

PHY422/820: Classical Mechanics

FS 2019

Final Exam Preparation

December 7, 2019

Problem P13 – Conserved Quantities of the Kepler Problem

The Hamiltonian of the Kepler problem is given by

$$H(\vec{x}, \vec{p}) = \frac{\vec{p}^2}{2m} - \frac{\kappa}{r}, \quad \kappa > 0, r = |\vec{x}|. \quad (1)$$

Compute the Poisson brackets $\{l_i, H\}$ and $\{A_i, H\}$, to show that the angular momentum and the Laplace-Runge-Lenz vector

$$\vec{A} = \frac{\vec{p} \times \vec{l}}{m\kappa} - \frac{\vec{r}}{r} \quad (2)$$

are conserved.

HINT: Show that

$$\{f(r), p_i\} = \frac{\partial f}{\partial r} \frac{x_i}{r} \quad (3)$$

and use the product rule for Poisson brackets.