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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Homework #6 (Due: Oct 11)

October 5, 2019

Problem 21 – Cycloidal Pendulum

[**15 Points**] An ideal cycloidal pendulum consists of a mass that oscillates under gravity along a frictionless cycloidal track that is parameterized by the following expressions:

$$x = R(\theta - \sin\theta)\,, \quad y = R(1 - \cos\theta)\,, \tag{1}$$

where the vertical y-axis points downward.

1. Show that the Lagrangian for this system is given by

   $$L = 2mR^2 \sin^2\left(\frac{\theta}{2}\right)\dot{\theta}^2 + mgR(1 - \cos\theta)\,. \tag{2}$$

   Hint:

   $$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

2. Make a point transformation to the new generalized coordinate $u = \cos\left(\frac{\theta}{2}\right)$ and derive the Lagrangian in u.

3. Derive the Lagrange equations and show that the period of oscillation is

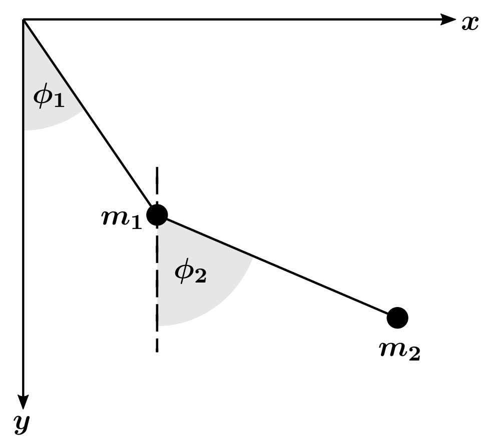
   $$\mathcal{T} = 4\pi\sqrt{\frac{R}{g}}\,, \tag{3}$$

   independent of the amplitude. C. Huygens recognized this property of the cycloid in 1659 in his attempt to come up with an improved design for a pendulum clock.

A Jupyter notebook that visualizes the oscillations of the ideal and cycloidal pendula as a function of the amplitude will be distributed with the computational homework C06.

Problem 22 – Double Pendulum

[15 Points] Consider a planar double pendulum with lengths l_1, l_2 and masses m_1, m_2 (see figure).



1. Construct the Lagrangian of the system and derive the equations of motion.

2. What are the normal frequencies for small angles ($\phi_1, \phi_2 \ll 1$)?

   HINT: Use the ansatz $\phi_k = \varphi_{0,k} \exp(-i\omega t)$ with identical ω for $k = 1, 2$, and determine ω such that the resulting system of equations has a non-trivial solution for $\phi_{0,1}, \phi_{0,2}$!

Problem C6 – Driven Oscillators

[15 Points] You can find the Jupyter notebook with comments and code fragments in the Homework section of the course website (`http://people.nscl.msu.edu/ hergert/phy820`), or by pulling from the course material repository. Follow the procedure described in the Computation section of the website to submit your homework when you are ready.