# Noether's Theorem and Symmetries in the Hamiltonian Formalism

→ Noether's Theorem connects symmetries and conservation laws – how does this translate to Hamiltonian mechanics?

→ Infinitesimal canonical transformation $G$:

$$\delta H = \sum_j \left( \frac{\partial H}{\partial q_j} \delta q_j + \frac{\partial H}{\partial p_j} \delta p_j \right)$$

$$= \sum_j \left( \frac{\partial H}{\partial q_j}\left(\alpha \frac{\partial G}{\partial p_j}\right) - \frac{\partial H}{\partial p_j}\left(\alpha \frac{\partial G}{\partial q_j}\right)\right) = \alpha \{H, G\}$$

If $G$ is the generator of a symmetry of the Hamiltonian, we must have

$$\delta H = \alpha \{H, G\} = 0$$

But we also saw (previous lecture & homework) that

$$\dot{G} = \{G, H\}$$

(for $\frac{\partial G}{\partial t} = 0$, which would be the case for the generator of a symmetry)

So

$$\{H, G\} = 0 \quad \text{from invariance of } H \text{ under symmetry}$$

$$\Leftrightarrow \dot{G} = 0$$

---

## Generating Functions

Prototypes of canonical transformations from $(q_j, p_j) \to (Q_j, P_j)$

Consider $F(q_j, Q_j)$ and (cf. Legendre)

$$p_j \equiv \frac{\partial F(q_k, Q_k)}{\partial q_j} \rightarrow \text{solve for } Q_j(q_k, p_k)$$

What is the canonical momentum, then?