H. Hergert
Facility for Rare Isotope Beams
and Department of Physics & Astronomy

MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2021

Homework #3 (Due: Sep 24)

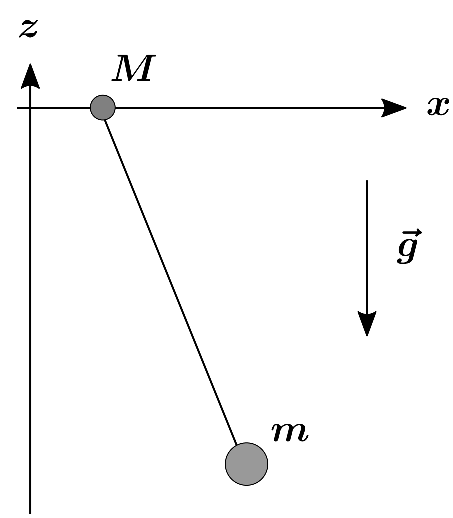
September 17, 2021

Problem H7 – Pendulum with Moving Suspension

[**15 Points**] A mass m can swing on a string of length l around its suspension, which has mass M and can move freely in x direction itself (see figure).

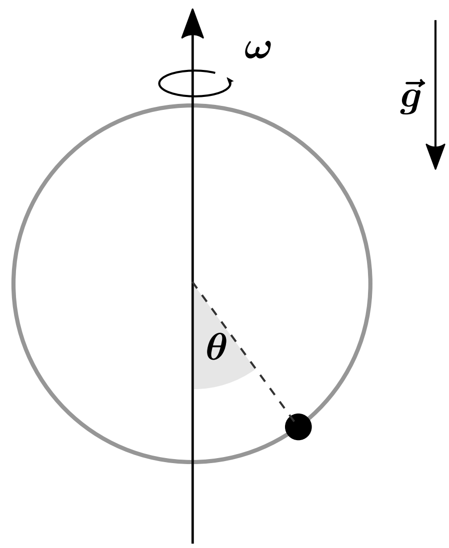
1. Determine the Lagrangian and the Lagrange equations.
2. Which quantities are conserved?
3. Determine the frequency of the pendulum for small initial displacement. What do you obtain for $M \gg m$?

   Hint: Use the conservation law from step 2 to decouple the equations of motion.



Problem H8 – Bead on a Rotating Hoop

[15 Points] A bead of mass m can glide on a frictionless hoop with radius R under the influence of gravity. The hoop rotates around the z axis with a constant angular velocity ω (see figure). The motion of the bead can be parameterized by the angle θ.



1. Construct the Lagrangian and the equation of motion for the bead.

2. Determine the equilibrium positions of the bead on the wire, distinguishing the cases $\omega^2 > g/R$ and $\omega^2 < g/R$. Under which conditions are the equilibria stable under small perturbations $\theta \to \theta \pm \epsilon$?

3. Show that the Lagrangian and the resulting equation of motion are invariant under the transformation $\theta \to -\theta$, i.e., they possess reflection symmetry. Which of the equilibria still have this symmetry?

4. The angular velocity ω is a controllable parameter of the system. Make a qualitative sketch of the *stable* equilibria as a function of ω. What happens at the critical value $\omega_c = \sqrt{g/R}$? Have you encountered such a behavior before, perhaps in other branches of physics?

5. Finally, assume that the bead is in a stable equilibrium for $\omega < \omega_c$, and the angular velocity is then slowly increased above the critical value. What would you expect to happen in an ideal and realistic mechanical system, respectively?