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U N I V E R S I T Y

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Exam Preparation

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Problem P3 – Atwood Machines II

[**cf. problem G6**] Consider the Atwood machine shown in the figure, consisting of two masses m_1, m_2, an ideal pulley and a string of fixed length l.

1. Show that the *unconstrained* Lagrangian of the machine is given by

$$L = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + m_1 g x_1 + m_2 g x_2, \quad (1)$$

   where x_1 and x_2 are the lengths indicated in the figure, increasing in downward direction.

2. Starting from the *holonomic* form of the constraint, use the Lagrange formalism of the first kind to show that the tension in the string is

$$T = |\vec{T}| = \frac{2m_1 m_2}{m_1 + m_2} g. \quad (2)$$

3. Repeat the analysis based on the nonholonomic form of the constraint, and show that you obtain the same result for T.