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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Homework #2 (Due: Sep 13)

September 7, 2019

Problem 4 – Invariances of the Euler-Lagrange Equations I

[10 Points] A system with n degrees of freedom satisfies a set of Euler-Lagrange equations with a Lagrangian L. Show by a direct substitution into the equations that the same system also satisfies the Euler-Lagrange equations with the Lagrangian

$$L' = L + \frac{\mathrm{d}\, F(q_1, \ldots, q_n, t)}{\mathrm{d}\, t}\,, \tag{1}$$

where F is any arbitrary, but differentiable, function of the n generalized coordinates.

Problem 5 – Invariances of the Euler-Lagrange Equations II

[10 Points] Let $q_1, \ldots q_n$ be a set of independent generalized coordinates for a system of n degrees of freedom, with a Lagrangian $L(q, \dot{q}, t)$. Suppose we transform to another set of independent coordinates $s_1, \ldots, s_n$ by means of transformation equations

$$q_i = q_i(s_1, \ldots, s_n, t)\,, \qquad i = 1, \ldots, n\,. \tag{2}$$

Show that if the Lagrangian function is expressed as a function of s_j, $\dot{s}_j$, and t through the equations of transformation, then L satisfies Lagrange's equations with respect to the s coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{s}_j}\right) - \frac{\partial L}{\partial s_j} = 0\,. \tag{3}$$

In other words, the *form* of Lagrange's equations is invariant.

Problem 6A – Spherical Pendulum (PHY422)

[15 Points] We consider a mass m that is suspended from the ceiling by a (massless) string of length l, which is swinging under the influence of gravity.

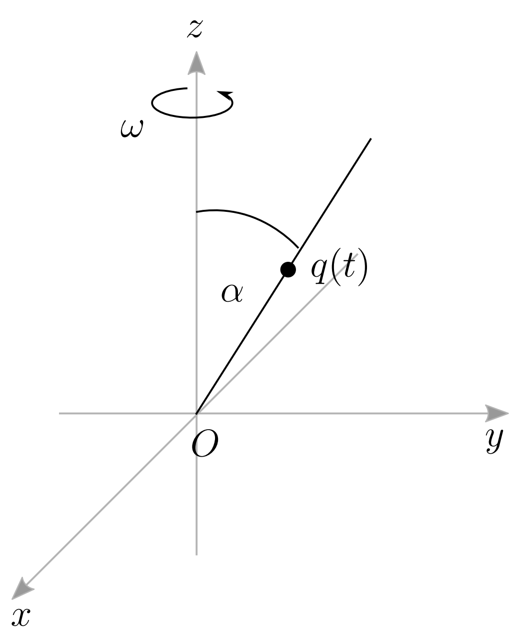
1. Write down the constraint equation(s).

2. Compute the Lagrangian.
3. Determine the Euler-Lagrange equations.
4. Notice that one of the equations can be solved immediately. Briefly discuss the properties of your solution, in particular the units/dimensions, its physical meaning, and the implications for the motion of the pendulum!
5. What happens to *both* equations of motion if the pendulum is restricted to a plane by fixing $\phi = \text{const.}$?

HINT: Choose appropriate coordinates before you do anything else!

Problem 6B – Bead on a Rotating Wire (PHY820)

[15 Points] Consider a bead of mass m that can slide without friction along a wire that rotates with constant angular velocity ω around an axis through the origin. The angle α between the wire and the axis is fixed.



1. Compute the Lagrangian.
2. Determine the Euler-Lagrange equation(s).
3. Show that the equilibrium position of the bead is given by $q_0 = \frac{g\cos\alpha}{\omega^2 \sin^2\alpha}$.
4. What is the general solution of the equation(s) of motion for the bead? (**Hint:** Recall what you know about inhomogeneous ODEs, and consider how the result of the previous part of the problem could be useful.) Briefly discuss the motion for large times, assuming infinitesimally small coefficients in your ODE solution. Distinguish the various sign cases.

HINT: Choose appropriate coordinates before you do anything else!

Problem C2 – Ideal Pendulum

[15 Points] You can find the Jupyter notebook with comments and code fragments in the Homework section of the course website (`http://people.nscl.msu.edu/ hergert/phy820`), or by pulling from the course material repository. Follow the procedure described in the Computation section of the website to submit your homework when you are ready.