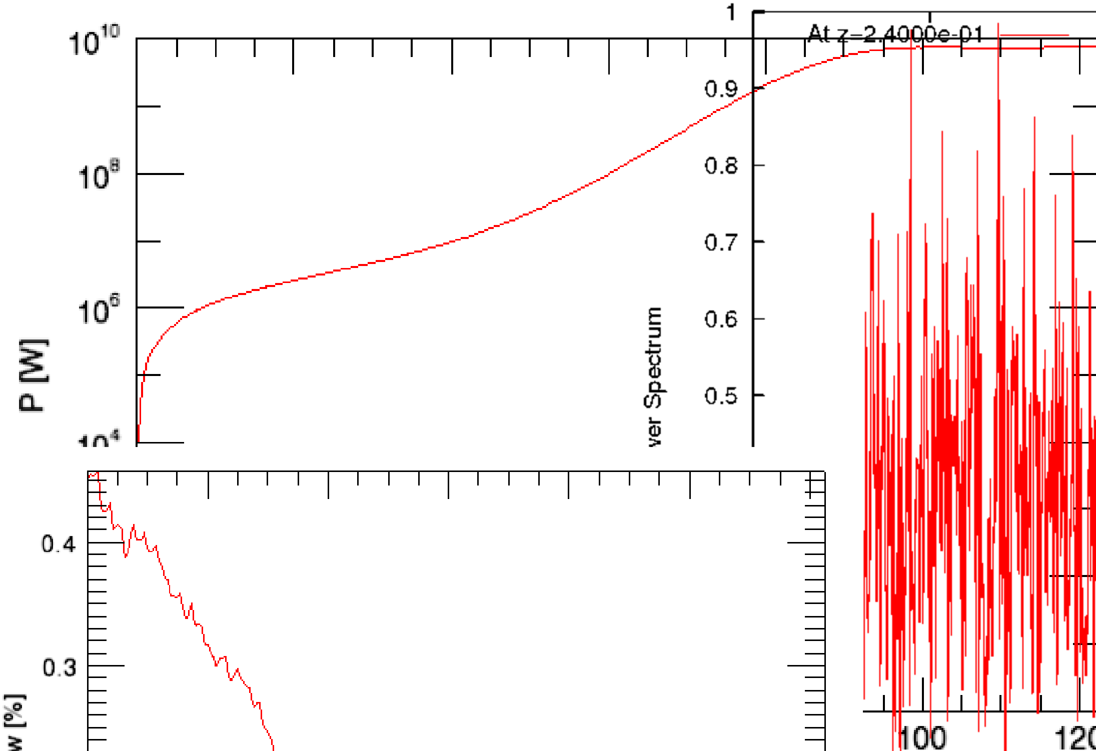


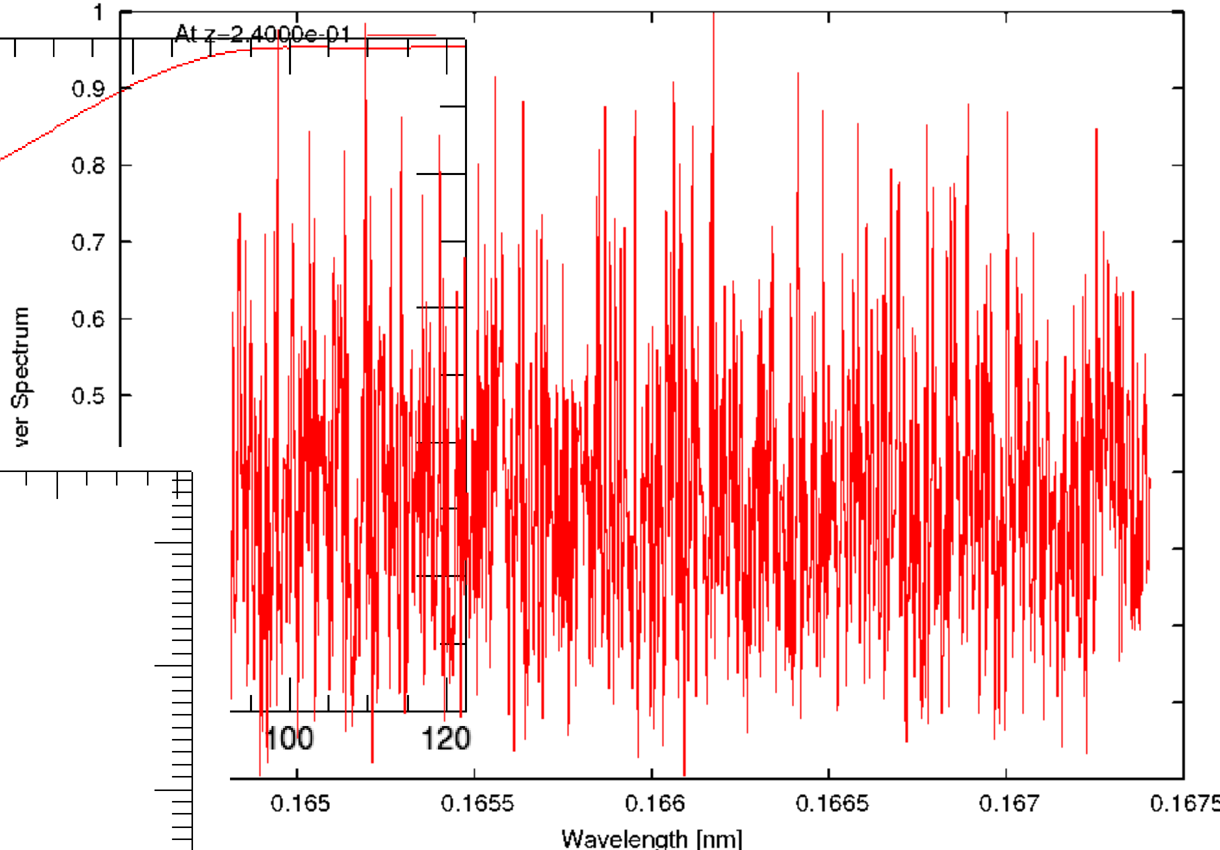
Energy spread becomes too large to maintain reasonable gain.

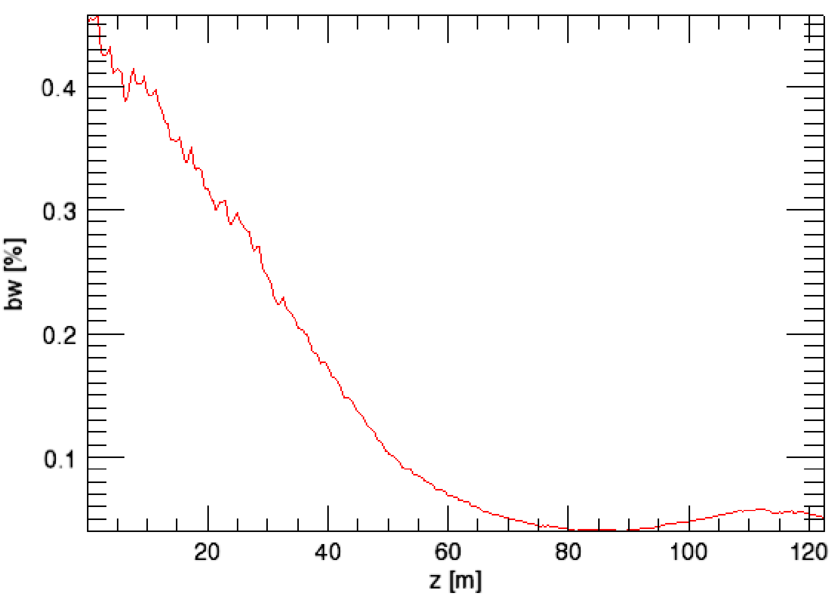
Our theory also break since the Vlasov approach assumes the distribution perturbation is small.

$$L_s \sim \lambda_u/\rho \sim 20L_G$$

Frequency Spectrum







The SASE frequency spread evolves as

$$\frac{\delta\omega}{\omega_0} \sim \sqrt{\frac{\rho}{N_u}}$$

Transverse Mode

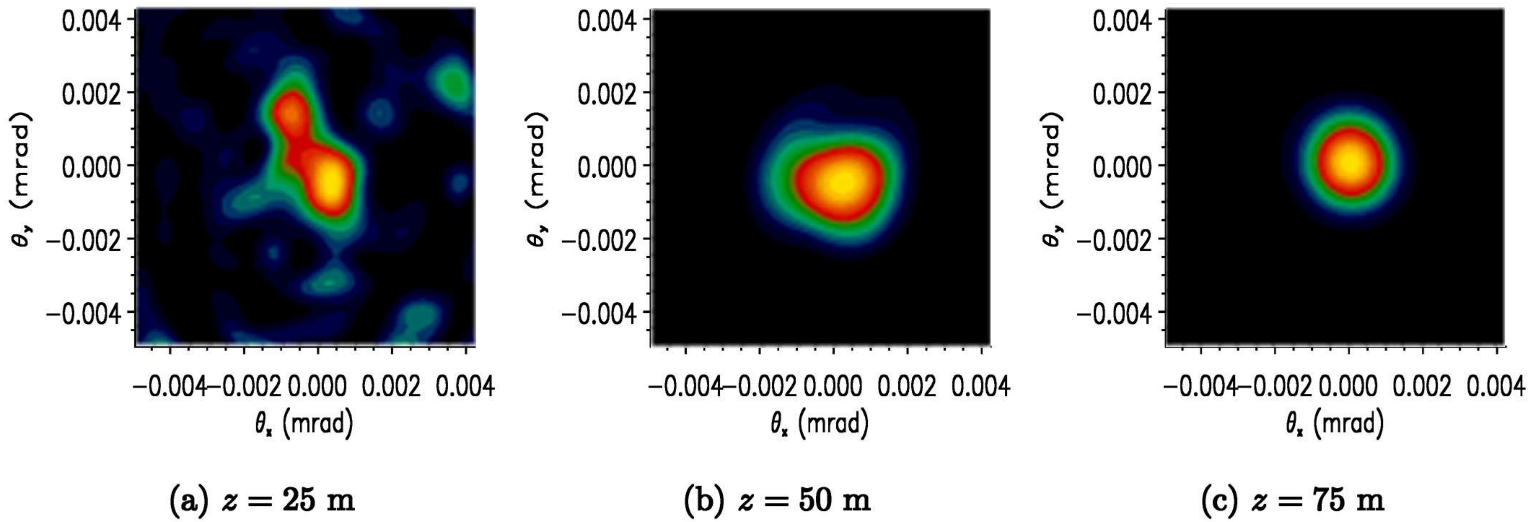
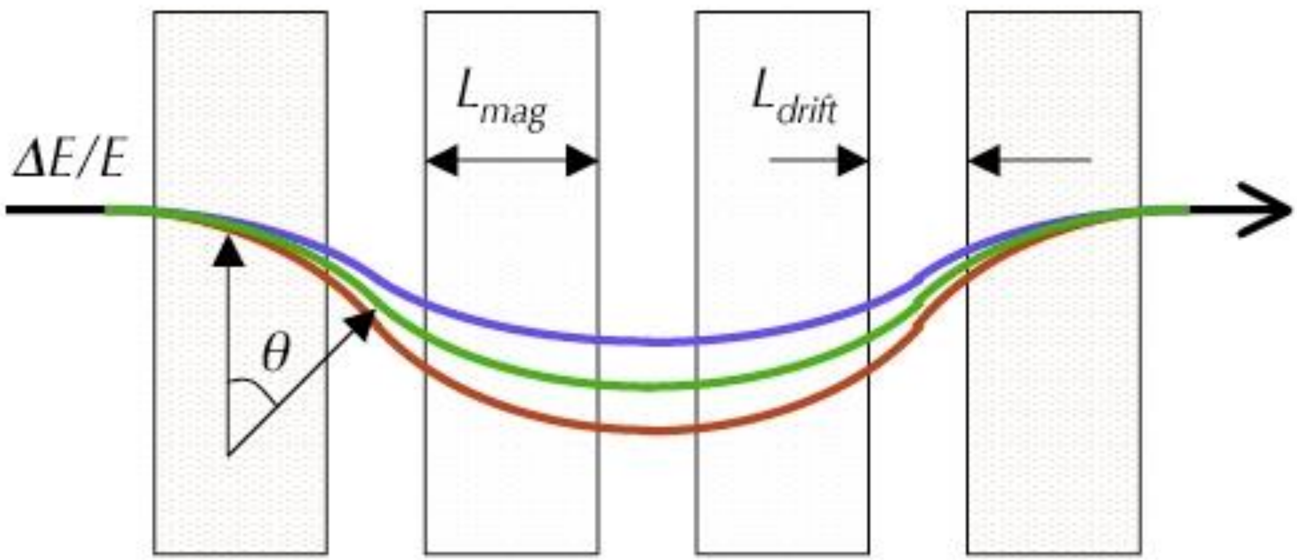


FIG. 9. (Color) Evolution of the LCLS transverse profiles at different z locations (courtesy of Sven Reiche, UCLA).

The lowest transverse mode has largest gain and becomes dominant. This is usually referred as 'optical guiding'.

SASE has transverse coherence at or near saturation.

Higher Harmonic Generation
Useful tool: Chicane



$$R_{56} = \frac{dz}{d\delta}$$

$$R_{56} \approx -2\theta_B^2 \left(L_1 + \frac{2}{3L_B} \right).$$

High Gain Harmonic Generation

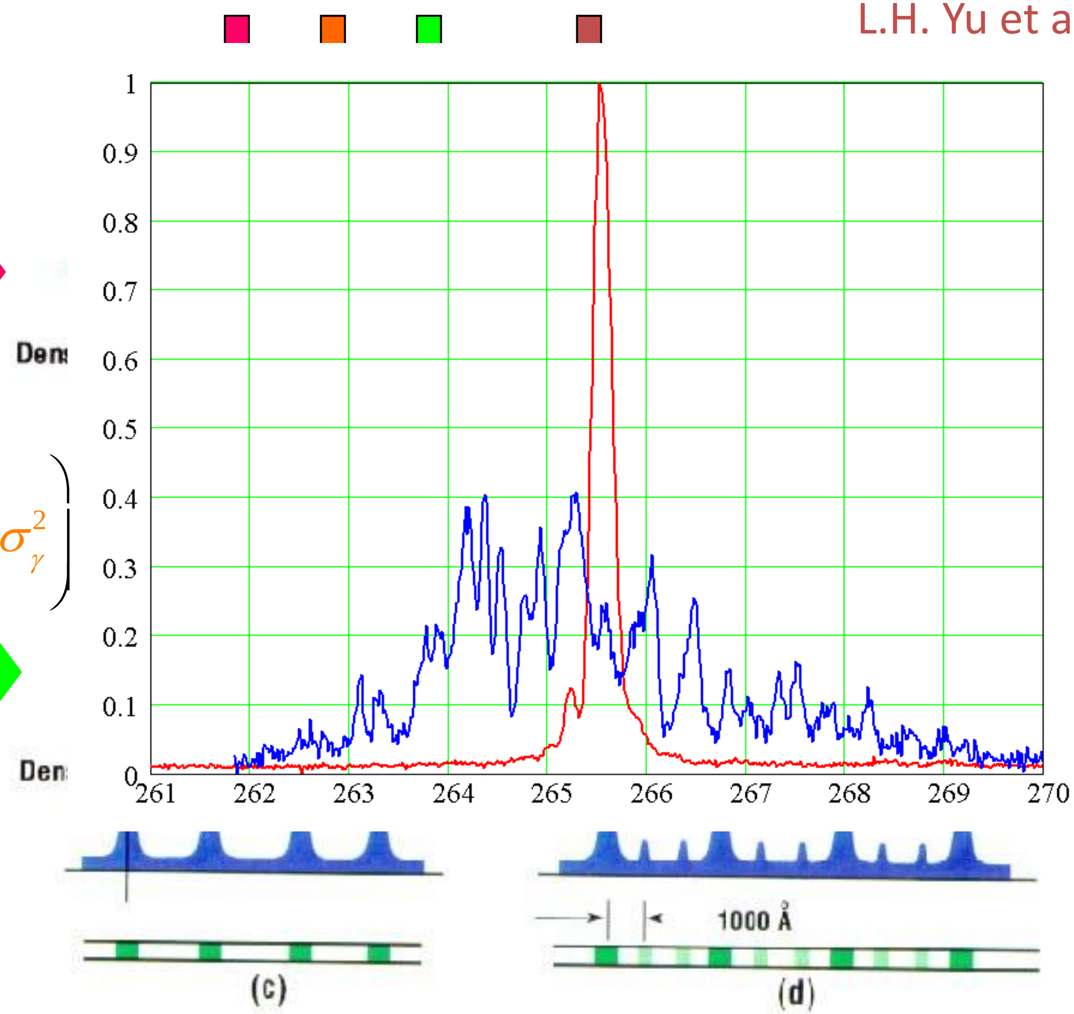
L.H. Yu et al

Dispersion: $d\theta / d\gamma$

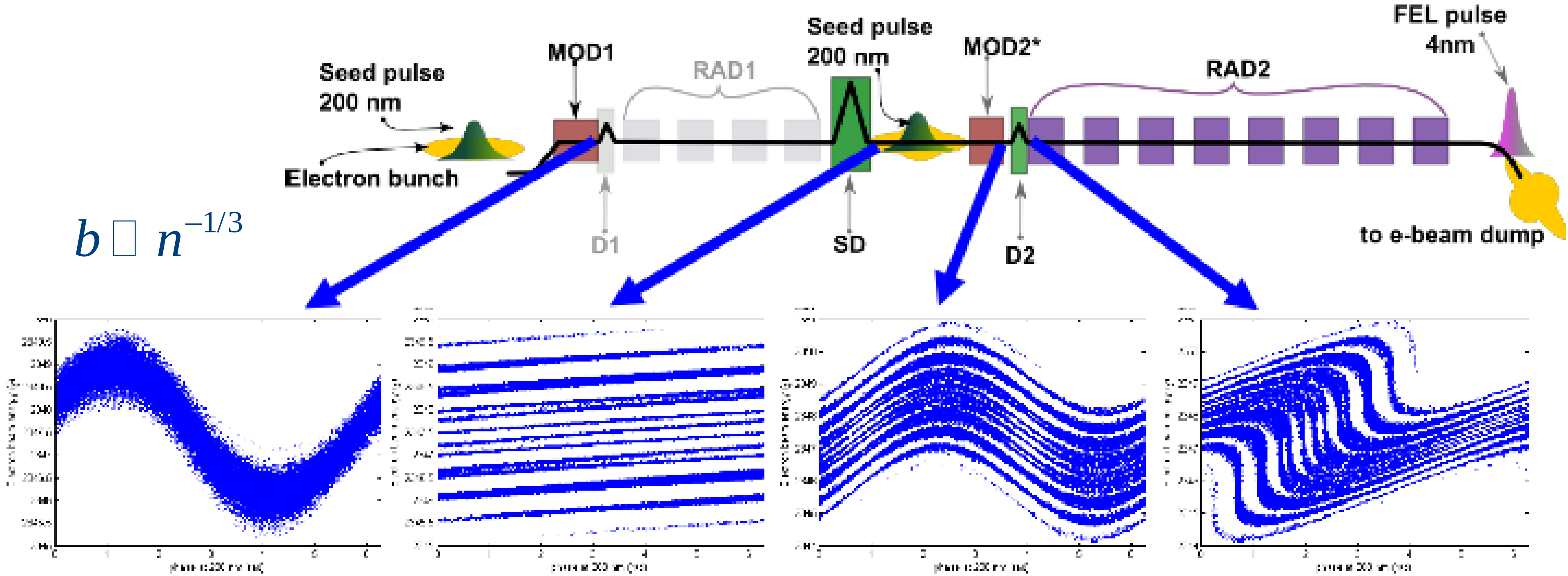
Energy Modu.: $\Delta\gamma$

Energy Spread: σ_γ

$$b_n = J_n\left(-n\frac{d\theta}{d\gamma}\Delta\gamma\right)\exp\left(-\frac{1}{2}n^2\left(\frac{d\theta}{d\gamma}\right)^2\sigma_\gamma^2\right)$$

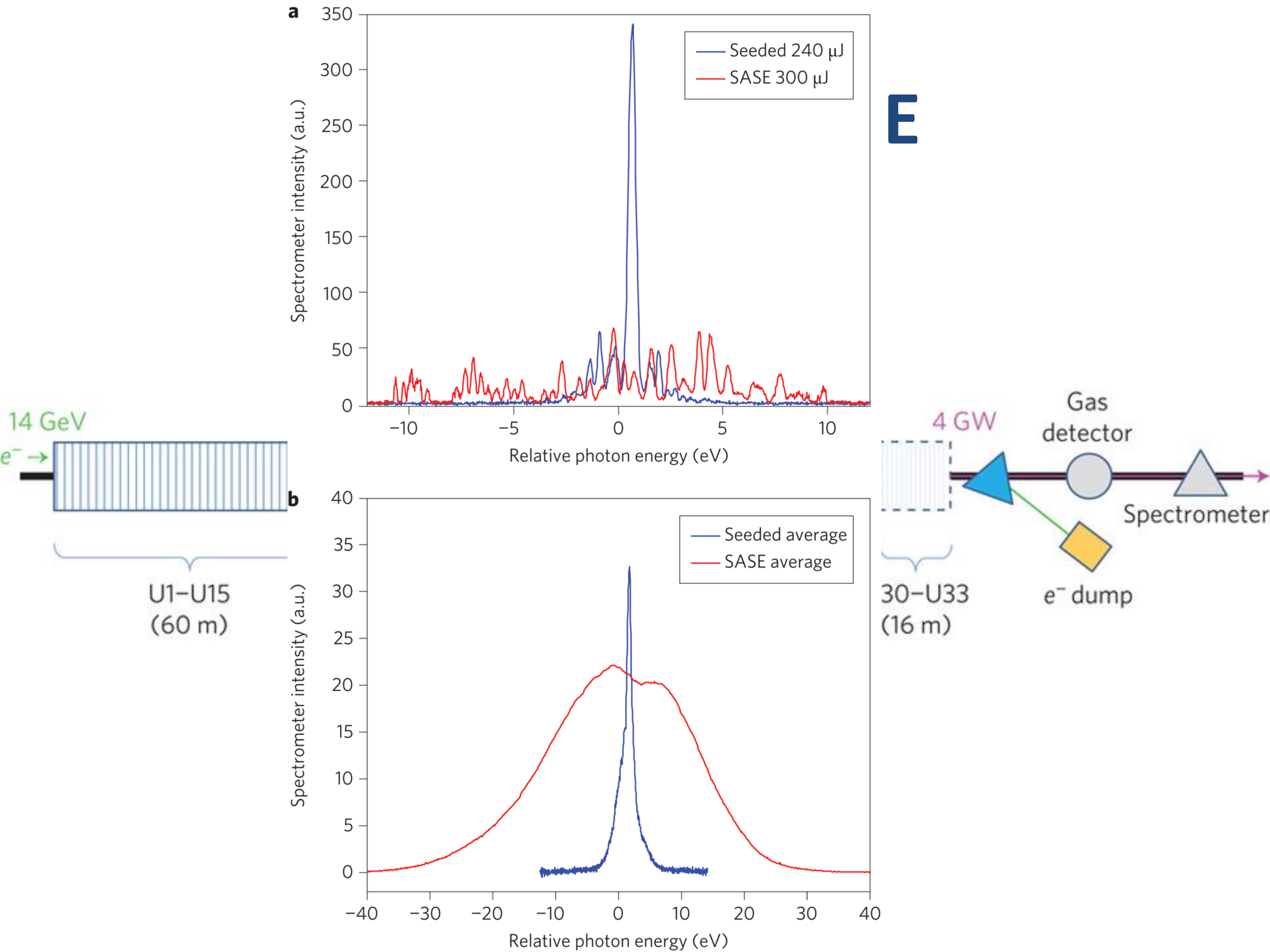


Echo-Enabled Harmonic Generation

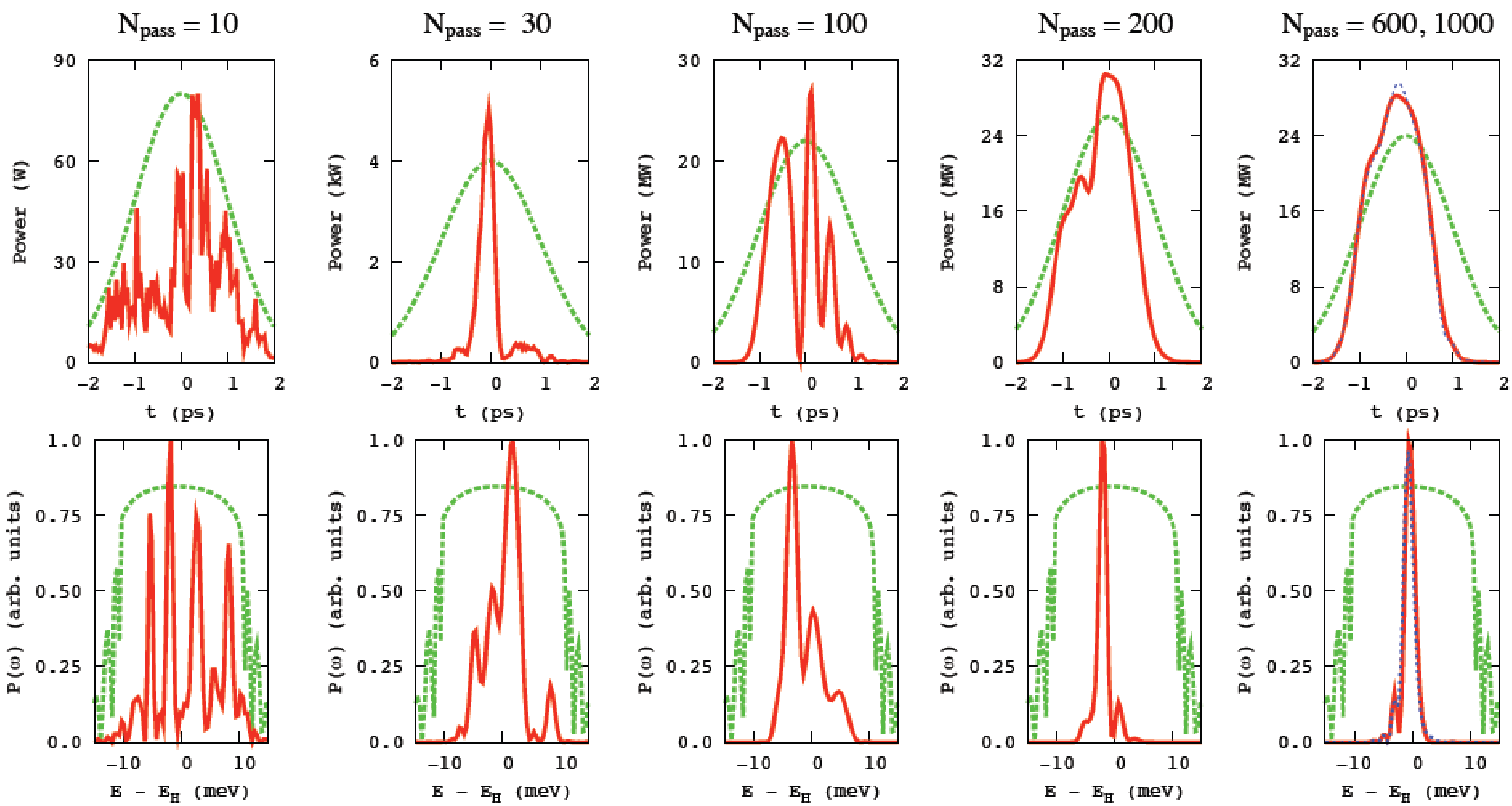


$b \sim n^{-1/3}$

@G. Stupakov, Phys. Rev. Lett. 102, 074801 (2009).
Picture from FEL 2010 presentation


E
a
Seeded 240 μJ
SASE 300 μJ
Spectrometer intensity (a.u.)
Relative photon energy (eV)
b
Seeded average
SASE average
Spectrometer intensity (a.u.)
Relative photon energy (eV)
14 GeV
e⁻ →
U1–U15
(60 m)
4 GW
30–U33
(16 m)
Gas
detector
Spectrometer
e⁻ dump

X-ray FEL amplifier

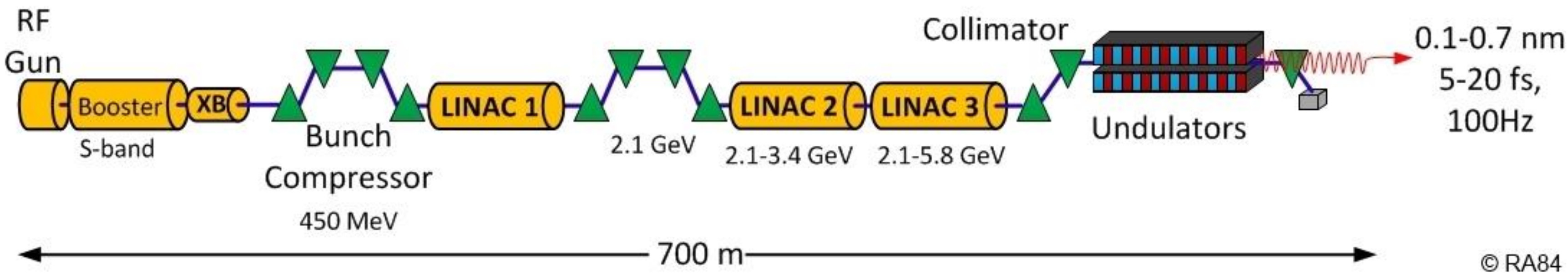


@ K-J Kim

Comparison of 3 X-ray sources

	FLASH European XFEL	LCLS	SACLA
Wavelength X-ray energy	450 –1 Å 0.3 –12 keV	25 –1.2 Å 0.48 – 10 keV	2.3 – 0.8 Å 5.4 – 15 keV
Beam energy	0.23 –17.5 GeV	3.3 –15 GeV	8 GeV
Linac type Frequency Length	SRF 1.3 GHz 2.1 km	NCRF 2.856 GHz 1 km	NCRF 5.712 GHz 0.4 km
Gun type, frequency Cathode	NCRF, 1.3 GHz Cs_2Te photocathode	NCRF, 2.856 GHz Cu photocathode	Pulsed DC gun CeB_6 thermionic
Bunch charge	130 – 1,000 pC	20 – 250 pC	200 pC
Bunch length	70 – 200 fs	5 - 500 fs	100 fs
rms emittance	0.4 –1 μm	0.13 – 0.5 μm	0.6 μm
Bunches per second	27,000	120	60
Undulator period Maximum K	2.7 cm 1.2	3 cm 3.7	1.8 cm 2.2

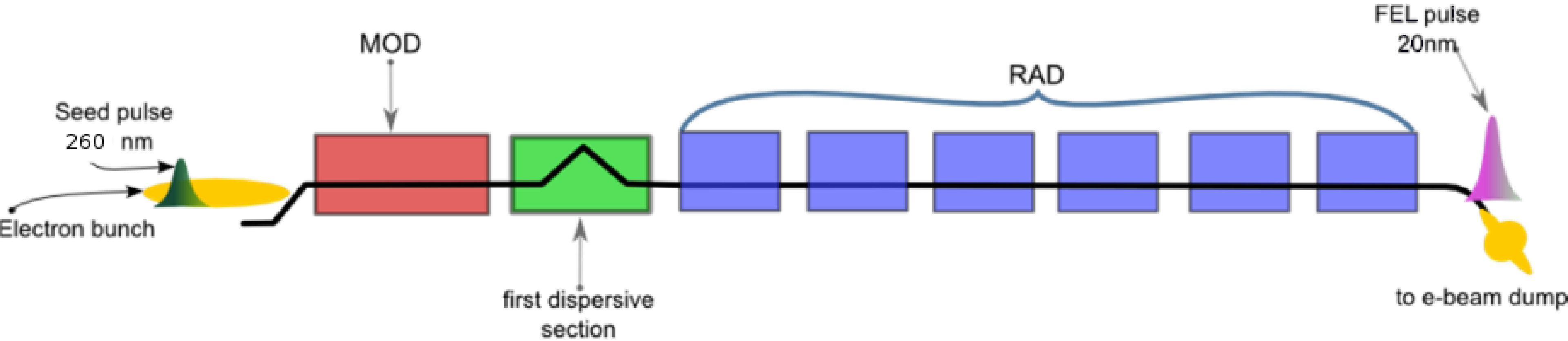
SwissFEL, 2016

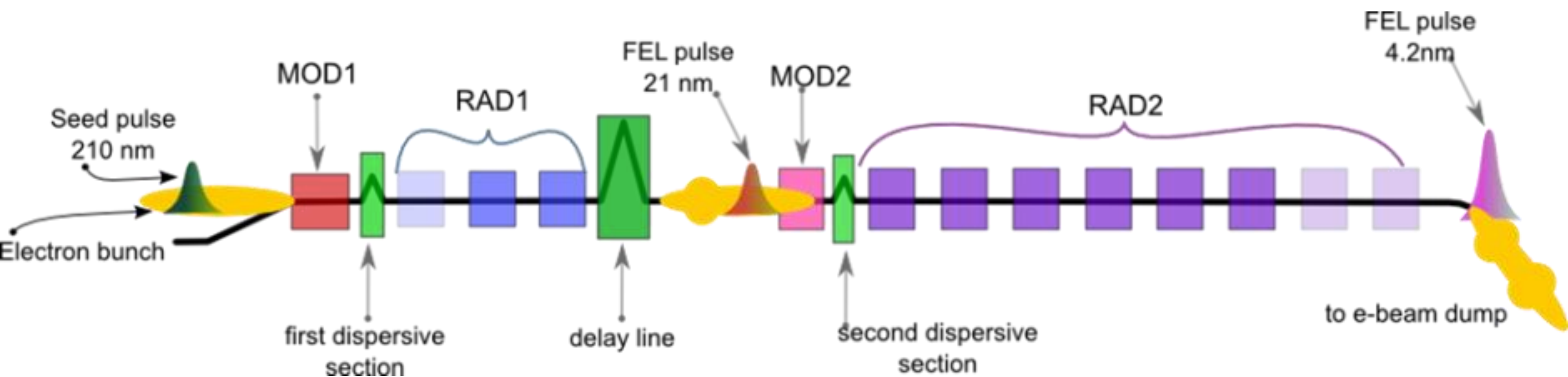


Project	Start of operation	Electron beam energy	Minimum photon wavelength (λ_{min})	Peak brilliance @ λ_{min}	Repetition rate	Number of X-ray pulses/ macropulse
		GeV	Å	10^{33} ph / s mm^2 mrad^2 0.1 % b.w.	macro-pulses/sec.	
LCLS	2009	13.6	1.5	1	120	1
SACLA	2010	8	1.0	0.5	60	1
European X-FEL	2014	17.5	1.0	5	10	3250
SwissFEL	2016	5.8	1.0	0.1 – 1	100	2

FERMI @ Elettra

- Cascade seeded FEL





References

- K-J. Kim and Z. Huang USPAS Lecture Notes
- D. Nguyen and Q. Marksteiner USPAS Lecture Notes