Homework 9

March 13, 2020

Problem 1. (10 pts)

The following conversion is frequently used in our calculation: please calculate the relative relation between $\Delta v/v$, $\Delta p/p$ and $\Delta E/E$. Note that, v is the velocity of the particle, p is the amplitude of the momentum and E is the energy of the particle.

Problem 2. (15 pts)

In class, we transform the longitudinal map

$$\begin{aligned} \delta_{n+1} - \delta_n &= \frac{eV}{\beta^2 E_0}(\sin\phi_n - \sin\phi_s) \\ \phi_{n+1} - \phi_n &= 2\pi h\eta\delta_{n+1} \end{aligned}$$

to longitudinal effective Hamiltonian. Actually we can also establish the one turn matrix for longitudinal motion if assume $\phi_n = \phi_s + \Delta\phi_n$, where $|\Delta\phi_n| \ll 1$. Find this matrix for $(\delta_{n+1}, \Delta\phi_{n+1})$ from $(\delta_n, \Delta\phi_n)$. Find the tune for this map, by assuming the tune is very close to zero, which is true in ring accelerator. Also please find the 'beta function' for the longitudinal motion.

Problem 3. (25 pts)

Let's use RHIC as an example. RHIC is a heavy ion collider, which accelerate the proton beam from 24 GeV to 250 GeV, or the ion (Gold, Au_{79}^{197}) from 10GeV/u to 100GeV/u. The lattice determines that the transition $\gamma_T = 22.8$.

1. The circumference of the RHIC ring is 3833.845 m. The RF system is design to have harmonic number 360. Please find required RF frequency.

2. The voltage of the cavity is limited to 300KV. The acceleration rate is designed to be $\frac{d\gamma}{dt} = 0.5\ \mathrm{s}^{-1}$. Please find the proper synchronous phase for the ion and proton beam. Also calculate the synchrotron tune of both beams at the 50 GeV.

3. The rms longitudinal emittance is 0.5eV-s. Please find the proton beam's bunch length and energy spread at top energy (250 GeV).