Nuclear Structure Theory I

Elena Litvinova

Western Michigan University
• Major problems and challenges in nuclear structure theory
• Basic approaches to nuclear many-body problem
• “First order” approach: Nuclear Shell Model and Density Functional Theory

• Fermionic propagators in the strongly-correlated medium: spectroscopic factors and response functions

• Exotic nuclear phenomena:
  - Changing/disappearance of magic numbers
  - Physics of neutron skin
  - Isospin-transfer excitations and beta decay
  - The onset of pion condensation
  - Continuum and finite temperature effects

• Literature
Major problems in nuclear structure theory

- Nuclear scales: Hierarchy problem
- No connection between the scales in the traditional NS models

**QCD**

“ab initio”

**Configuration Interaction (CI)**

**Density Functional Theory (DFT)**

**Collective coordinates (CC)**

Expanding, but still limited domains of microscopic theories

Tremendous error propagation in the astrophysical modeling with phenomenological nuclear structure input:

Building blocks for nuclear structure theories

Degrees of freedom

at ~1-50 MeV excitation energies:
  single-particle & collective (vibrational, rotational)
NO complete separation of the scales!
  -Coupling between single-particle and collective:
  -Coupling to continuum
as nuclei are open quantum systems

Symmetries -> Eqs. of motion

Galilean inv. -> Schrödinger Equation
Lorentz inv. -> Dirac, Klein-Gordon Equations

Interaction $V_{NN}$ : 3 basic concepts

Ab initio: from vacuum $V_{NN}$ -> in-medium $V_{NN}$
Configuration interaction: matrix elements for in-medium $V_{NN}$
Density functional: an ansatz for in-medium $V_{NN}$

(Relativistic) Nuclear Field Theory: connecting scales
Nuclear forces: meson exchange

Pion (Yukawa, 1935), heavy mesons 1950-s

Quantum Hadrodynamics

Nuclear “forces”

- The nucleons in the interior of the nuclear medium do not feel the same bare force $V$.
- They feel an effective force $G$ (calculated from $V$ in “ab initio” methods).
- The Pauli principle prohibits the scattering into states, which are already occupied in the medium.
- Therefore this force $G(\rho)$ depends on the density.
- This force $G$ is much weaker than bare force $V$.
- Nucleons move nearly free in the nuclear medium and feel only a strong attraction at the surface (shell model).
Theories based on the meson-exchange interaction

\[ \sum_{\text{EMF}} = \sum_{\text{HF}} + \sum_{\text{RPA}} \]

Hartree-Fock (HF)

\[ \sum_{\text{EMF}} = \sum_{\text{HF}} + \sum_{\text{RPA}} \]

Random Phase Approximation (RPA)

\[ \Phi^{(n+1)} = R^{(n)} + R^{(n)} + R^{(n)} + R^{(n)} \]

Quasiparticle-vibration coupling (QVC)

\[ \Phi^{(n+1)} = R^{(n)} + R^{(n)} + R^{(n)} + R^{(n)} \]

\[ \sum_{\text{EMF}} = \sum_{\text{HF}} + \sum_{\text{RPA}} \]

Emergent collective degrees of freedom: 'phonons'

New order parameter: phonon coupling vertex

Relativistic Mean Field (P. Ring et al.) Covariant DFT (8-10 parameters fixed) = Extended Walecka model

Hierarchy of contributions:
- mean field
- line corrections
- vertex corrections

Nuclear Field Theory: Copenhagen - Milano (P.F. Bortignon, G. Bertsch, R. Broglia, G. Colo, E. Vigezzi et al.): NFT
- St. Petersburg-Juelich (J. Speth, V. Tselyaev, S. Kamedzhiev et al.): Extension of Landau-Migdal theory: ETFFS

Finite size & angular momentum couplings

Relativistic NFT:
- EL, P. Ring, PRC 73, 044328 (2006);
- EL, P. Ring, V. Tselyaev, PRC 78, 014312 (2008)

RNFT*
The many-body problem is mapped onto a one-body problem:

Density functional theory starts from the Hohenberg-Kohn theorem:

„The exact ground state energy $E[p]$ is a universal functional for the local density $\rho(r)$“

Kohn-Sham theory starts with a density dependent self-energy:

$$h(r) = \frac{\delta E[\rho]}{\delta \rho(r)}$$

and the single particle equation:

$$h(r) |\varphi_i\rangle = \epsilon_i |\varphi_i\rangle$$

with the exact density:

$$\rho(r) = \sum_i^A |\varphi_i(r)|^2$$

Problems:

For Coulombic systems the density functional is derived ab initio: for nuclei it is much more complicated!

DFT in nuclei can not provide a precise description, but rather only the first approximation to the nuclear many-body problem.
Relativistic Hartree (Fock): mean field approximation (=DFT)

\[
E_{RMF}[\hat{\rho}, \phi] = Tr[(\alpha p + \beta m)\hat{\rho}] + \sum_m \left\{ Tr[(\beta \Gamma_m \phi_m)\hat{\rho}] + \int \left[ \frac{1}{2}(\nabla \phi_m)^2 + U(\phi_m) \right] d^3r \right\}
\]

\[
\left\{ \begin{aligned}
\mathcal{H}_{RHB} |\psi^\eta_k\rangle &= \eta E_k |\psi^\eta_k\rangle, \quad \eta = \pm 1 \\
-\Delta \phi_m(r) + U'(\phi_m(r)) &= \mp \sum_k V^T_k(r) \beta \Gamma_m V^*_k(r)
\end{aligned} \right.
\]

Nucleons

Mesons

\[
\hat{\mathcal{H}}_{RHB} = \frac{\delta E_{RHB}}{\delta \mathcal{R}} = \begin{pmatrix}
\hbar^p - m - \lambda & \Delta \\
-\Delta^* & -\hbar^p* + m + \lambda
\end{pmatrix}
\]

Dirac Hamiltonian

\[
h^p = \alpha p + \beta (m + \sum_m \Gamma_m \phi_m(r))
\]

\[
|\psi^+_k(r)\rangle = \begin{pmatrix} U_k(r) \\ V^*_k(r) \end{pmatrix}
\]

\[
|\psi^-_k(r)\rangle = \begin{pmatrix} V^*_k(r) \\ U_k(r) \end{pmatrix}
\]

Relativistic Hartree (Fock)-Bogoliubov (HFB) Hamiltonian

RMF self-energy

\tilde{\Sigma}(r)

Eigenstates
Uncorrelated ground state as the zero-th approximation: 
(Relativistic) Mean Field Approximation

Fermi sea

Dirac sea (relativistic)
Beyond Mean Field: coherent oscillations

Fermi sea

Dirac sea (relativistic)

Continuum

Vibrational modes (phonons $J^\pi$): particle-hole quasi bound configurations

"No sea" approximation
“Naive” Shell-model (independent particle model)
J. Jensen, M. Goeppert-Mayer: Nobel Prize 1963

Configuration Interaction shell-model:
core + valence space => interaction
=> diagonalization
(B.A. Brown, F. Nowacki, A. Poves et al.)

No-core shell-model:
bare NN potential => effective interaction
(G-matrix theory, SRG etc.)
=> exact diagonalization
(B. Barrett, E. Ormand, P. Navratil)
Single-quasiparticle Green's function

One-body Green’s function in (N+1)-body system

\[ G(x, x') = -i \langle \Phi_0^N | \hat{T} \hat{a}(x) \hat{a}^\dagger(x') | \Phi_0^N \rangle \]

\[ x = \{ \xi, t \} \]

Lehmann expansion (Fourier transform):

\[ G(\xi, \xi'; \varepsilon) = \sum_n \frac{(\Psi(\xi))_{0m}(\Psi^\dagger(\xi'))_{n0}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \]

\[ + \sum_m \frac{(\Psi^\dagger(\xi'))_{0m}(\Psi(\xi))_{m0}}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta} \]

Excited state (N+1)

\[ (\Psi^\dagger(\xi))_{n0} = \langle \Phi_n^{(N+1)} | \Psi^\dagger(\xi) | \Phi_0^{(N)} \rangle \],

\[ (\Psi(\xi))_{n0} = \langle \Phi_n^{(N-1)} | \Psi(\xi) | \Phi_0^{(N)} \rangle \],

Ground state (N)

“Free quasiparticle” propagator in the mean field:

\[ \tilde{G}_k^\eta(\varepsilon) = \frac{1}{\varepsilon - \eta E_k + i\eta \delta} \]

Basis states (spherical):

\[ k = |n_k, j_k, l_k, m_k \rangle \]

Interacting (fragmented):

\[ G_k^\eta(\varepsilon) = \sum_{\nu, \nu'} \frac{S_k^{\eta'(\nu)}}{\varepsilon - \eta \eta' E_k^{(\nu)}} \]

\[ \eta = \pm 1 \]

Spectroscopic factors (occupancies)

Quasiparticle Energies
Systematic expansion in meson-exchange interaction: one-fermion self-energy

Order
1 (HF)

\[ \Sigma^{(e)}(e) = k_1 k_2 \]

Coupling

Correlated particle-hole (vibration)

\[ \sum = \text{Dyson Equation} \]

Superfluidity: Gor'kov's GF

Doubled quasiparticle space:

\[ \Sigma^{(e)}_{\eta_1 \eta_2}(\varepsilon) = \sum_{\eta=\pm 1} \sum_{k, \mu} \frac{\gamma^{\eta_1 \eta_2 \eta, \eta_1 \eta_2 \eta^*}_{\mu k_{1k} \mu k_{2k}}}{\varepsilon - \eta(E_k + \Omega_{\mu} - i\delta)} \]

\[ (\varepsilon - \mathcal{H}_{RHB} - \Sigma^{(e)}(\varepsilon))G(\varepsilon) = 1 \]

\[ \eta = \pm 1 \]

\[ \text{Dyson Equation} \]
Spectroscopic factors $S^{(ν)}_k$

$$E_k = \sum_{k} E^{(ν)}_k S^{(ν)}_k$$

$$\sum_{ν} S^{(ν)}_k = 1$$

Dominant level

Strong fragmentation

Single-particle structure

Response
Quasiparticle-vibration coupling:
Pairing correlations of the superfluid type + coupling to phonons

Spectroscopic factors in $^{120}$Sn:

<table>
<thead>
<tr>
<th>(nlj) $\nu$</th>
<th>$S_{th}^{*}$</th>
<th>$S_{exp}^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2d_{5/2}$</td>
<td>0.32</td>
<td>0.43</td>
</tr>
<tr>
<td>$1g_{7/2}$</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>$2d_{3/2}$</td>
<td>0.53</td>
<td>0.45</td>
</tr>
<tr>
<td>$3s_{1/2}$</td>
<td>0.43</td>
<td>0.32</td>
</tr>
<tr>
<td>$1h_{11/2}$</td>
<td>0.58</td>
<td>0.49</td>
</tr>
<tr>
<td>$2f_{7/2}$</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>$3p_{3/2}$</td>
<td>0.58</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Spectroscopic factors in $^{132}$Sn:

<table>
<thead>
<tr>
<th>(nlj) $\nu$</th>
<th>$S_{th}^{*}$</th>
<th>$S_{exp}^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2f_{7/2}$</td>
<td>0.89</td>
<td>0.86±0.16</td>
</tr>
<tr>
<td>$3p_{3/2}$</td>
<td>0.91</td>
<td>0.92±0.18</td>
</tr>
<tr>
<td>$1h_{9/2}$</td>
<td>0.88</td>
<td>1.1±0.3</td>
</tr>
<tr>
<td>$3p_{1/2}$</td>
<td>0.91</td>
<td>1.1±0.2</td>
</tr>
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<td>$2f_{5/2}$</td>
<td>0.89</td>
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</table>

Shapes in superheavy mass region

Magic numbers in superheavy mass region

S. Ćwiok, P.-H. Heenen & W. Nazarewicz
Nature 433, 705 (2005)

http://asrc.jaea.go.jp/soshiki/gr/HENS-gr/index_e.html
Dominant neutron states in superheavy $Z = 120$ isotopes

PC+QVC: Formation of the shell gap!

Comparable Spectroscopic strengths

shell gap ???
Shell evolution in superheavy $Z = 120$ isotopes: Quasiparticle-vibration coupling (QVC) in a relativistic framework

1. Relativistic Mean Field: **spherical minima**
2. $\pi$: collapse of pairing, **clear shell gap**
3. $\nu$: survival of pairing coexisting with the shell gap
4. Very **soft** nuclei: large amount of low-lying collective vibrational modes ($\sim$100 phonons below 15 MeV)

**Vibration corrections to binding energy (RQRPA)**

$$E_{VC} = - \sum_{\mu} \Omega_{\mu} \sum_{k_1 k_2} |Y_{k_1 k_2}^\mu|^2$$

**Vibration corrections to $\langle -\rangle$-decay Q-values**

- Impact on the shell gaps
- Smearing out the shell effects

**Shell stabilization & vibration stabilization/destabilization (?)**

E.L., PRC 85, 021303(R) (2012)
3D Harmonic Oscillator:

\[ V_{\text{HO}}(r) = \frac{1}{2} m \omega^2 r^2 \]

\[ \varepsilon_\alpha = \hbar \omega \left( 2n_\alpha + l_\alpha + \frac{3}{2} \right) \]

Degeneracy \( N_\alpha \)

Woods-Saxon (WS) potential:

\[ V_{\text{WS}}(r) = -\frac{V_0}{1 + \exp \left( r - R \right)/a} \]

WS + spin-orbit interaction:

\[ V_{\text{WS}+ls}(r) = -\frac{V_0}{1 + \exp \left( r - R \right)/a} + V_{ls}(r) l_s \]
Nuclear Structure Theory II

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Excited states: nuclear response function

Bethe-Salpeter Equation (BSE):

\[ R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega) \]

\[ W(\omega) = \Phi(\omega) - \Phi(0) \]

\[ V = \frac{\delta \Sigma^{\text{RMF}}}{\delta \rho} \]

\[ U^e = i \frac{\delta \Sigma^e}{\delta G} \]

Consistency on 2p2h-level

E.L., V. Tselyaev, PRC 75, 054318 (2007)
**Problem:**
'Melting' diagrams

**Solution:**

**Time-projection operator:**

\[
\begin{align*}
\delta_{\sigma_1,\sigma_2} \theta(\sigma_{12}) &= 1 \overset{\theta_{12}}{\longrightarrow} 2' \\
\delta_{\sigma_2,\sigma_1} \theta(\sigma_{1'2}) &= 2' \overset{\theta_{12}}{\longleftarrow} 1'
\end{align*}
\]

V.I. Tselyaev, Yad. Fiz. 50,1252 (1989)

**Allowed terms:** 1p1h, 2p2h

**Blocked terms:** 3p3h, 4p4h, ...

- Separation of the integrations in the BSE kernel
- R has a simple-pole structure (spectral representation)
  - Strength function is positive definite!

**Perturbative schemes:**

Unphysical result: negative cross sections

Partially fixed

Included on the next step
Response function in the neutral channel: relativistic quasiparticle time blocking approximation (RQTBA)

\[ R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)W(\omega)R(\omega) \]

Interaction

\[ W(\omega) = V_\sigma + V_\omega + V_\rho + V_e + \Phi(\omega) - \Phi(0) \]

Subtraction to avoid double counting

Static: RQRPA

\[
\begin{align*}
    v_\sigma(1, 2) &= -g_\sigma^2 \gamma_1^0 D_\sigma(1, 2) \gamma_2^0 \\
    v_\omega(1, 2) &= +g_\omega^2 (\gamma^0 \gamma_\mu)_1 D_\omega^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu)_2 \\
    v^V_\rho(1, 2) &= +g_\rho^2 (\gamma^0 \gamma_\mu \vec{r}_1) \cdot \vec{r}_2 D_\rho^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu \vec{r})_2
\end{align*}
\]

Dynamic (retardation):

Quasiparticle-vibration coupling

in time blocking approximation

\[
\Phi_{k_1 k_4 k_2 k_3}(\omega) = 
\sum_{\mu \xi} \delta_{k_1 k_3} \delta_{k_2 k_4} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2} \gamma_{\mu; k_6 k_4} \gamma_{\eta'; k_1 k_5} \gamma_{\eta'; k_3 k_5} \gamma_{\eta; k_1 k_5} \gamma_{\eta; k_3 k_5}}{\eta_\omega - E_{k_1} - E_{k_6} - \Omega_\mu} 
+ \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5} \gamma_{\mu; k_3 k_5} \gamma_{\eta; k_1 k_5} \gamma_{\eta; k_3 k_5}}{\eta_\omega - E_{k_1} - E_{k_2} - \Omega_\mu} 
\]
Response to an external field: strength function

Nuclear Polarizability:

\[ \Pi_{PP}(\omega) = P^\dagger R(\omega)P := \sum_{k_1 k_2 k_3 k_4} P_{k_1 k_2}^* R_{k_1 k_4, k_2 k_3}(\omega) P_{k_3 k_4} \]

External field

\( \Pi = \) \( \text{Diagram} \)

Strength function:

\[ S(E) = -\frac{1}{\pi} \lim_{\Delta \to +0} \text{Im} \ \Pi_{PP}(E + i\Delta) \]

Transition density:

\[ \rho_{k_1 k_2}^\nu = \langle 0 | \psi_{k_2}^{\dagger} \psi_{k_1} | \nu \rangle \]

Response function:

\[ R_{k_1 k_4, k_2 k_3}^\nu(\omega) \approx \frac{\rho_{k_1 k_2}^\nu \rho_{k_3 k_4}^{\nu \ast}}{\omega - \Omega_{\nu}^\nu} \]

\( \omega \to \Omega_{\nu}^\nu \)
<table>
<thead>
<tr>
<th>Nuclear excitation modes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monopole</strong> ( \otimes L = 0 )</td>
</tr>
<tr>
<td>[ \begin{align*} \otimes T = 1, \ \otimes S = 0 \end{align*} ]</td>
</tr>
<tr>
<td><strong>Dipole</strong> ( \otimes L = 1 )</td>
</tr>
<tr>
<td>[ \begin{align*} \otimes T = 1, \ \otimes S = 1 \end{align*} ]</td>
</tr>
<tr>
<td><strong>Quadrupole</strong> ( \otimes L = 2 )</td>
</tr>
<tr>
<td>[ \begin{align*} \otimes T = 1, \ \otimes S = 1 \end{align*} ]</td>
</tr>
</tbody>
</table>

* M. N. Harakeh and A. van der Woude: Giant Resonances
Dipole response in medium-mass and heavy nuclei within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

**Test case: E1 (IVGDR) stable nuclei**

- **Sn**
  - 120 Sn
  - 116 Sn

- **Zr**
  - 90 Zr

- **Sr**
  - 88 Sr

**Neutron-rich Sn**

- 132 Sn
- 130 Sn

**Response of superheavy nuclei:**

From giant resonances' widths to transport coefficients

- Giant & pygmy dipole resonances

**Input for r-process nucleosynthesis:**


Microscopic structure of pygmy dipole resonance is extremely important for stellar (n,γ) reaction rates.
Exotic modes of excitation: pygmy dipole resonance in neutron-rich nuclei

**Exotic nuclei** (nuclei with unusual N/Z ratios: neutron-rich or proton-rich) are characterized by weak binding of outermost nucleons, diffuse neutron densities, formation of the neutron skin and halo effect on multipole response → new exotic modes of excitation

**Pygmy dipole resonance (PDR):**

N. Paar et al.

Nucleonic density:

\[ \rho(r,t) = \rho_0(r) + \delta \rho(r,t) \]

Neutron skin oscillations
RQTBA dipole transition densities in $^{68}$Ni at 10.3 MeV

**Theoretical:**


**Experimental:**

- Coulomb excitation of $^{68}$Ni at 600 AMeV
- O. Wieland et al., PRL 102, 092502 (2009)

Mathematical expression:

$$ \rho(\mathbf{r}) = \rho(r) Y_{10}(\hat{r}), $$

where $\rho(r)$ is the density function and $Y_{10}(\hat{r})$ is the spherical harmonic.
RQTBA dipole transition densities in $^{68}$Ni at 10.3 MeV

\[ \rho(\mathbf{r}, t) = \rho(r) Y_{10}(\hat{r}) e^{i\omega t} \]

Neutrons

Protons


O. Wieland et al., PRL 102, 092502 (2009)
Experimental vs theoretical systematics of the pygmy dipole resonance

Experimental systematics:
- various measurements

Theoretical systematics:
- Consistent calculations within the same framework
- Accurate separation of PDR from GDR by transition density analysis:

- "Plateau" shell effects

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\[ \frac{(N-Z)}{A}^2 \]

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D. Savran, T. Aumann, A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013)

A. Tamii et al., PRL 107, 062502 (2011)

A. Krumbholtz, PLB 744, 7 (2015)


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EM vs hadron probes

polarized (p, p')
Isospin transfer response function: proton-neutron RQTBA (pn-RQTBA)

Response
\[ R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)\bar{W}(\omega)R(\omega) \]

Interaction
\[ \bar{W}(\omega) = V_\rho + V_\pi + V_{\delta\pi} + \Phi(\omega) - \Phi(0) \]

Static: RRPA
\[ V_\rho(1, 2) = g_\rho^2 \bar{\tau}_1 \bar{\tau}_2 (\beta\gamma^\mu)_1 (\beta\gamma^\mu)_2 D_\rho(r_1, r_2) \]
\[ V_\pi(1, 2) = -\left(\frac{f_\pi}{m_\pi}\right)^2 \bar{\tau}_1 \bar{\tau}_2 (\Sigma_1 \nabla_1)(\Sigma_2 \nabla_2) D_\pi(r_1, r_2), \]

Dynamic (retardation):
quasiparticle-vibration coupling
in time blocking approximation

\[ \Phi_{k_1 k_2 k_3}^{\eta} (\omega) = \sum_{\mu, \xi} \delta_{\eta, \xi} \left[ \delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2} \gamma_{\mu; k_4}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5} \gamma_{\mu; k_3 k_5}}{\eta\omega - E_{k_1} - E_{k_2} - \Omega_\mu} - \left(\frac{\gamma_{\mu; k_1 k_3} \gamma_{\mu; k_2 k_4}}{\eta\omega - E_{k_1} - E_{k_2} - \Omega_\mu} \right) \right] \]
Gamow-Teller resonance from closed-shell to open-shell: superfluid pairing and phonon coupling

Closed shell, stable

\[ P = \sum_i \sigma^{(i)} T^{(i)} \]

\[ ^{208}\text{Pb} \]

pn-RRPA
pn-RTBA
GT+IVSM

Open shell, neutron rich

Isovector Spin Monopole (IVSM) Resonance


\[ ^{68}\text{Ni} \]
\[ ^{70}\text{Ni} \]
\[ ^{72}\text{Ni} \]
\[ ^{74}\text{Ni} \]
\[ ^{76}\text{Ni} \]
\[ ^{78}\text{Ni} \]


pn-RQRPA
pn-RQTBA
Low-energy GT strength

\[
\frac{1}{T_{1/2}} = \sum_m \lambda_{i_f}^m = D^{-1} g_A^2 \sum_m \int dE_e \left| \sum_{pn} < 1^+_x | \sigma_{\tau_-} | 0^+ > \right|^2 \frac{dn_m}{dE_e}
\]

- For the 1st time a quantitative self-consistent description of \( T_{1/2} \) without artificial quenching or other parameters is achieved
- Both phases of r-process can be computed within the same framework of high predictive power
- Description of the rp-process (on the proton-rich side) is available

Spin-dipole resonance: beta-decay, electron capture

\[ P_\pm^\lambda = \sum_i r(i) [\sigma(i) \otimes Y_1(i)]_\lambda \ t_\pm(i) \]

\[ \Delta L = 1 \]
\[ \Delta T = 1 \]
\[ \Delta S = 1 \]
\[ \lambda = 0, 1, 2 \]


Earlier studies;

W.H. Dickhoff et al., PRC 23, 1154 (1981)

Existence of low-lying unnatural parity states indicates that nuclei are close to the pion condensation point. However, it is not clear which observables are sensitive to this phenomenon.

Only nuclear matter and doubly-magic nuclei were studied...

Now: In some exotic nuclei 2- states are found at very low energy. Similar situation with 0-, 4-, 6-... states.
Isovector part of the interaction: diagrammatic expansion

IV interaction: pion + ρ-meson

Free-space pseudovector coupling + RMF-Renormalized Landau-Migdal contact term (g'-term)

Fixed strength

Infinite sum:

Transverse (cross) channel

Longitudinal (direct) channel

Self-energy contribution

Isovector phonon
Low-lying states in $\Delta T=1$ channel and nucleonic self-energy

In spectra of neighboring odd-odd nuclei we see low-lying (collective) states with natural and unnatural parities: $0^+, 0^-, 1^+, 1^-, 2^+, 2^-, 3^+, 3^-, \ldots$ Their contribution to the nucleonic self-energy is expected to affect single-particle states:

Nucleonic self-energy beyond mean-field:

$$\sum^{(e)}_{(N,Z)} \rightleftharpoons \sum^{(e)}_{(N+1,Z-1)}$$

Forward  
Backward

Underlying Mechanism for pn-pairing or $T=0$ pairing

Matrix element in Nambu space

$$\Sigma^{(e)}_{k_1 k_2}(\varepsilon) = \sum_{\eta=\pm 1} \sum_{k,\mu} \frac{\gamma_{\mu; k_1 k} \gamma_{\mu; k_2 k}^{*}}{\varepsilon - \eta(E_k + \Omega_{\mu} - i\delta)}$$
Single-particle states in $^{100}$Sn: effects of pion dynamics

Truncation scheme: phonons below 20 MeV
Phonon basis: $T=0$ phonons: $2^+, 3^-, 4^+, 5^-$, $T=1$ phonons: $0, \pm 1^\pm, 2^\pm, 3^\pm, 4^\pm, 5^\pm, 6^\pm$

Converged


Next step: pionic ground state correlations (backward going diagrams), in progress
Nuclear dipole response at finite temperature

\[ \tilde{n}_i(E_i, T) = (1 - v_i^2(T))n_i(E_i, T) \]

\[ \tilde{n}_i(E_i, T) = v_i^2(T)(1 - n_i(E_i, T)) \]

1. Saturation of the strength with \( \Delta \) at \( \Delta = 10 \text{ keV} \) for \( T > 0 \)

2. The low-energy strength is not a tail of the GDR and not a part of PDR

3. The nature of the strength at \( E_\gamma \to 0 \) is continuum transitions from the thermally unblocked states

4. Spurious translation mode should be eliminated exactly
Low-energy limit of the RSF in even-even Mo isotopes

\[ T = \sqrt{(E^* - \delta)/a} \]

\[ a = a_{\text{EGSF}} \Rightarrow T_{\text{min}} \quad (\text{RIPL-3}) \]

\[ a = \frac{\pi^2 (g_\nu + g_\pi)}{6} \quad (\text{microscopic}) \]

Exp-1: NLD norm-1, M. Guttormsen et al., PRC 71, 044307 (2005)

Theory: E. L., N. Belov, PRC 88, 031302(R)(2013)

Books and Topical Reviews:

- Rowe, Nuclear Collective Motion Models and Theory, (World Scientific, 2010).

Lecture notes and slides:

- TALENT Courses 2013-2016: [http://www.nucleartalent.org](http://www.nucleartalent.org)
- P. Ring: [http://indico2.riken.jp/indico/getFile.py/access?contribId=8&sessionId=4&resId=0&materialId=slides&confId=1450](http://indico2.riken.jp/indico/getFile.py/access?contribId=8&sessionId=4&resId=0&materialId=slides&confId=1450)