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MICHIGAN STATE UNIVERSITY

PHY422/820: Classical Mechanics

FS 2019

Homework #8 (Due: Oct 30)

October 25, 2020

Problem H15 – Scattering off Constant Potentials

[**10 points**] A particle is scattering off the potential well

$$V(r) = \begin{cases} -V_0 & R \leq 0\,, \\ 0 & R > 0 \end{cases}, \tag{1}$$

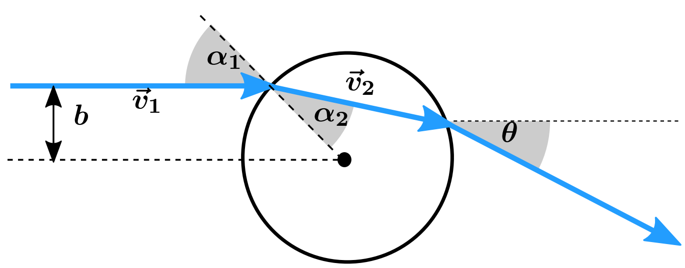
where V_0 and R are positive constants.

1. Show that

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{v_2}{v_1} = \sqrt{\frac{E+V_0}{E}} \equiv n\,. \tag{2}$$

This is identical to what we would find for the refraction of light rays at the boundary of two substances with relative index of refraction n.

Hint: Consider which quantities will be conserved or not conserved at the boundary of the potential region.



2. Show that the scattering angle θ is given by

$$\theta(b) = 2\left(\arcsin\frac{b}{R} - \arcsin\frac{b}{nR}\right)\,. \tag{3}$$

Problem H16 – Scattering from a Repulsive Inverse-Square Potential

[**10 points**] Show that the differential cross section for the repulsive inverse-square potential

$$V(r) = \frac{k}{r^2}\,,\quad k > 0\,, \tag{4}$$

is given by

$$\frac{d\sigma}{d\Omega} = \frac{k\pi^2}{E}\frac{\pi-\theta}{\theta^2\left(2\pi-\theta\right)^2\sin\theta}\,. \tag{5}$$

Hint: Determine the distance of closest approach from energy conservation, and use

$$\int du \frac{1}{\sqrt{1-\alpha^2 u^2}} = \frac{1}{\alpha} \arcsin(\alpha u) + c \tag{6}$$

(cf. hard-sphere scattering) or your favorite computer-algebra system.

Problem H17 – Inverse Scattering Problem

[10 points] In the following, we will use inverse-scattering techniques to extract the potential (4) from the differential cross section (5).

1. Determine $b(\theta)$ by rearranging and integrating

   $$\frac{d\sigma}{d\Omega} = -\frac{b}{\sin\theta}\frac{db}{d\theta}\,. \tag{7}$$

   How must the integration limits be chosen?

2. Invert $b(\theta)$ to obtain $\theta(b)$ and compute the function $T(y)$.

   Note: The integration is straightforward with an appropriate substitution. It is not necessary to work with the partial derivative trick discussed in the example in the lecture notes.

3. Determine $r(y) = y \exp T(y)$ and extract the potential $V(r)$.