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PHY422/820: Classical Mechanics

FS 2020

Worksheet #11 (Nov 9 – Nov 13)

November 8, 2020

1 Preparation

- Lemos, Chapter 4
- Goldstein, Chapter 5

2 Group Exercises

Problem G27 – Rotating Cuboid

Consider a homogenous rotating cuboid with side lengths a, b, c and mass M.

1. Compute the principal moments of inertia with respect to the cuboid's center of mass.

   Hint: The diagonalization of the moment-of-inertia tensor can be avoided through an appropriate choice of coordinate system.

2. Determine the cuboid's equations of motion in the body-fixed frame, the **Euler equations for the rigid body**, by starting from

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{N}\,, \quad \vec{L} = \boldsymbol{I}\vec{\omega} = (A\omega_{x'}, B\omega_{y'}, C\omega_{z'})^T\,, \tag{1}$$

   where all vectors are expressed in the **body-fixed frame**, and A, B and C denote the principal moments of inertia.

3. Consider the force-free rotation of the cuboid around a principal axis, e.g., $\vec{\omega}_0 = (\omega_0, 0, 0)^T =$ const. Under which conditions is a rotation around this axis stable?

   Hint: Assume a small perturbation of the rotational axis,

$$\vec{\omega} = \vec{\omega}_0 + \vec{\epsilon} = \vec{\omega}_0 + (\epsilon_{x'}, \epsilon_{y'}, \epsilon_{z'})^T\,, \tag{2}$$

   and determine the conditions under which the amplitude of the perturbation $\vec{\epsilon}$ remains small. Omit terms of order $O(\epsilon^2)$ and higher.

Problem G28 – Rotating Platelet

[cf. problem G27] Consider a thin rectangular platelet of mass m with side lengths a, b and a homogeneous mass distribution. Choose a coordinate system whose origin is the platelet's center of mass.

1. Express the platelet's mass density $\rho(x, y, z)$ using δ and step functions.

2. Determine the moment of inertia tensor $\boldsymbol{I}$ in the chosen center-of-mass frame, and determine the principal axes.

   HINT: You can use your results from problem G27, or compute $\boldsymbol{I}$ explicitly for practice.

3. Derive the Euler equations for the platelet in the body-fixed frame.

4. Compute the torque $\vec{N}$ that is required to make the platelet rotate with a *constant* angular velocity around its *diagonal*. What happens if the platelet is quadratic, i.e., $a = b$?