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PHY422/820: Classical Mechanics

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Midterm #2 Preparation

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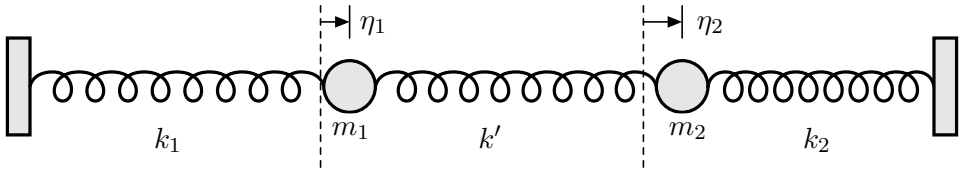
Problem P2 – Two Coupled Masses



Consider the system shown in the figure: Two masses m_1 and m_2 are connected to the walls by springs with constants k_1 and k_2, respectively, and to each other with a spring with a different constant k'.

1. Construct the Lagrangian and write it in the usual quadratic form:
$$L = \frac{1}{2}\dot{\vec{\eta}}^T \boldsymbol{T} \dot{\vec{\eta}} - \frac{1}{2}\vec{\eta}^T \boldsymbol{V} \vec{\eta}. \tag{1}$$

2. Determine the normal modes for the special case where $m_1 = m, m_2 = 2m, k_1 = k, k_2 = 2k, k' = 2k$.

Now consider the case $m_1 = m_2 = m, k_1 = k_2 = k$, and $k' = \epsilon k$ for weak coupling, $\epsilon \ll 1$.

3. Find the normal modes and use them to state a general solution.

4. Assume that the system is set in motion at $t = 0$ by displacing the left mass out of rest by an amount d while the right mass is held fixed at its equilibrium position. Show that the motion can be expressed as
$$\eta_1(t) = d \cos\left(\frac{\omega_2 - \omega_1}{2} t\right) \cos\left(\frac{\omega_1 + \omega_2}{2} t\right), \tag{2}$$
$$\eta_2(t) = d \sin\left(\frac{\omega_2 - \omega_1}{2} t\right) \sin\left(\frac{\omega_1 + \omega_2}{2} t\right). \tag{3}$$
Thus, the masses exhibit *beats* with a slow frequency $\frac{\omega_2 - \omega_1}{2}$ and a fast frequency $\frac{\omega_1 + \omega_2}{2}$.