

PHY422/820: Classical Mechanics

FS 2020

Exam Preparation

December 1, 2020

Problem P6 – Friction

A dumbbell consisting of two equal masses that are connected by a massless rod of length l can move in a horizontal plane. The dumbbell is subject to a frictional force that is linear in the velocity.

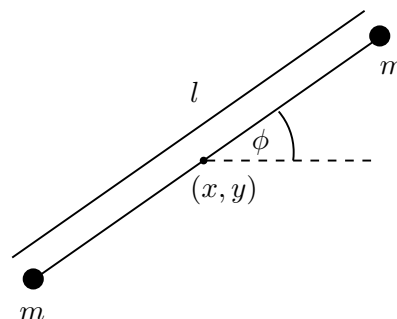
1. Show that the Lagrangian for the motion without friction is

$$L = m(\dot{x}^2 + \dot{y}^2) + \frac{1}{4}ml^2\dot{\phi}^2 \quad (1)$$

and Rayleigh's dissipation function

$$D = \beta(\dot{x}^2 + \dot{y}^2) + \frac{1}{4}\beta l^2\dot{\phi}^2, \quad (2)$$

where x, y are the coordinates of the center of mass and ϕ is the angle indicated in the figure.



2. Compute the generalized forces Q_x, Q_y and Q_ϕ for the case with friction, and derive the equations of motion. State the general solutions in terms of the initial values for the coordinates (x_0, y_0, ϕ_0) and velocities $(v_{x0}, v_{y0}, \omega_0)$.