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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Homework #9 (Due: Nov 1)

October 29, 2019

Problem 28 – Laplace-Lenz-Runge Vector in Rutherford Scattering

[**15 points**] In our discussion of Rutherford scattering we derived the following relationship between the scattering angle θ and the impact parameter b:

$$b(\theta) = \pm\frac{\kappa}{2E}\cot\frac{\theta}{2}\,, \quad \kappa = -\frac{q_1 q_2}{4\pi\varepsilon_0}\,. \tag{1}$$

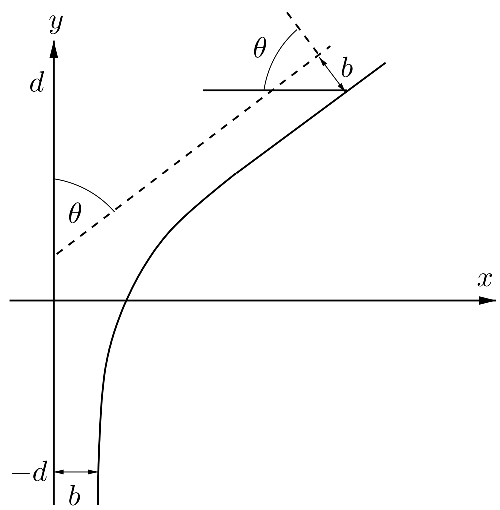
Validate this relationship using the conservation of the Laplace-Runge-Lenz vector by computing $\vec{A}$ for the incoming and outgoing particle(s):

$$t\to-\infty: \quad \vec{r}(-\infty) = (b, -d, 0)^T\,, \qquad \dot{\vec{r}} = (0, v_\infty, 0)^T\,, \tag{2}$$

$$t\to\infty: \quad \vec{r}(\infty) = (d\tan\theta + \frac{b}{\cos\theta}, d, 0)^T\,, \qquad \dot{\vec{r}} = (v_\infty\sin\theta, v_\infty\cos\theta, 0)^T\,, \tag{3}$$

where we can assume $d \gg 1$ at an appropriate stage of our calculation (see figure).

Hint: It is sufficient to consider a single component of $\vec{A}$.



Problem 29 – Scattering from a Repulsive Inverse-Square Potential

[**15 points**] Show that the differential cross section for the repulsive inverse-square potential

$$V(r) = \frac{\kappa}{r^2}\,, \quad \kappa > 0\,, \tag{4}$$

is given by

$$\frac{d\sigma}{d\Omega} = \frac{\kappa\pi^2}{E}\frac{\pi-\theta}{\theta^2\,(2\pi-\theta)^2\sin\theta}\,. \tag{5}$$

HINT: Determine the distance of closest approach from energy conservation, and use

$$\int dr\, \frac{1}{r^2\sqrt{1-\alpha^2/r^2}} = -\frac{1}{\alpha}\arcsin\frac{\alpha}{r} + C\,. \tag{6}$$

(cf. hard-sphere scattering) or your favorite computer-algebra system.

Problem C9 – Solar System Simulator

[15 Points] You can find the Jupyter notebook with comments and code fragments in the Homework section of the course website (`http://people.nscl.msu.edu/ hergert/phy820`), or by pulling from the course material repository. Follow the procedure described in the Computation section of the website to submit your homework when you are ready.