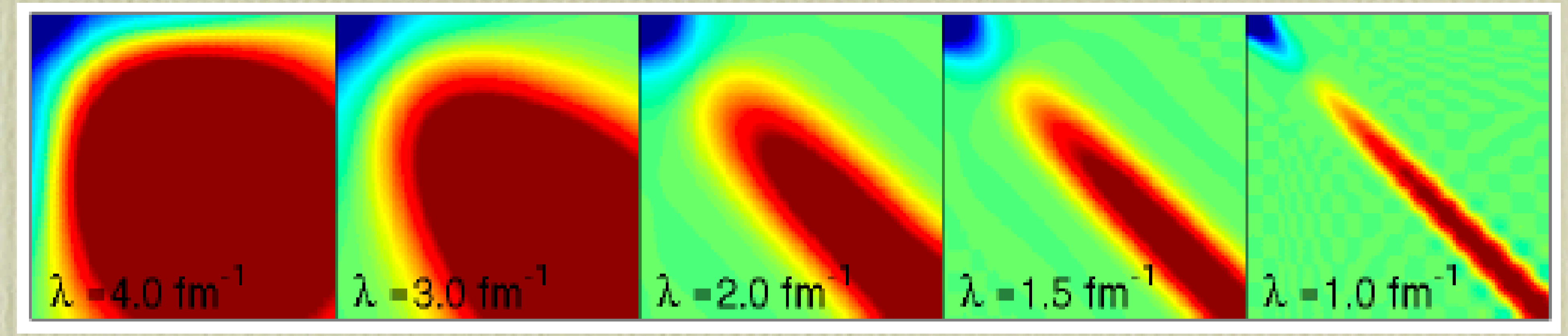


Renormalization Group Methods in Nuclear Theory or Putting on Blurry Glasses to Model Nuclei

Scott Bogner
NSCL/FRIB

The Nuclear Landscape and the Big Questions

- How did (visible) matter come into being and how does it evolve?
- How are the nuclei of atoms made and organized?
- What are the fundamental particles and forces at work inside atomic nuclei?

TIMESCALES

- ➥ from QCD transition (color singlets formed; 10 ms after Big Bang) till today (13.8 billion years later)

DISTANCE SCALES

- ➥ from 10^{-15} m (proton's radius) to ~12 km (neutron star radius)

Phenomenological Models vs. Microscopic Models

Phenomenological Models

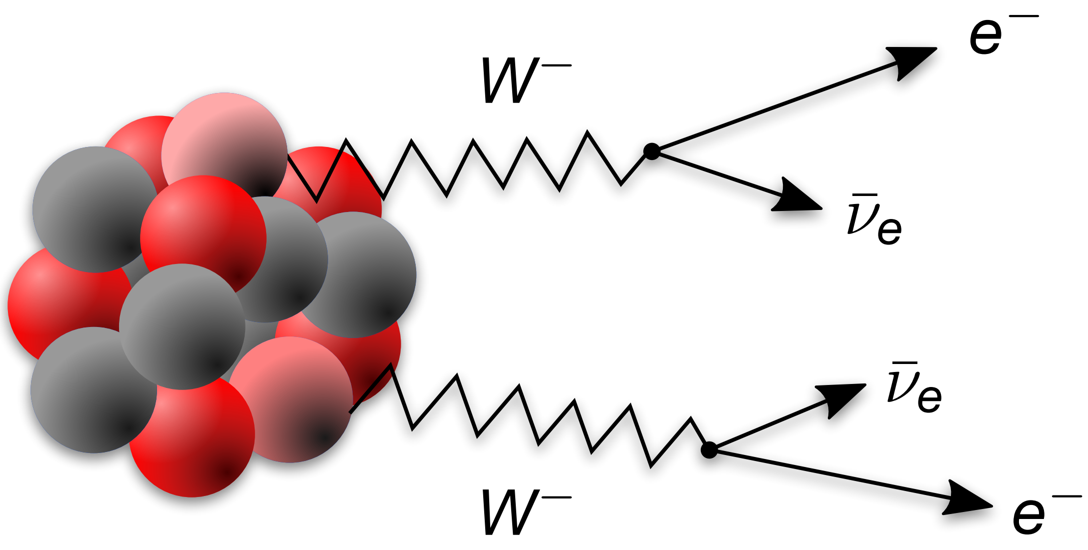
- Shell Model, Energy Density Functional theory, etc...
- **Data Driven** (fit to some mass region)
- Very accurate generally, but **uncontrolled extrapolations** and not systematically improvable
- Computationally cheap (cover most nuclei)

Microscopic Models

- Lattice QCD, ab-initio many-body theory
- Start from **fundamental interactions**
- Less global accuracy, but more controlled extrapolations and systematically improvable
- Computationally expensive (cover fewer nuclei)

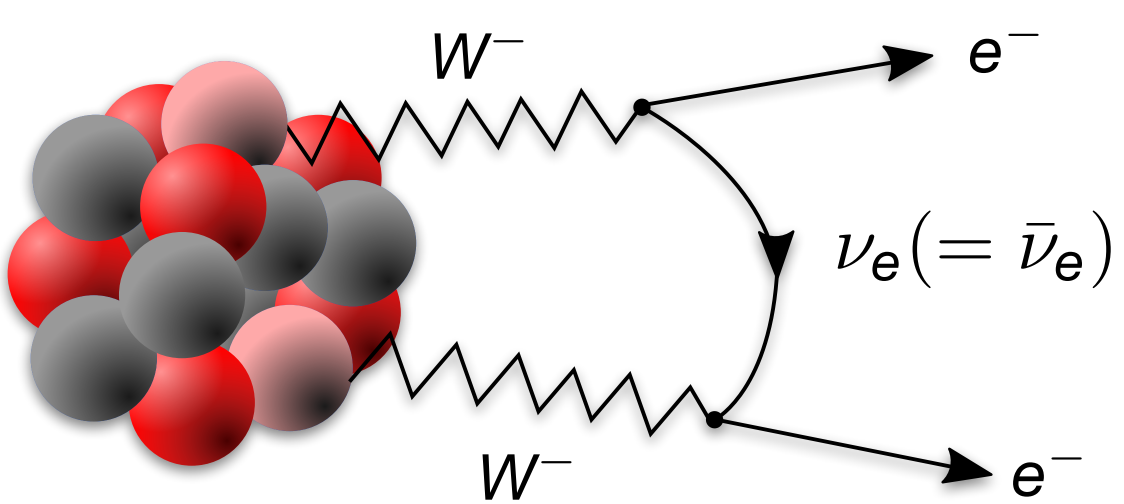
Is the Standard Model Complete?

"Standard" Double Beta Decay



- observed and well understood
- consistent w/the Standard Model

Neutrinoless Double Beta Decay



- neutrinos are **Majorana** particles
- beyond Standard Model: **new physics (and Nobel prizes!) if observed**

Decay Rate

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)\right)^{-1} = G|M^{0\nu\beta\beta}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

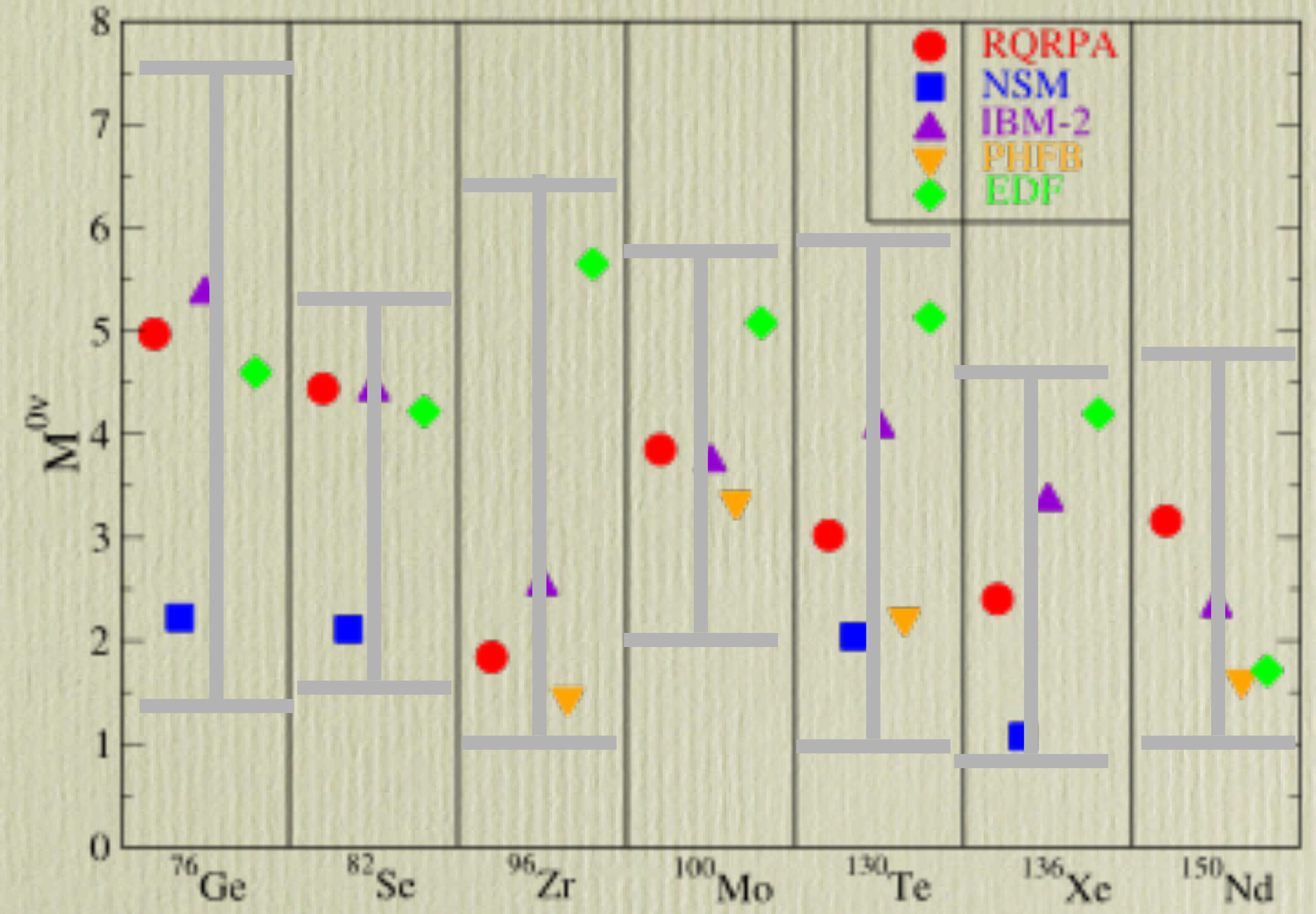
- G : kinematic factor
- m_e : *electron mass*
- effective Majorana mass:

$$m_{\beta\beta} = \sum_{i=1}^{3} m_i U_{ei}^2$$

  - m_i : neutrino mass eigenvalue
  - U_{ei} : neutrino flavor mixing matrix
- $M^{0\nu\beta\beta}$: **nuclear matrix element**

need accurate nuclear matrix elements!

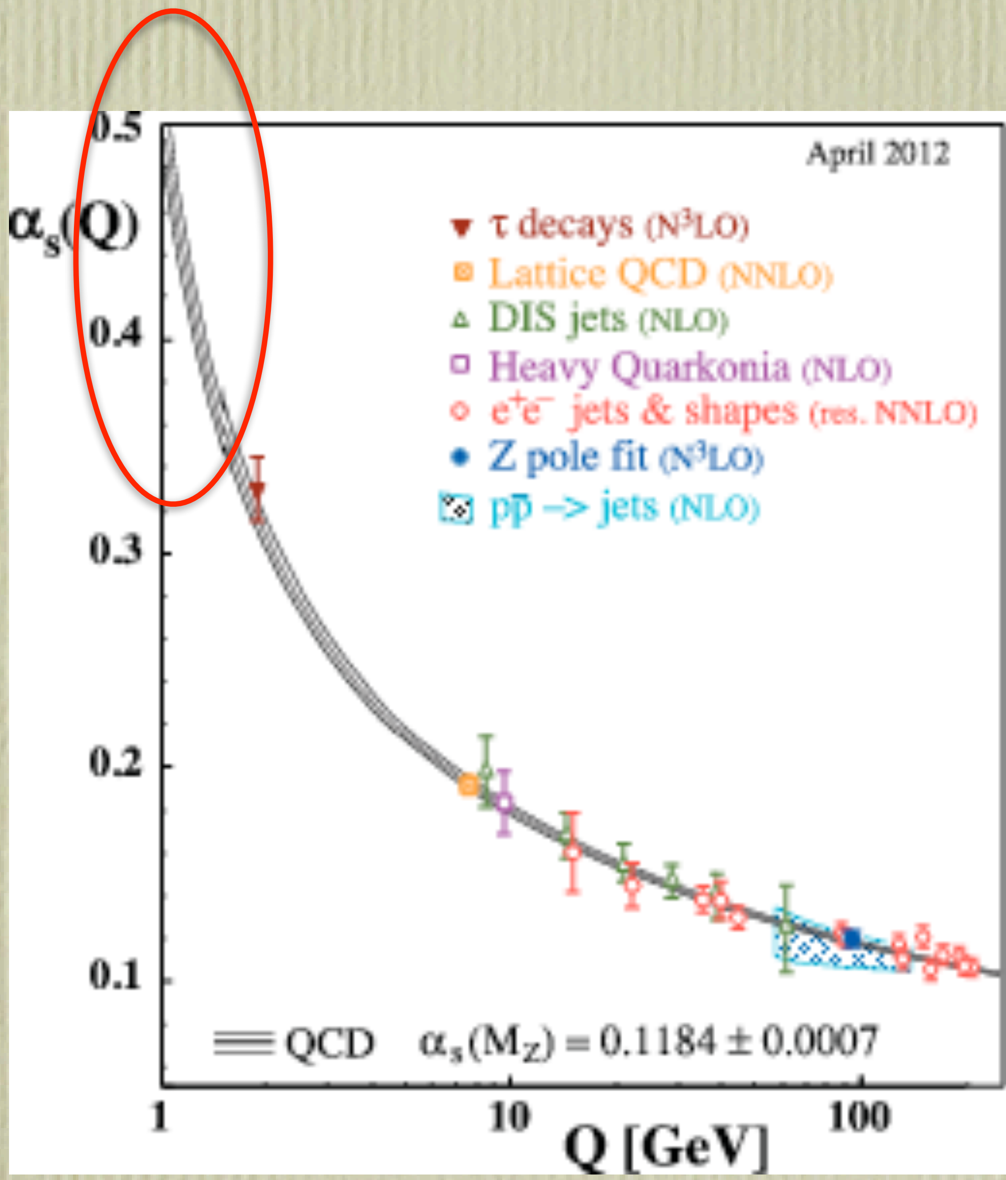
Nuclear Matrix Elements $M^{0\nu\beta\beta}$



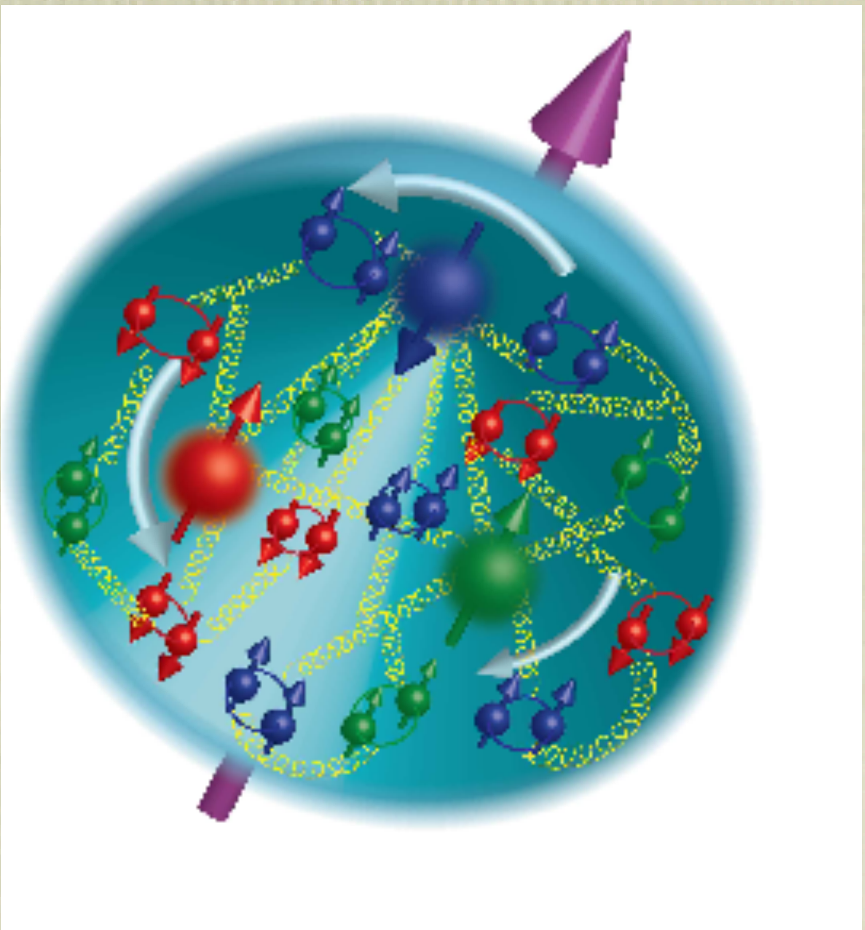
"There is generally significant variation among different calculations of the nuclear matrix elements for a given isotope. For consideration of future experiments and their projected sensitivity it would be very desirable to reduce the uncertainty in these nuclear matrix elements." (Neutrinoless Double Beta Decay NSAC Report 2014)

Expect microscopic calculations can improve this!

Nuclei from QCD?

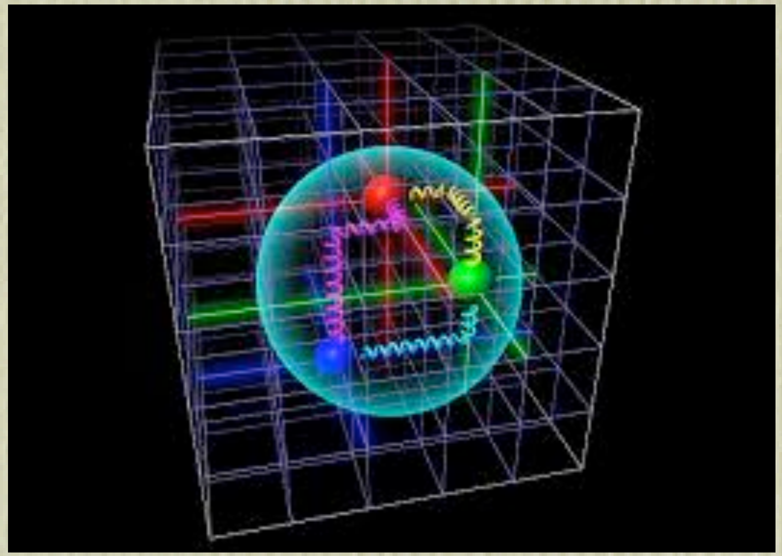


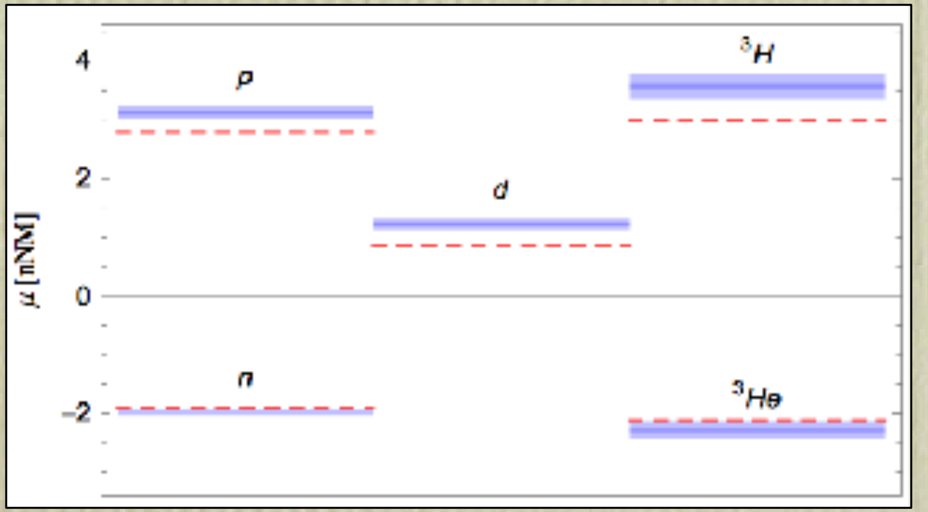
QCD coupling "constant" gets big at low energies



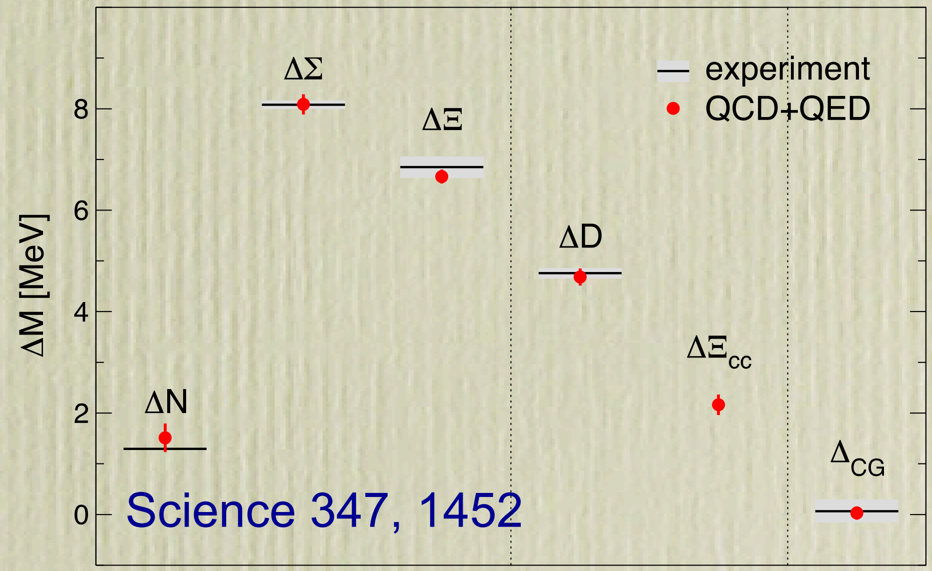
How it actually looks

Towards computing nuclei from QCD

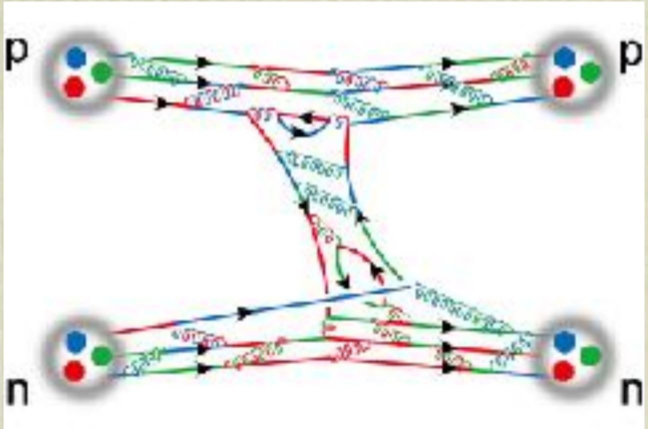
Lattice QCD =  + 



LQCD predictions for magnetic moments A<4
PRL113, 252001 (2014)



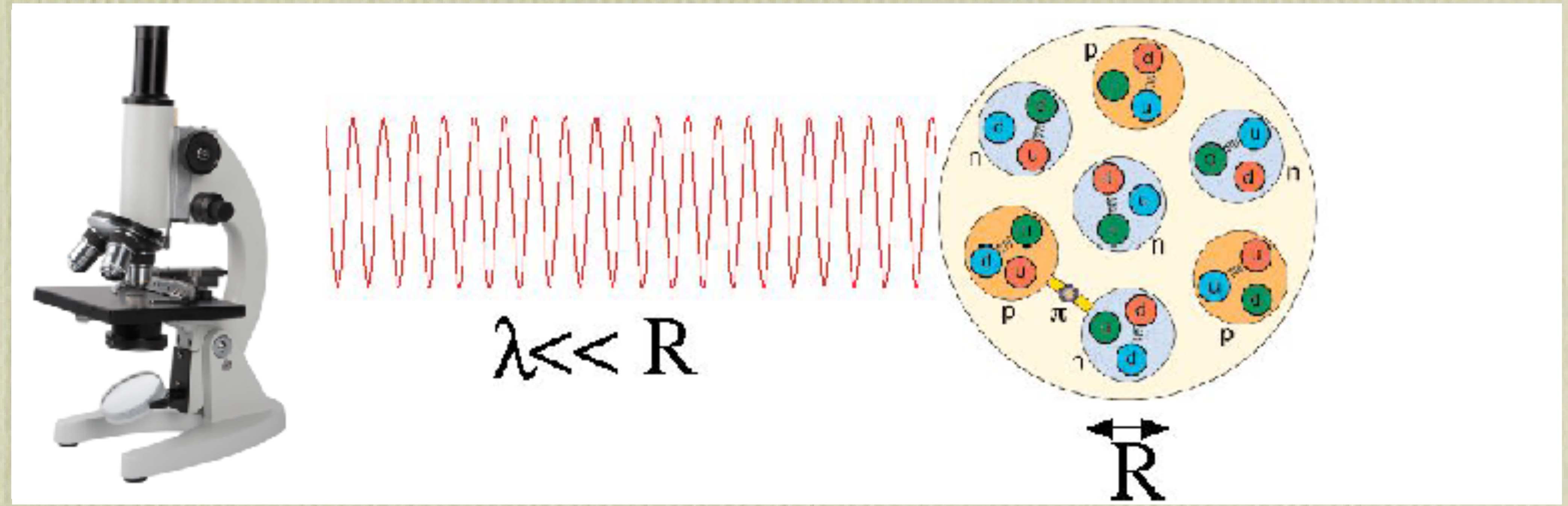
n-p mass difference from LQCD



Nuclear Force from lattice QCD
PRL 111, 112503 (2013)

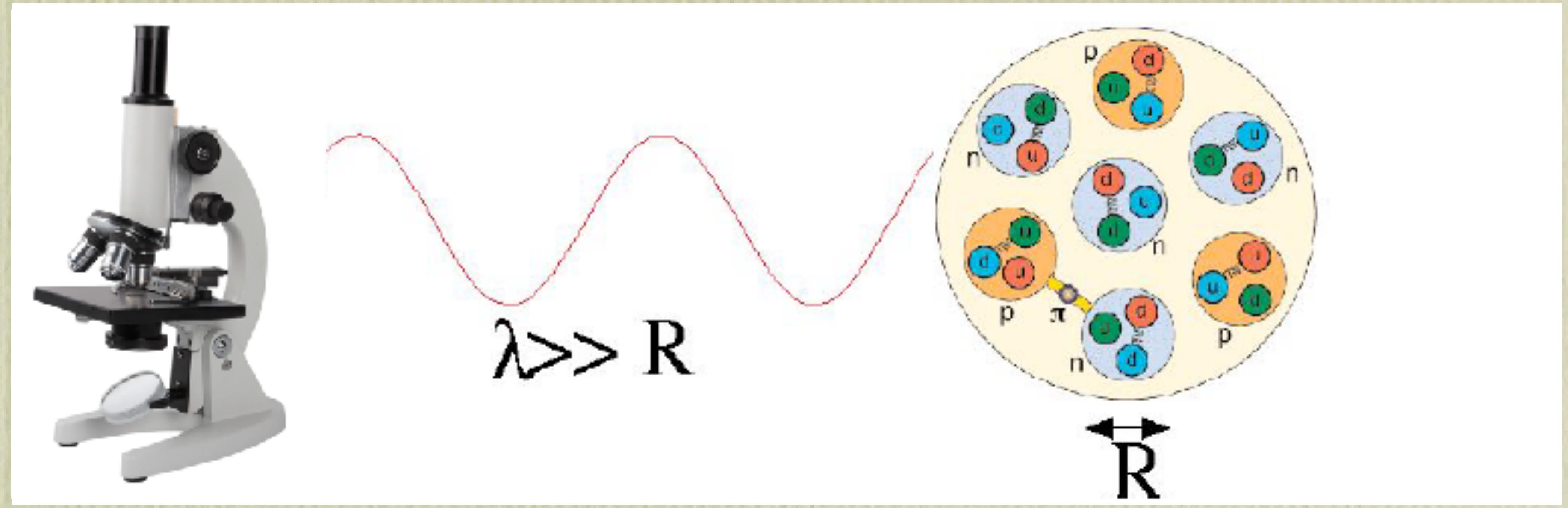
Looks great, but lattice QCD calculations limited to light (A=1,2,3) systems thus far….is there a more efficient way?

Principle of Low-Energy Effective Theories



- Short-distance structure resolved; need QCD degrees of freedom

Principle of Low-Energy Effective Theories



- Nucleus probed at low energies, fine details not resolved
  - Use convenient DOF (protons/neutrons **instead** of quarks/gluons)
  - Complicated short-distance dynamics replaced by something **simpler**

Scale Separation and effective theories

$$F = G\frac{M_E M_A}{R^2}$$

Scale Separation and effective theories

$$F = G\frac{M_E M_A}{R^2}$$

$$R \approx R_E$$

$$g \equiv G\frac{M_E}{R_E^2} \approx 9.81\frac{m}{s^2}$$

$$F = M_A g$$

Claim: you can likewise "reduce" QCD to an effective theory of neutrons and protons

Quantum Mechanics in 1 slide

$H|\psi_n\rangle = E_n|\psi_n\rangle$ Schrodinger Equation to find the quantized energy levels E_n for a system

$H = T + V$ the Hamiltonian of the system comprised of kinetic energy T and potential energy V

Quantum Mechanics in 1 slide

$H|\psi_n\rangle = E_n|\psi_n\rangle$ Schrodinger Equation to find the quantized energy levels E_n for a system

$H = T + V$ the Hamiltonian of the system comprised of kinetic energy T and potential energy V

Can be cast as a linear algebra problem

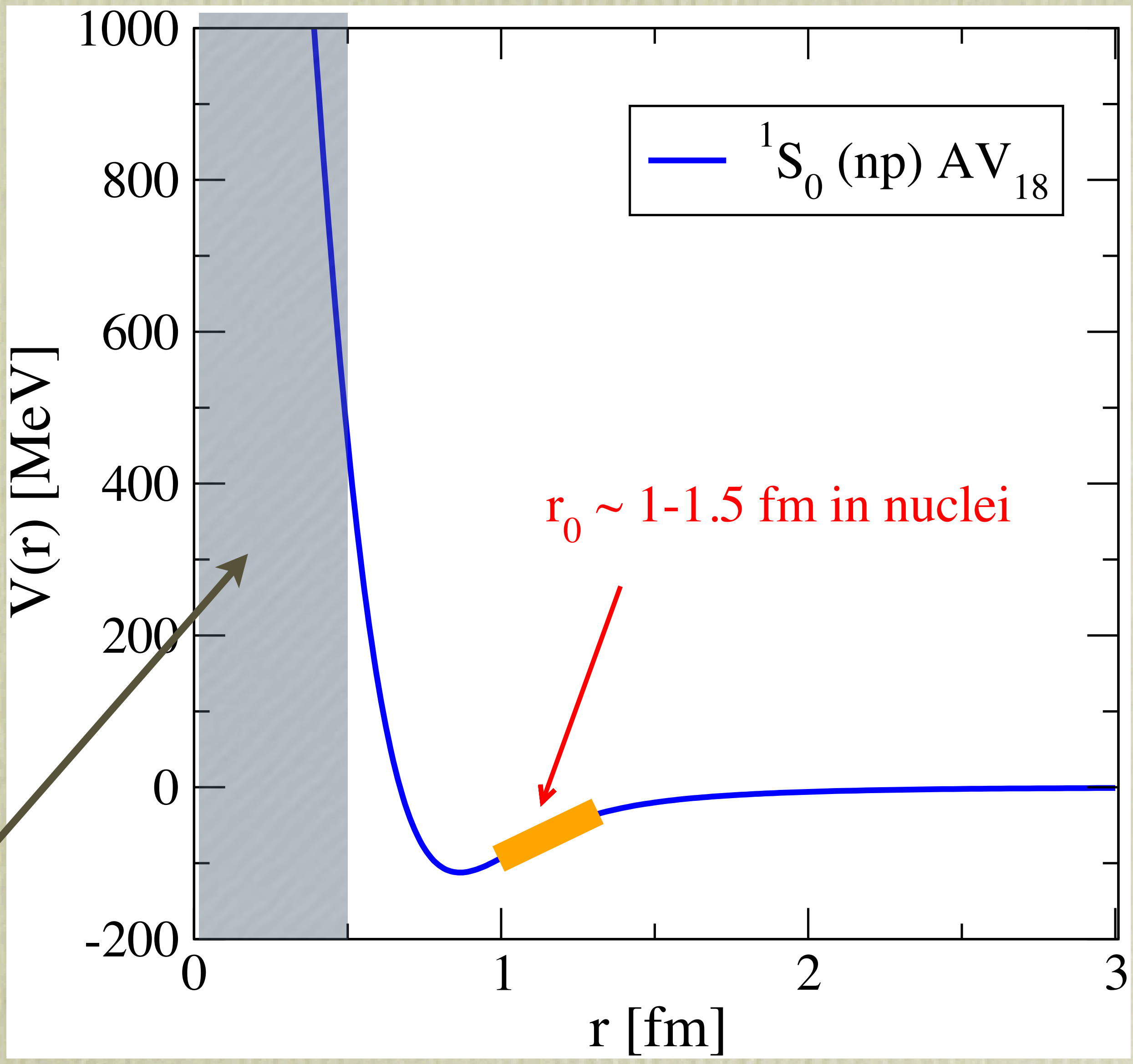
H represented as a N x N array of numbers ("matrix")

$|\psi_n\rangle$ represented a N-component column of numbers

E_n are the "eigenvalues" of the matrix H

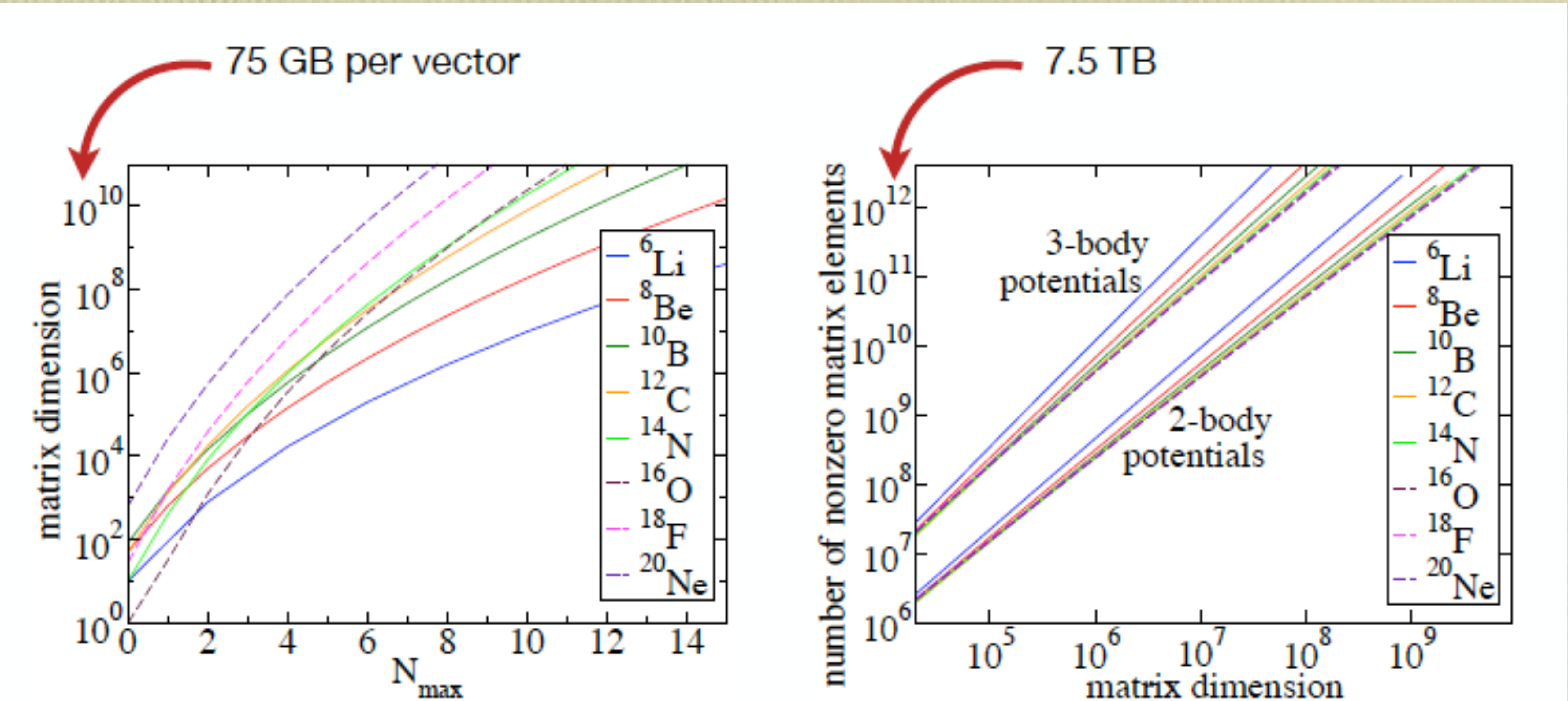
Even with nucleons as our DOF, this is still hard!

- Why? Because we have strong interactions!



repulsive
"hard core"

Life is still hard, even with the "right" degrees of freedom!

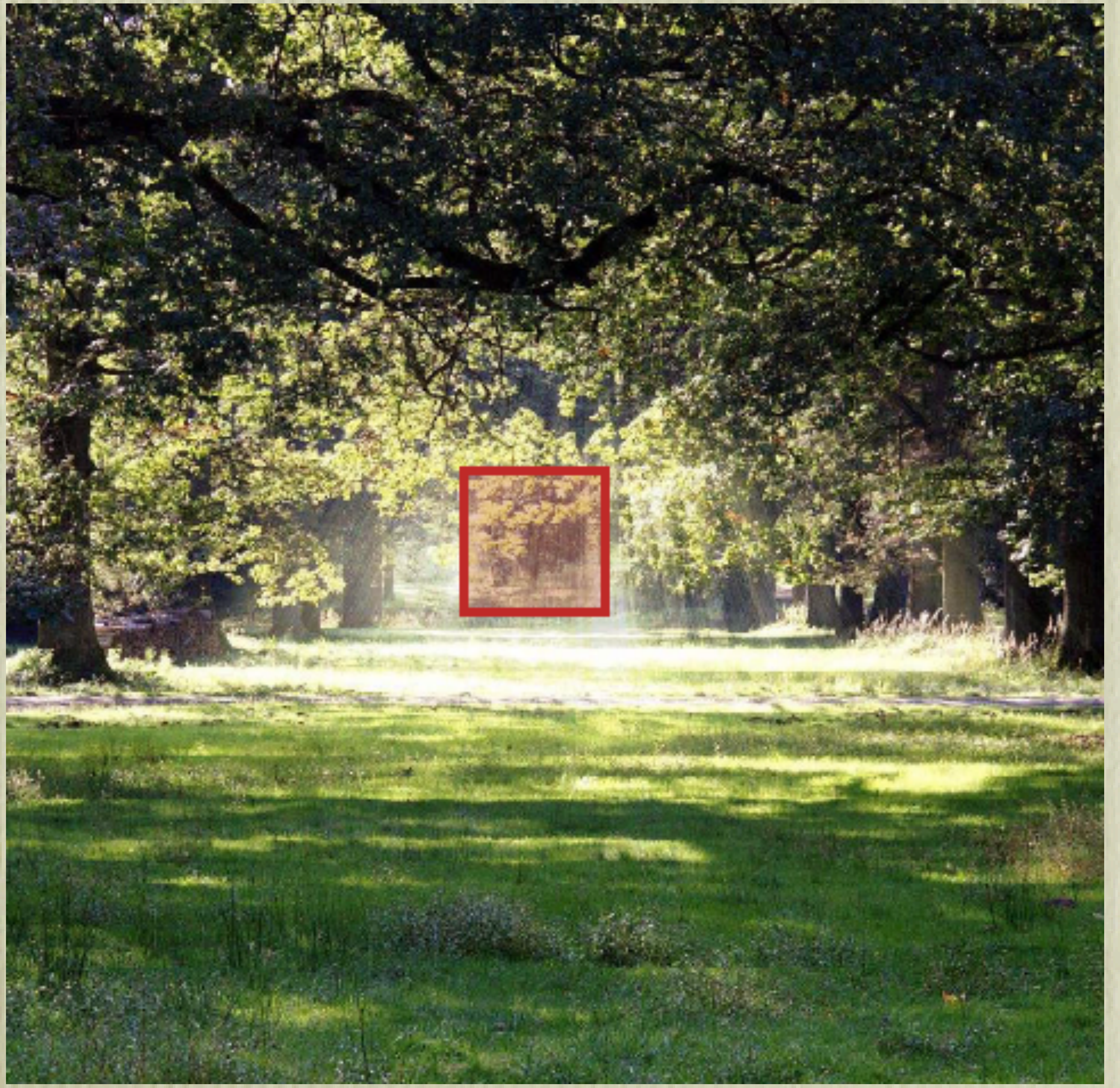


from: C. Yang, H. M. Aktulga, P. Maris, E. Ng, J. Vary, Proceedings of NTSE-2013

Nuclear interactions are **large** matrices
Huge memory/computational demands

Is this necessary? (Hint: We are mostly interested in low E)

Renormalization Group: Image Processing Analogy



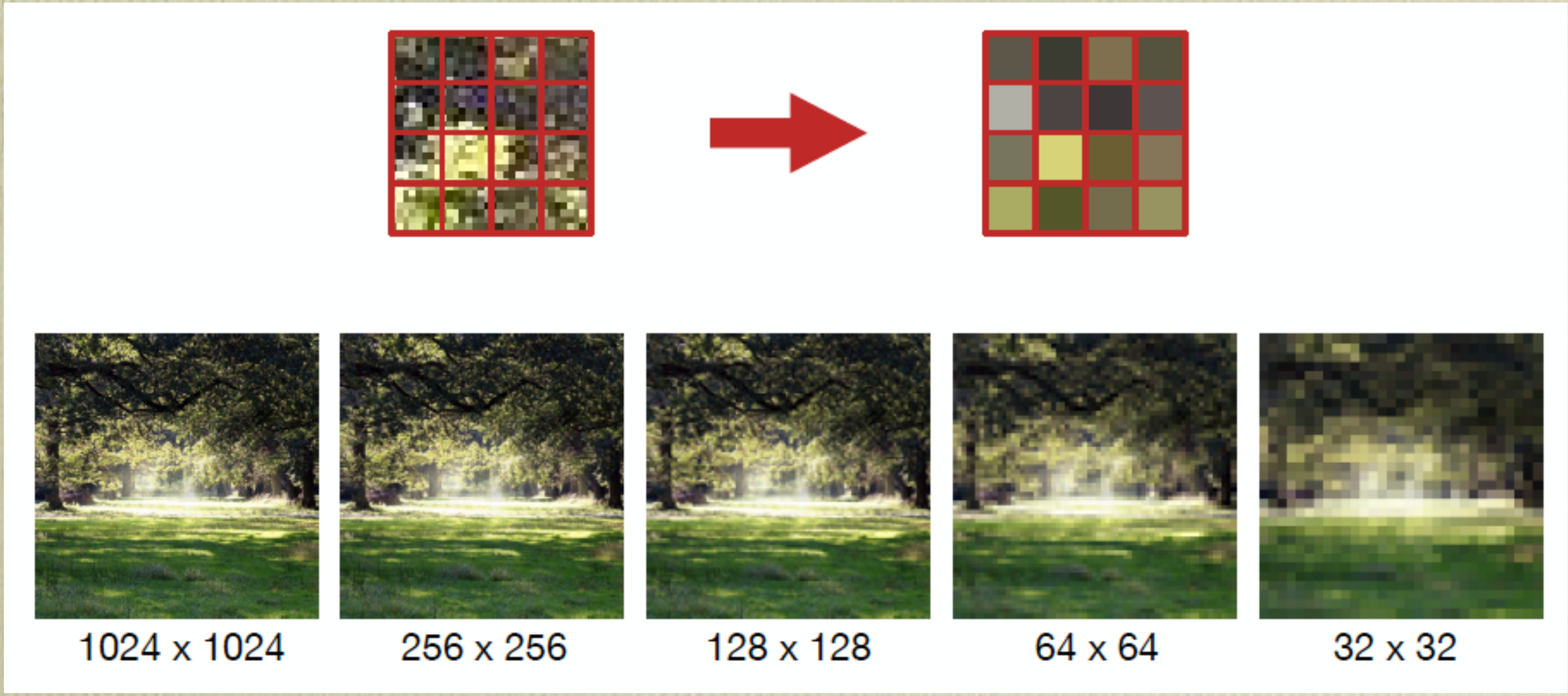
high resolution image

memory/computing power is limited!

What if we only care about the gross structure?

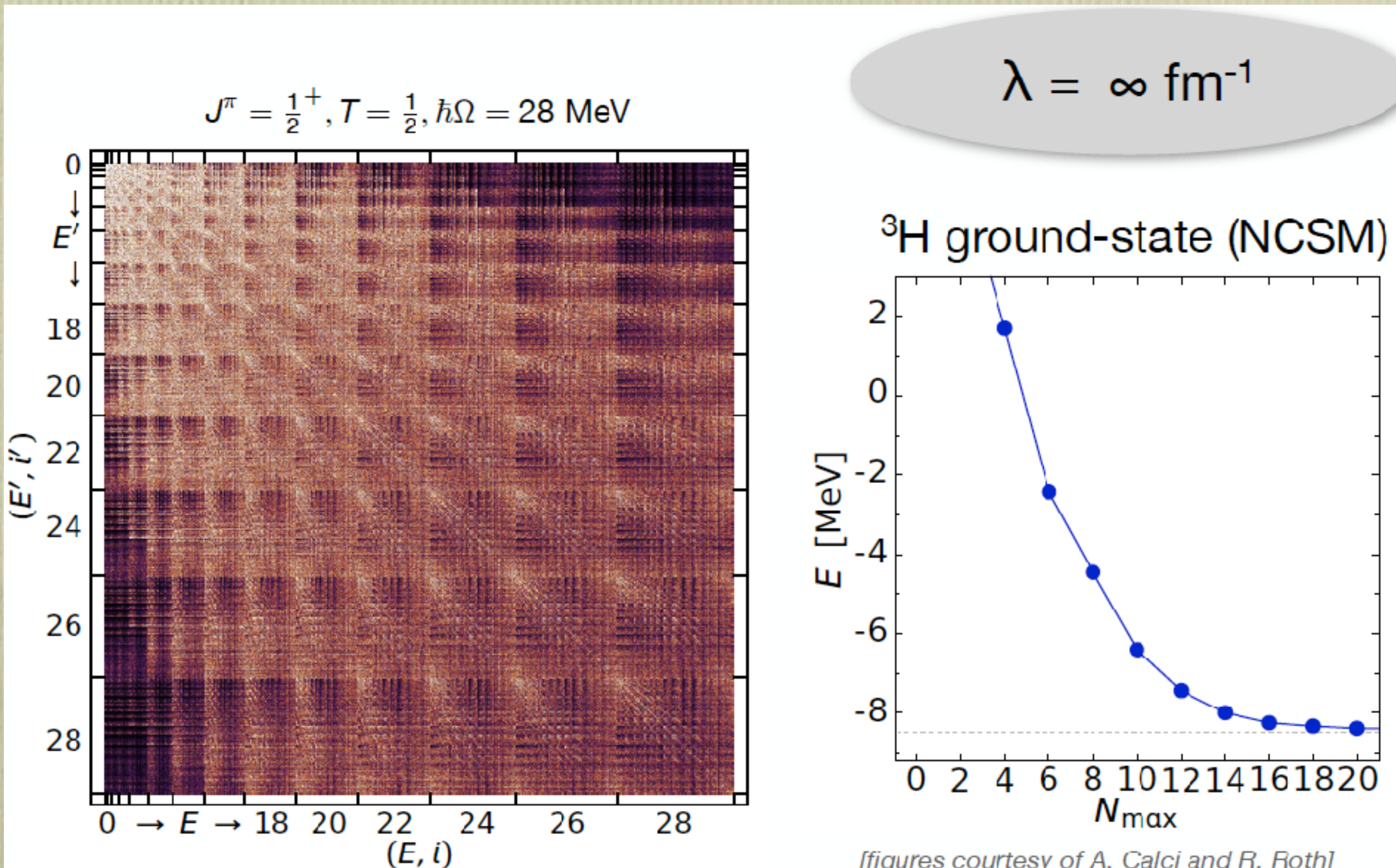
Renormalization Group: Image Processing Analogy

Compress data by **"coarse graining"** (i.e.. averaging over blocks of pixels)



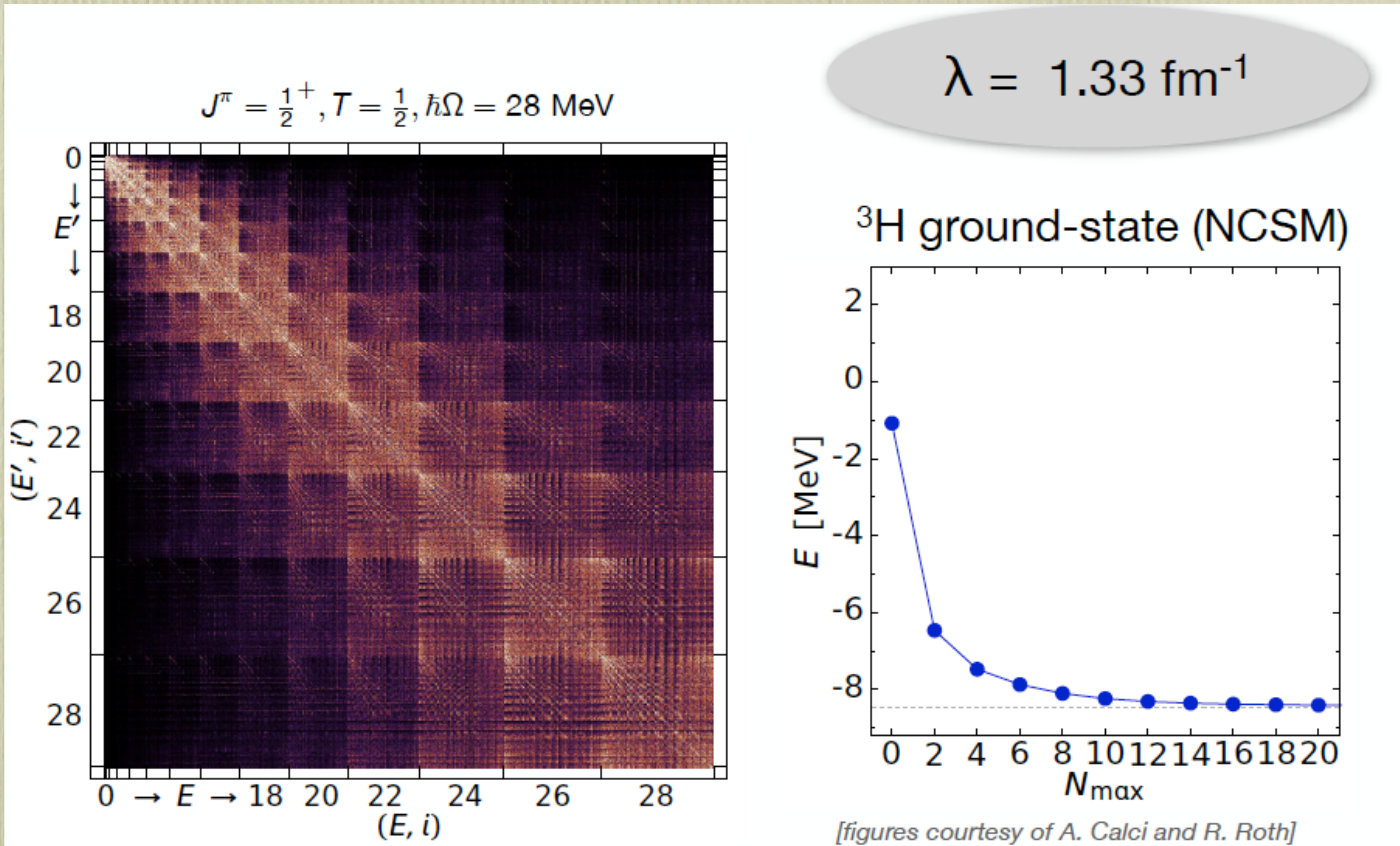
Analogously, in nuclear physics we "coarse grain" by averaging out irrelevant high-energy DOF

"Coarse Graining" nuclear interactions



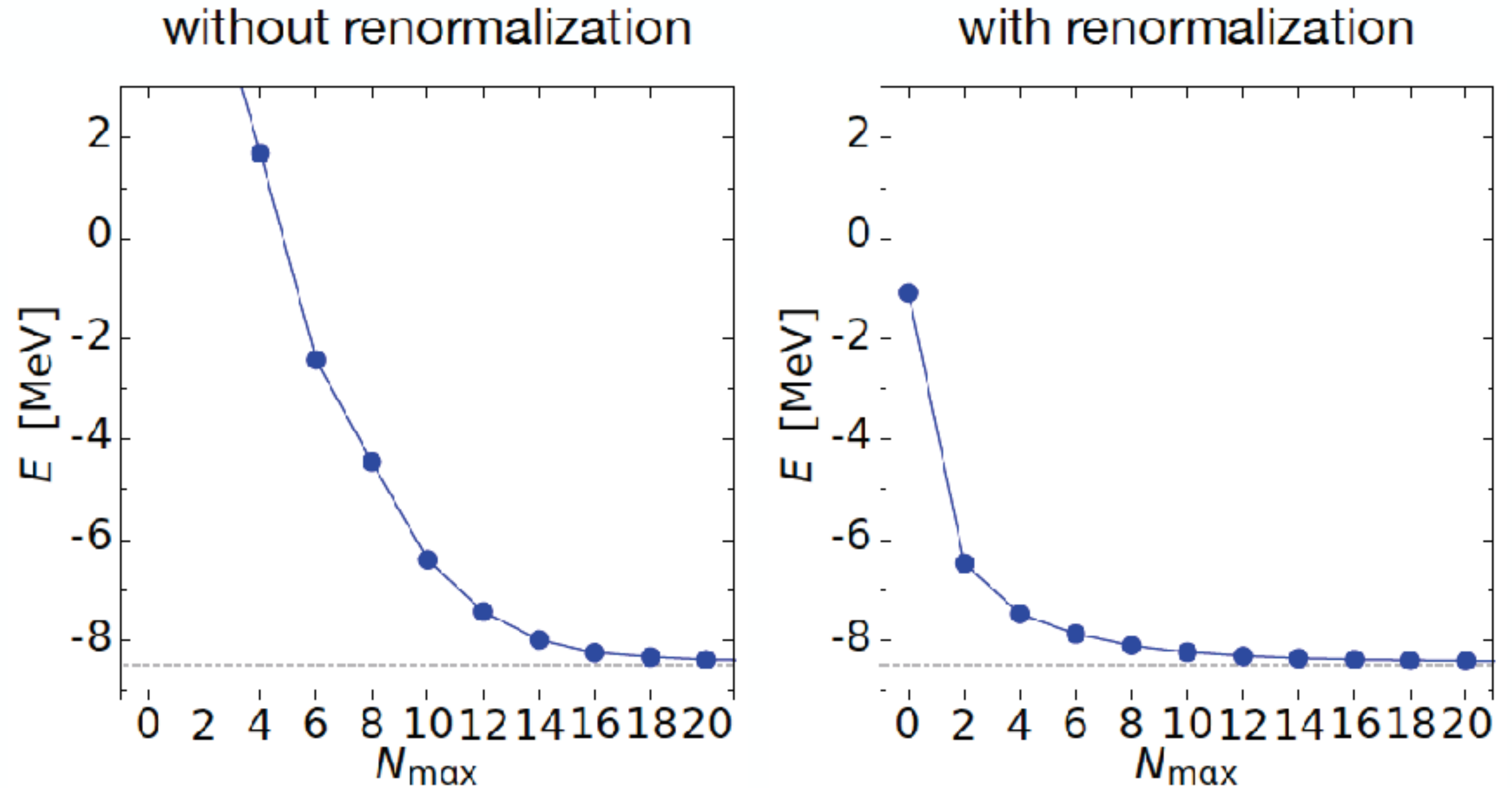
Huge matrices, "too much resolution"

“Coarse Graining” nuclear interactions



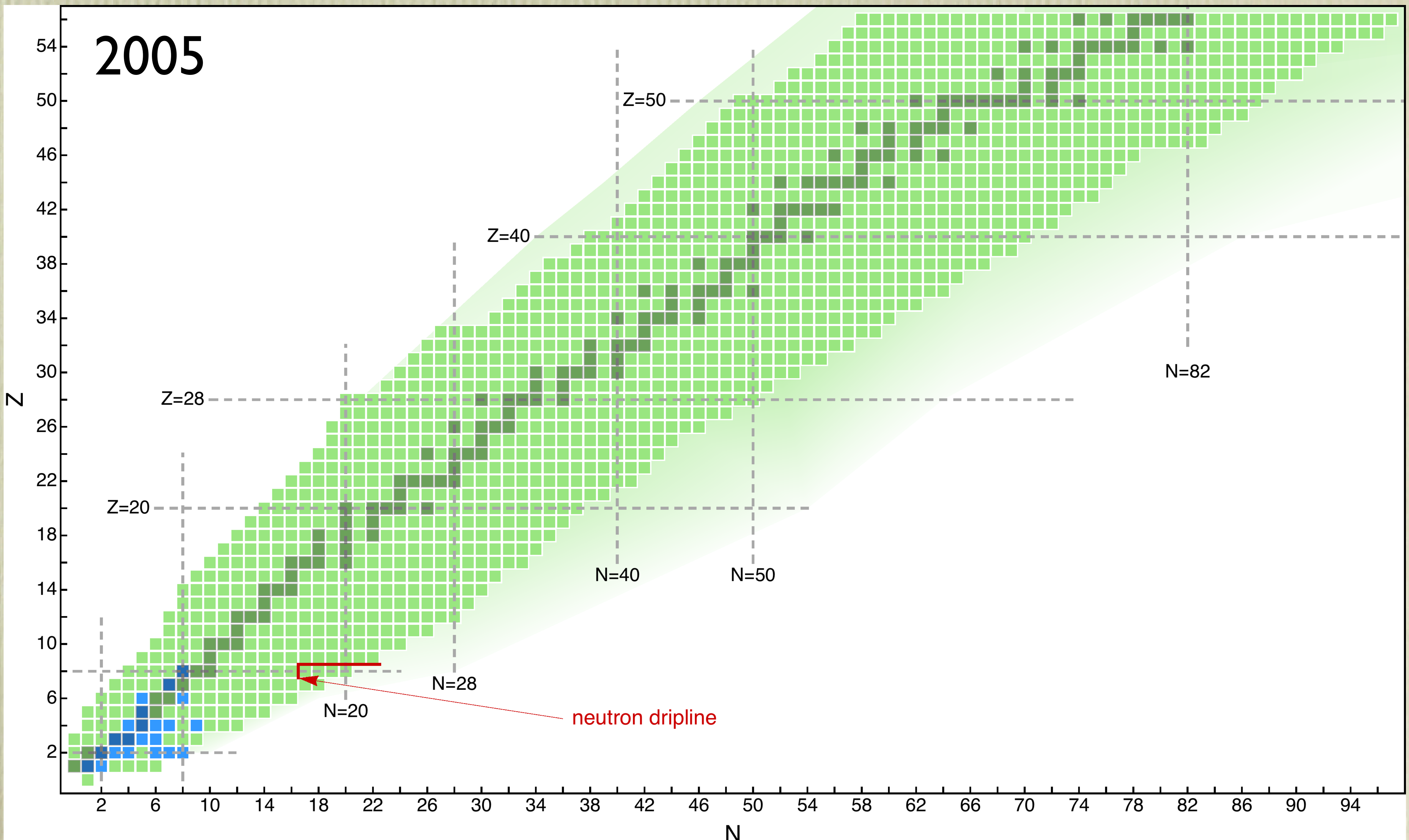
“course grained” matrix much smaller!

"Coarse Graining" nuclear interactions

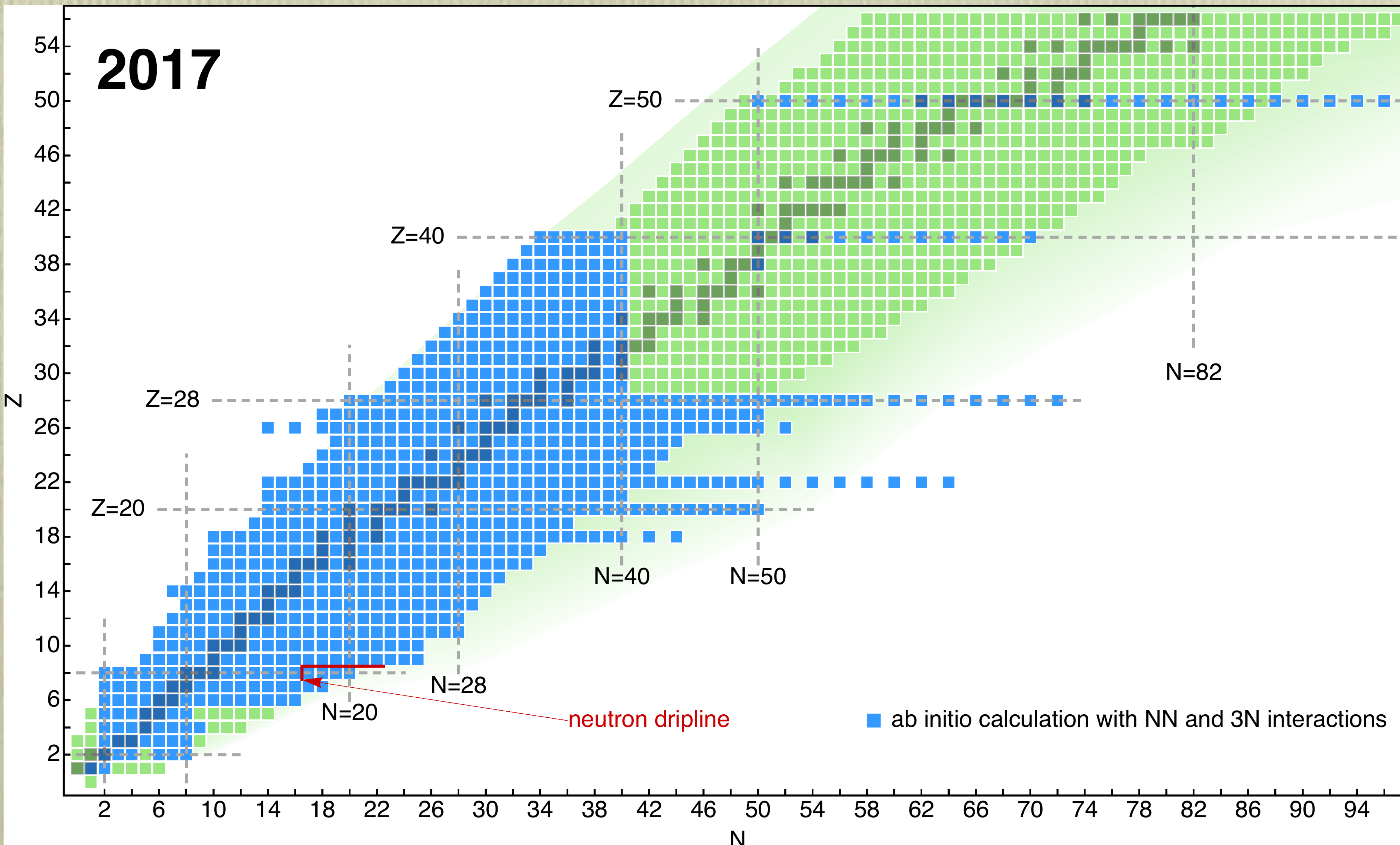


renormalization **reduces effort by orders of magnitude,**
allows our methods to **reach heavier nuclei**

Progress in Ab- Initio calculations



Progress in Ab- Initio calculations



Project

- Study different ways to renormalize or "course grain" matrix models of nuclear dynamics
- Write simple codes (Python, Matlab) and analyze results of calculations
  - great if you've coded before, but NOT essential as we'll have sample codes to learn from
- Don't be intimidated by unfamiliar math (matrices, eigenvalues, etc.) You don't have to become an expert, and you'll be shielded from gory details using high-level software packages.