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PHY422/820: Classical Mechanics

FS 2021

Homework #2 (Due: Sep 17)

September 10, 2021

Problem H4 – Invariances of the Lagrange Equations

[**10 Points**] A system with n degrees of freedom satisfies a set of Lagrange equations with a Lagrangian L. Show by a direct substitution into the equations that the same system also satisfies the Euler-Lagrange equations with the Lagrangian

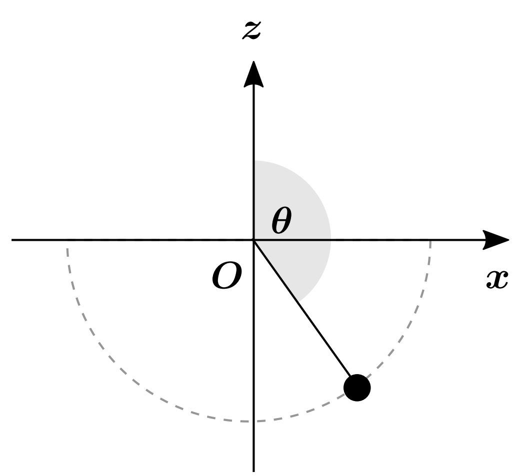
$$L' = L + \frac{\mathrm{d}\, F(q_1, \ldots, q_n, t)}{\mathrm{d}\, t}\,, \tag{1}$$

where F is any arbitrary, but differentiable, function of the n generalized coordinates.

Problem H5 – Spherical Pendulum

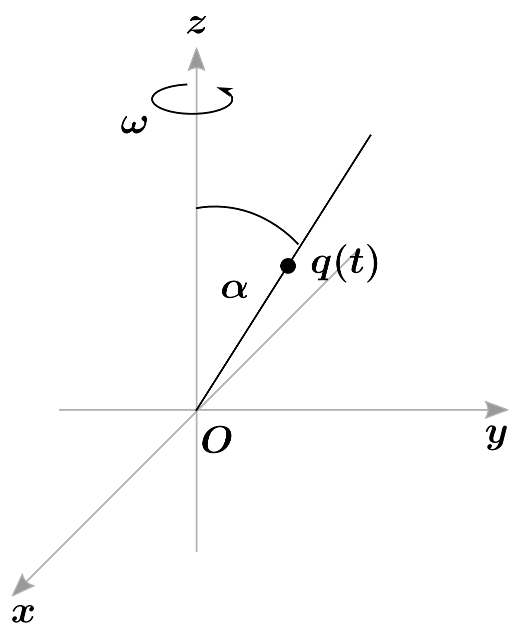
[**10 Points**] We consider a mass m that is suspended from the ceiling by a (massless) string of length l, which is swinging under the influence of gravity.

1. Write down the constraint equation(s).
2. Construct the Lagrangian.
3. Determine the Lagrange equations.
4. Notice that one of the equations can be solved immediately. Briefly discuss the properties of your solution, in particular the units/dimensions, its physical meaning, and the implications for the motion of the pendulum!
5. What happens to *both* equations of motion if the pendulum is restricted to a vertical plane by fixing the *azimuthal* angle $\phi = \text{const.}$?



Problem H6 – Bead on a Rotating Wire

[**10 Points**] Consider a bead of mass m that can slide without friction along a wire that rotates with constant angular velocity ω around an axis through the origin. The angle α between the wire and the axis is fixed.



1. Construct the Lagrangian.
2. Derive the Lagrange equations.
3. Show that the equilibrium position of the bead is given by $q_0 = \frac{g\cos\alpha}{\omega^2 \sin^2\alpha}$.
4. What is the general solution of the equation(s) of motion for the bead? Briefly discuss the motion for large times, assuming infinitesimally small coefficients in your ODE solution. Distinguish the various sign cases.

   Hint: Mind the signs in the equation(s) of motion to avoid misinterpretations! Recall that the solution of an inhomogeneous ODE is $x(t) = x_{\mathrm{p}}(t) + x_{\mathrm{hom}}(t)$, where $x_p(t)$ is a particular solution and $x_{\mathrm{hom}}(t)$ the general solution of the homogenous version of the ODE. Use the result of the previous part!