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MICHIGAN STATE UNIVERSITY

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Exam Preparation

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Problem P9 – Precessing Orbits

In the following, we want to study the precession of nearly-circular orbits in the potentials

$$V_1(r) = -\frac{k}{r} + \frac{\alpha}{r^2}\,, k > 0, \alpha \in \mathbb{R}\,, \tag{1}$$

$$V_2(r) = -\frac{k}{r} e^{-r/a}\,, \quad a > 0\,, k > 0\,, \tag{2}$$

using intermediate results from the proof of Bertrand's theorem that relate the change in angle per round trip between the apses, $\Delta\phi$, to the potential $V(r)$.

Note: $V_2(r)$ is a so-called **Yukawa potential**. It plays an important role in Quantum Field Theory, where it emerges for interactions that are involve the exchange of particles with non-zero mass.

1. Show that the approximate rate of precession for the perturbed Kepler potential is $-\frac{2\pi\alpha}{kR}$, where R denotes the radius of the circular trajectory.

2. Prove that the range of the Yukawa potential must exceed a critical value $a_c = (\sqrt{5} - 1)\frac{R}{2}$ to admit bounded orbits. Show that the approximate rate of precession for the nearly-circular orbit is $\pi\frac{R}{a}$. Discuss the limits of your expressions.

Hint: It is not necessary — and for the Yukawa potential, not even be possible — to determine the radius of the circular trajectory analytically.