Homework 2

January 16, 2020

Problem 1. (15 pts)

We have a long straight wire place in vacuum along $\hat{z}$, which carries current I_0. The wire locates at $w_0 = x_0 + iy_0$. Please prove that the magnetic field at location $w = x + iy$ reads:

$$B_y + iB_x = \frac{\mu_0 I}{2\pi\left(w - w_0\right)}$$

Problem 2. (25 pts)

Instead of a long straight wire, we have a long circular current sheet place in vacuum along $\hat{z}$, which carries current density J. The density reads in cylindrical coordinate (r, ϕ) as:

$$J(r, \phi) = \frac{I_0}{2a}\cos\phi\,\delta\left(r - a\right)$$

where I_0 is the current of half sheet and a is the radius of the circle. Please prove that the current sheet produce a dipole field inside the sheet in vertical direction:

$$B_y = -\frac{\mu_0 I_0}{4a}$$

Problem 3. (10 pts)

Prove that a pure n-pole, $b_m = a_m = 0$ when $m \neq n$, preserves under rotation with respective to the origin and along the longitudinal axis.

Problem 4. (Optional)

In many cases, the magnet design follows some symmetry. Under symmetry condition, some multipole components vanish (not allowed). Please indicate the allowed b_n and a_n terms for the inner magnetic field under the symmetry below:

1. Up-down symmetry: $J\left(r, \phi\right) = J(r, 2\pi - \phi)$
2. Up-down anti-symmetry: $J\left(r, \phi\right) = -J(r, 2\pi - \phi)$
3. Left-right symmetry: $J\left(r, \phi\right) = J(r, \pi - \phi)$

Hint: first prove the following relation using the result of problem 1.

$$b_n + ia_n \propto \int_0^{2\pi} e^{-i(n+1)\phi} J(r, \phi) d\phi$$