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Final Exam Preparation

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Problem P12 – Poisson Brackets and Conserved Quantities

The total time derivative of a function $f(\vec{q},\vec{p},t)$ can be written as

$$\frac{df}{dt} = \left\{f, H\right\} + \frac{\partial f}{\partial t}\,. \tag{1}$$

For conserved quantities $g(\vec{q},\vec{p})$, this implies

$$\frac{dg}{dt} = \left\{g, H\right\} = 0\,. \tag{2}$$

1. Show that the Poisson bracket of two conserved quantities g_1 and g_2,

   $$I = \left\{g_1, g_2\right\} \tag{3}$$

   is conserved as well.

2. Show that the angular momentum $l_i = \sum_{jk}\epsilon_{ijk}x_j p_k$ $(i,j,k = 1,\ldots,3)$ satisfies the following algebraic relation:

   $$\left\{l_i, l_j\right\} = \epsilon_{ijk} l_k\,, \tag{4}$$

   where ϵ is the Levi-Civita tensor. Use this relation to prove that a Hamiltonian that is invariant under rotations around the x and y axes is also invariant under rotations around the z axis.