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PHY422/820: Classical Mechanics

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Exam Preparation

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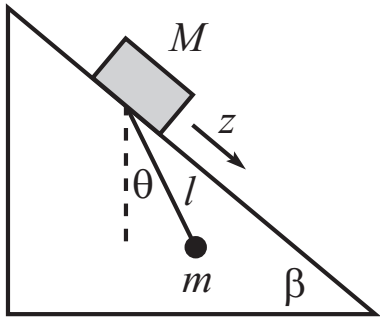
Problem P5 – Pendulum on Inclined Plane

A mass M is free to slide down a frictionless plane inclined at an angle β. A pendulum of length l and mass m hangs from M (see figure) (assume that M extends a short distance beyond the side of the plane, so the pendulum can hang down).

1. Show that the Lagrangian of the machine is given by

$$L = \frac{1}{2}M\dot{z}^2 + \frac{1}{2}m\left(\dot{z}^2 + l^2\dot{\theta}^2 + 2l\dot{z}\dot{\theta}\cos(\theta+\beta)\right) + Mgz\sin\beta + mg\left(z\sin\beta + l\cos\theta\right) , \qquad (1)$$



   where z is the distance traveled on the plane and θ is the angle between the pendulum and the vertical axis.

2. Find the equations of motion and determine the equilibrium positions. Solve them for small displacements from equilibrium. (Equivalently, you can determine the normal modes and frequencies.)