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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Homework #4 (Due: Sep 27)

September 22, 2019

Problem 10 – Mass Sliding Down a Hemisphere

[15 Points] A block of mass m is released from rest at the top of a frictionless hemisphere of radius R, and slides down the surface under the influence of gravity until it flies off.

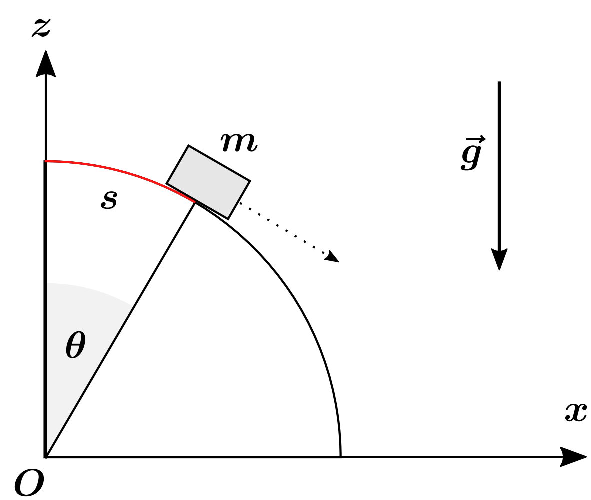
1. Construct the Lagrangian for the initial phase of the block's motion, coupling it to the constraint with a Lagrange multiplier. Use appropriate coordinates.

2. Use the Lagrange equations of the first kind to determine the angle at which the block flies off, and the length of the arc from the top to the point at which it launches.

   HINT: Consider what happens to the constraint when the block flies off. You will also find the following relation useful:

$$\frac{d}{dt}\dot{\theta}^2 = 2\dot{\theta}\ddot{\theta}\,. \tag{1}$$

3. Compute the block's point of impact on the ground.



Problem 11 – Damped Pendulum

[15+5 Points] We consider a pendulum of mass m on a string of length l that is subject to gravity and a drag force

$$\vec{F}_D = -\beta\dot{\vec{r}}\,. \tag{2}$$

1. Construct the Lagrangian, dissipation function, and the Lagrange equations.

2. Solve the equation of motion for a small initial displacement from rest, and determine the time after which the amplitude has been reduced by a factor $1/e$.

3. **Bonus:** Add the damping term to the notebook you developed for the ideal pendulum in C02. Compare the small angle approximation and exact solution for the damped and overdamped cases by generating figures and briefly discussing them. Submit the updated notebook via GitLab, along with C04.

Problem 12 – Gauge Symmetry for a Particle in an Electromagnetic Field

[**15 Points**] A particle with charge q and mass m is moving in an electromagnetic field. In SI units, the electric and magnetic fields, $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$, are related to the scalar and vector potentials $\phi(\vec{r},t)$ and $\vec{A}(\vec{r},t)$ via

$$\vec{E}(\vec{r},t) = -\nabla\phi(\vec{r},t) - \frac{\partial}{\partial t}\vec{A}(\vec{r},t), \qquad \vec{B}(\vec{r},t) = \nabla \times \vec{A}(\vec{r},t)\,. \tag{3}$$

The Lagrangian of the particle is given by

$$L(\vec{r},\dot{\vec{r}}) = \frac{1}{2}m\dot{\vec{r}}^{\,2} - q\left(\phi - \dot{\vec{r}}\cdot\vec{A}\right) \tag{4}$$

1. Show explicitly that the Lagrange equations lead to Lorentz's force law,

$$m\ddot{\vec{r}} = q\vec{E} + q\dot{\vec{r}}\times\vec{B}\,. \tag{5}$$

   HINT:

$$\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b}) \tag{6}$$

   (Be careful about the ordering of the terms when any of the vectors is a gradient!)

2. Now consider the **gauge transformation**

$$\phi(\vec{r},t) \longrightarrow \phi(\vec{r},t) - \frac{\partial}{\partial t}\Lambda(\vec{r},t)\,, \tag{7}$$

$$\vec{A}(\vec{r},t) \longrightarrow \vec{A}(\vec{r},t) + \nabla\Lambda(\vec{r},t)\,. \tag{8}$$

   with an arbitray twice differentiable function $\Lambda(\vec{r},t)$.

   How do the electromagnetic fields change? How does the gauge transformation affect the Lagrangian?

Problem C4 – Harmonic Oscillator Class and Visualization

[**15 Points**] You can find the Jupyter notebook with comments and code fragments in the Homework section of the course website (`http://people.nscl.msu.edu/ hergert/phy820`), or by pulling from the course material repository. Follow the procedure described in the Computation section of the website to submit your homework when you are ready.