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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Problem Set #1 (Due: Sep 6)

August 30, 2019

Problem 1 – Kinematics in Elliptical Coordinates

[15 Points] We can define an *elliptical* coordinate system as follows:

$$\begin{aligned} x &= l \sinh u \sin\theta \cos\phi\,, \qquad l, u > 0\,, \\ y &= l \sinh u \sin\theta \sin\phi\,, \qquad \theta \in [0,\pi)\,, \quad \phi \in [0, 2\pi)\,, \\ z &= l \cosh u \cos\theta\,. \end{aligned} \tag{1}$$

1. Show that for $l, u = \text{const.}$ the coordinates define a *rotational ellipsoid*,

$$\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1\,, \quad b > a\,. \tag{2}$$

   Use this expression to determine u as a function of a and b alone, and confirm that the condition $b > a$ is satisfied. (**Hint:** Notice that Eq. (2) must hold for all angles.)

2. Compute the velocity $\vec{v}$ and acceleration $\vec{a}$ of a point mass moving on the surface of the ellipsoid.

3. Determine the length of a curve with constant value θ by choosing a suitable parameter for the curve. What do you find for a closed-loop trajectory?

Now assume that u can vary as well.

4. Construct the unit vectors $\{\vec{e}_u, \vec{e}_\theta, \vec{e}_\phi\}$ of the elliptical coordinate system.

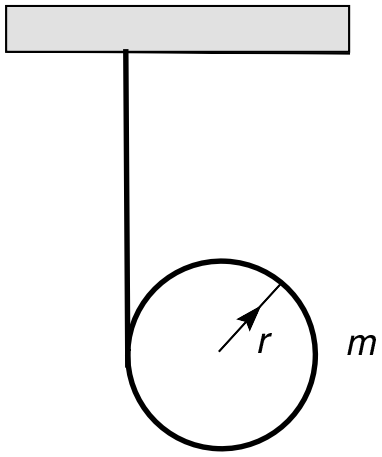
5. Consider the spherical limit, where $a = b = r$, and show that the unit vectors determined before reduce to the usual unit vectors for spherical coordinate systems.

HINT::

$$\sinh' x = \cosh x\,, \quad \cosh' x = \sinh x\,, \quad \cosh^2 x - \sinh^2 x = 1$$

Problem 2 – Dynamics: Cylinder on a String

[15 Points] A string is wrapped around a uniform homogeneous cylinder whose radius is r and mass is m. The free end of the string is tied to the ceiling and the cylinder is allowed to fall, see the figure, starting from rest. As the string unwraps, the cylinder rotates.



1. What is the linear acceleration of the center of mass of the cylinder?
2. What is the tension in the string?
3. What is the linear velocity of the cylinder after it has dropped down a distance h?

Problem 3 – Conservative Forces

[15 Points] Let $\vec{F} = C\left(k_1 xy,\, x^2 + k_2 z^2,\, yz\right)^T$ be a force field, with $C > 0$ having the units of a force, and k_1, k_2 being initially unspecified parameters.

1. For which values of the constants k_1 and k_2 does the force have a potential?

Use these values k_1 and k_2 that result in a conservative force $\vec{F}$ in the following.

2. Compute the work required to move a point mass m from the origin of the coordinate system to $\vec{a} = (a_x, a_y, a_z)^T$. Under which conditions does this transport require and generate energy, respectively?
3. Determine the potential and check that it indeed yields the correct expression for the force field.

Problem C1 – Getting Started with Jupyter

[20 Points] You can find the Jupyter notebook with comments and code fragments in the Homework section of the course website (`http://people.nscl.msu.edu/ hergert/phy820`), or by pulling from the course material repository. Follow the procedure described in the Computation section of the website to submit your homework when you are ready.