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Midterm #2 Preparation

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Problem P9 – Properties of the Two-Body Problem

Two particles with masses m_1 and m_2 interact with each other and are also subject to external forces. We denote the interparticle force $\vec{F}_{12} = -\vec{F}_{21}$, and the external forces $\vec{F}^{(e)}_{1,2}$. Assume that we describe the dynamics of the particles from an inertial frame.

1. Use the equations of motion for the individual masses to show that the equation of motion for the center of mass is
$$\frac{d\vec{P}}{dt} = M\frac{d^2\vec{R}}{dt^2} = \vec{F}_1^{(e)} + \vec{F}_2^{(e)}\,, \tag{1}$$
where $M = m_1 + m_2$.

2. Analogously, show that the equation of motion for the relative degree of freedom is
$$\frac{d\vec{p}}{dt} = \mu\frac{d^2\vec{r}}{dt^2} = \vec{F}_{12} + \mu\left(\frac{\vec{F}_1^{(e)}}{m_1} - \frac{\vec{F}_2^{(e)}}{m_2}\right)\,, \tag{2}$$
where
$$\vec{r} \equiv \vec{r}_2 - \vec{r}_1\,, \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2}\,. \tag{3}$$

3. Show that the total kinetic energy and total angular momentum of the two-particle system can be expressed as
$$T = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2 \tag{4}$$
and
$$\vec{L} = M\vec{R}\times\dot{\vec{R}} + \mu\vec{r}\times\dot{\vec{r}}\,. \tag{5}$$

4. Assume that the interparticle force is central, i.e., $\vec{F}_{12} = F_{12}(r)\vec{e}_r$ with $r = |\vec{r}|$. Show that
$$\frac{d\vec{L}}{dt} = \vec{N}^{(e)}\,, \tag{6}$$
where $\vec{L}$ is the total angular momentum and $\vec{N}^{(e)}$ is the torque exerted on the particles by the external forces:
$$\vec{N}^{(e)} \equiv \vec{r}_1\times\vec{F}_1^{(e)} + \vec{r}_2\times\vec{F}_2^{(e)}\,. \tag{7}$$