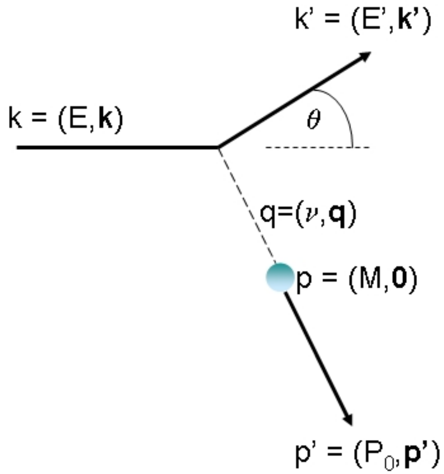
How to probe nuclear size?
⇒ Electron Scattering from nuclei



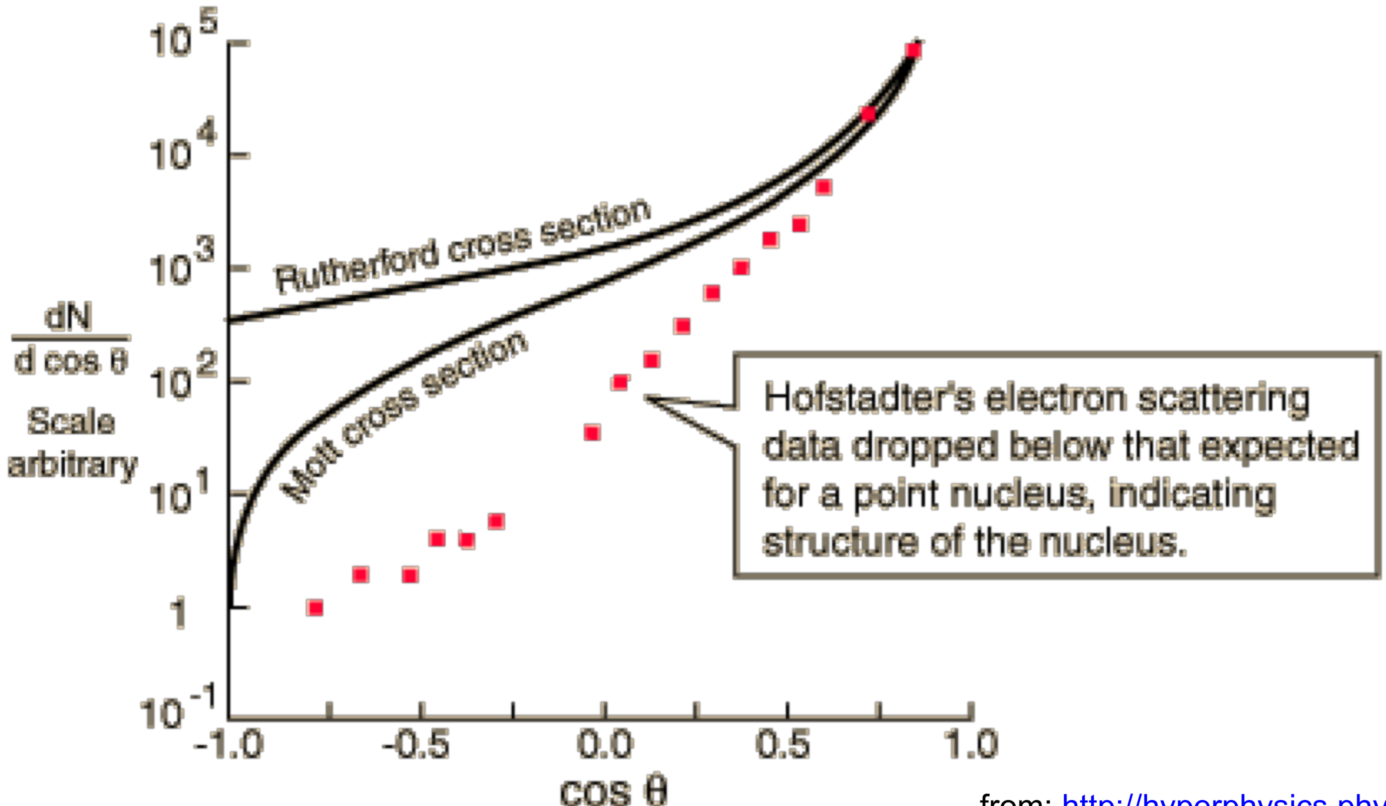
For low energies and under conditions where the electron does not penetrate the nucleus, the electron scattering can be described by the Rutherford formula. The *Rutherford formula* is an analytic expression for the differential scattering cross section, and for a projectile charge of *e*, it is

$$\left(\frac{d\sigma}{d\cos\theta}\right)_R = \frac{\pi}{2}\left[\frac{\hbar c Z\alpha}{T_{KE}(1-\cos\theta)}\right]^2$$

Kinetic energy of electron

As the energy of the electrons is raised enough to make them an effective nuclear probe, a number of other effects become significant, and the scattering behavior diverges from the Rutherford formula. The probing electrons are relativistic, they produce significant nuclear recoil, and they interact via their magnetic moment as well as by their charge. When the magnetic moment and recoil are taken into account, the expression is called the *Mott cross section*.

A major period of investigation of nuclear size and structure occurred in the 1950's with the work of Robert Hofstadter and others who compared their high energy electron scattering results with the Mott cross section. The illustration below from Hofstadter's work shows the divergence from the Mott cross section which indicates that the electrons are penetrating the nucleus - departure from point-particle scattering is evidence of the structure of the nucleus.



from: http://hyperphysics.phy-astr.gsu.edu/

The cross section from elastic electron scattering is:

$$\frac{d\sigma}{d\cos\theta} = \left(\frac{d\sigma}{d\cos\theta}\right)_{\rm point} |F(q)|^2$$

Mott cross section

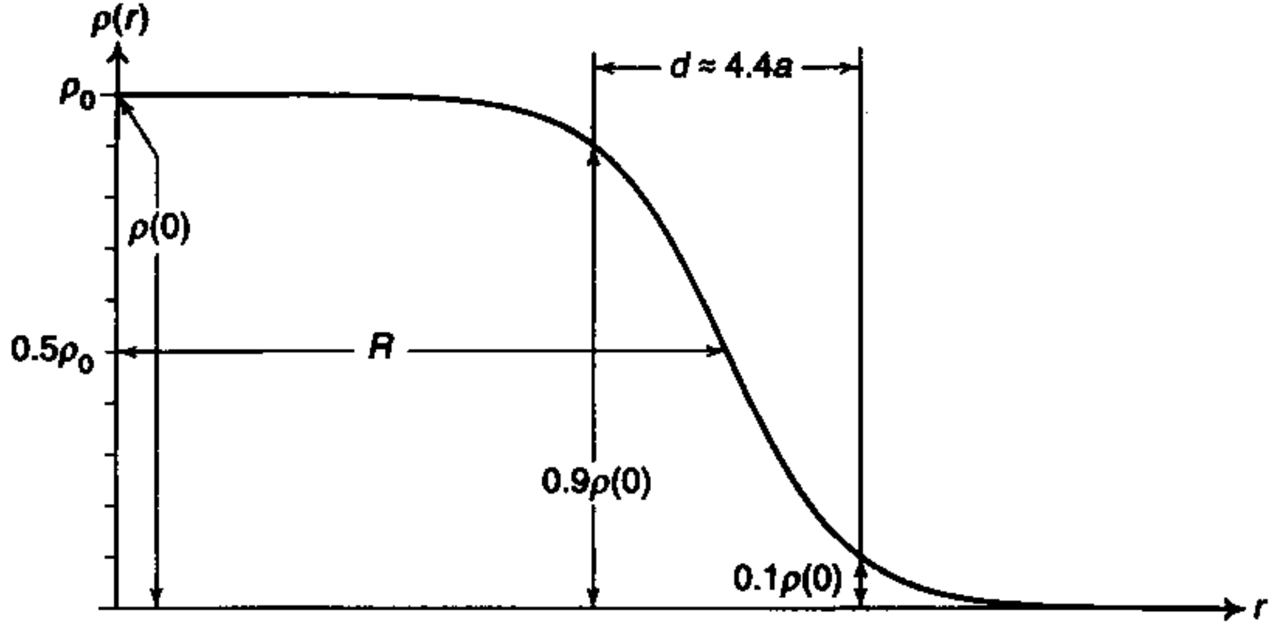
form factor

Form factor

$$F(\vec{q}) = \int d^3r\, \rho_{\rm ch}(\vec{r}) e^{i\vec{q}\vec{r}}$$

q – three momentum transfer of electron

Sizes

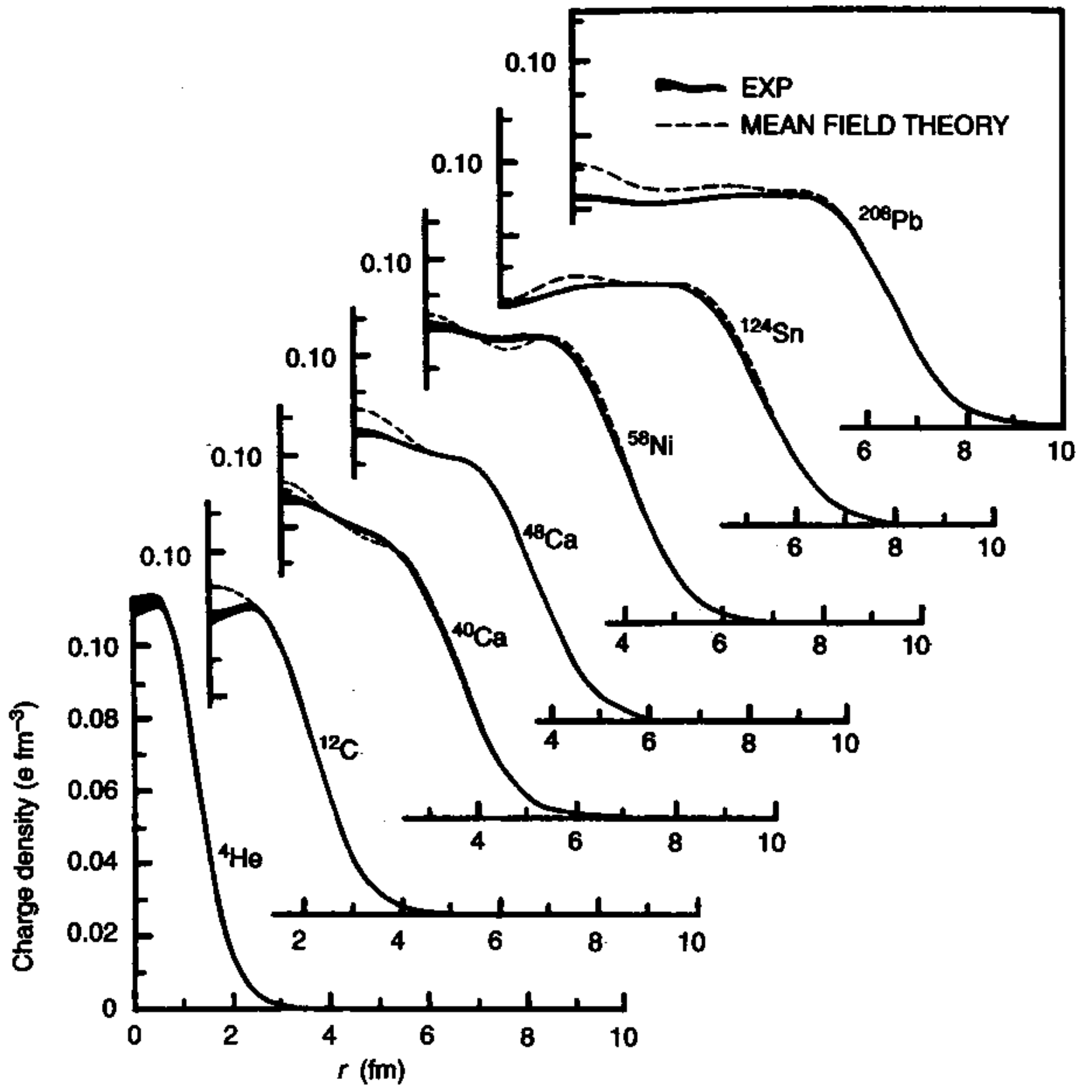


$$\rho(0) = 0.16 \text{ nucleons/fm}^3$$

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r - R}{a} \right) \right]^{-1}$$

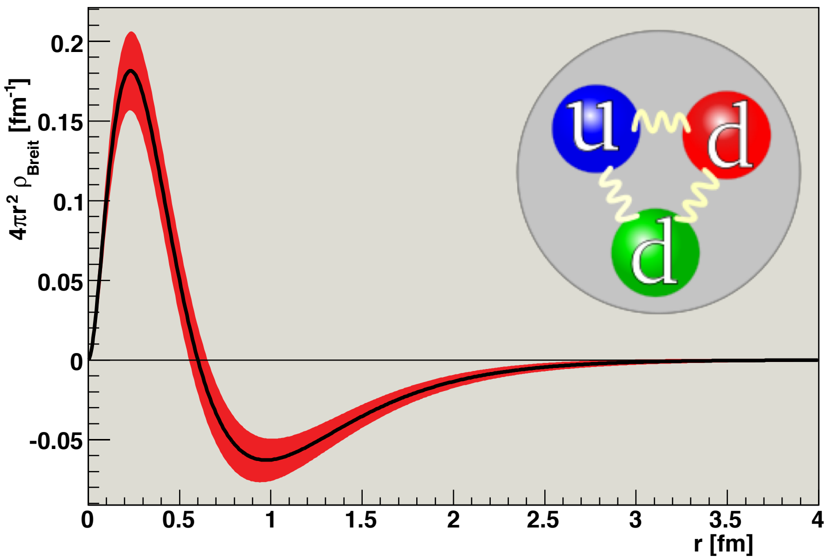
$$R \approx 1.2 A^{1/3} \text{fm}, \quad a \approx 0.6 \text{fm}$$

Calculated and measured densities

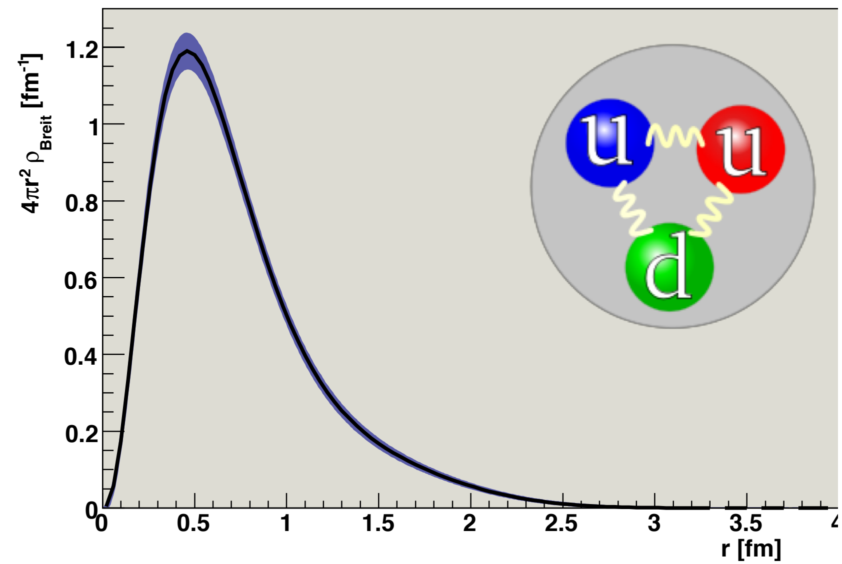


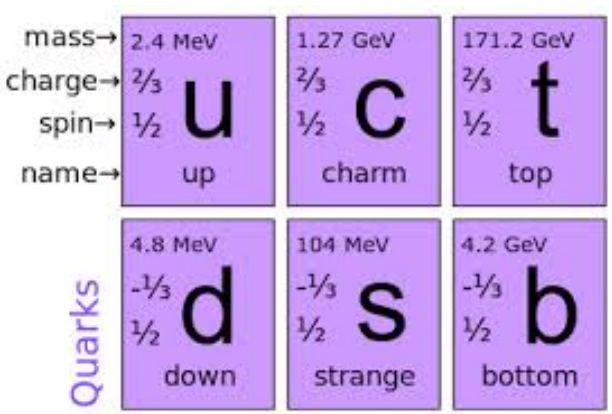
Protons and neutrons aren't point particles

charge distribution in the neutron



charge distribution in the proton





$$\langle R_{\rm ch}^2 \rangle = \langle R_{\rm pp}^2 \rangle + \langle r_{\rm p}^2 \rangle + \frac{N}{Z} \langle r_{\rm n}^2 \rangle + \frac{3\hbar^2}{4m_p^2 c^2}$$

relativistic Darwin-Foldy correction

$$\sqrt{\langle r_{\rm p}^2 \rangle} = 0.8775(51)\,{\rm fm}$$

$$\langle r_{\rm n}^2 \rangle = -0.1149(27)\,{\rm fm}^2$$

Proton size puzzle

http://www.newscientist.com/article/dn19141-incredible-shrinking-proton-raises-eyebrows.html

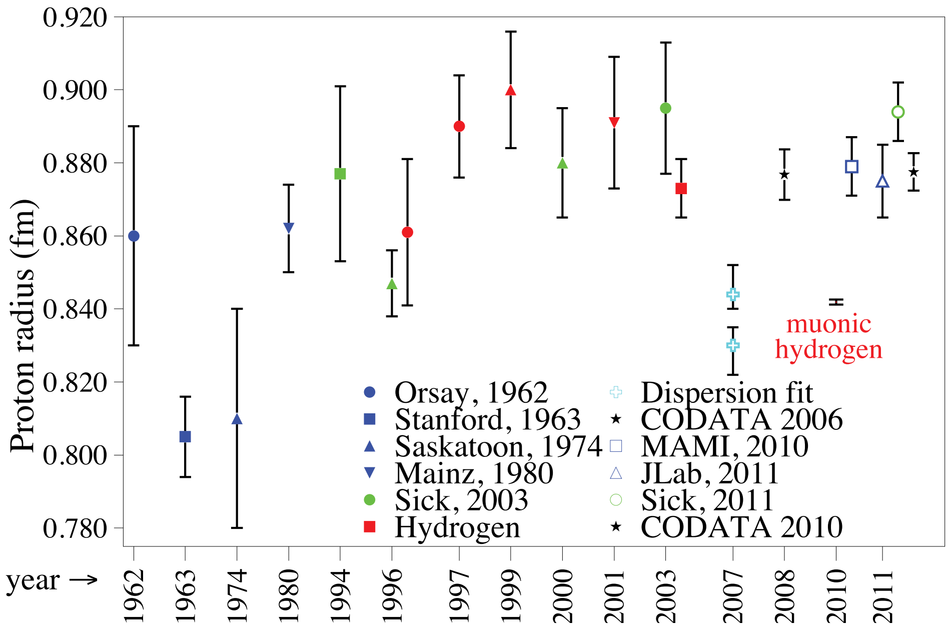
https://www.psi.ch/media/proton-size-puzzle-reinforced

Muon has a mass of 105.7 MeV, which is about 200 times that of the electron

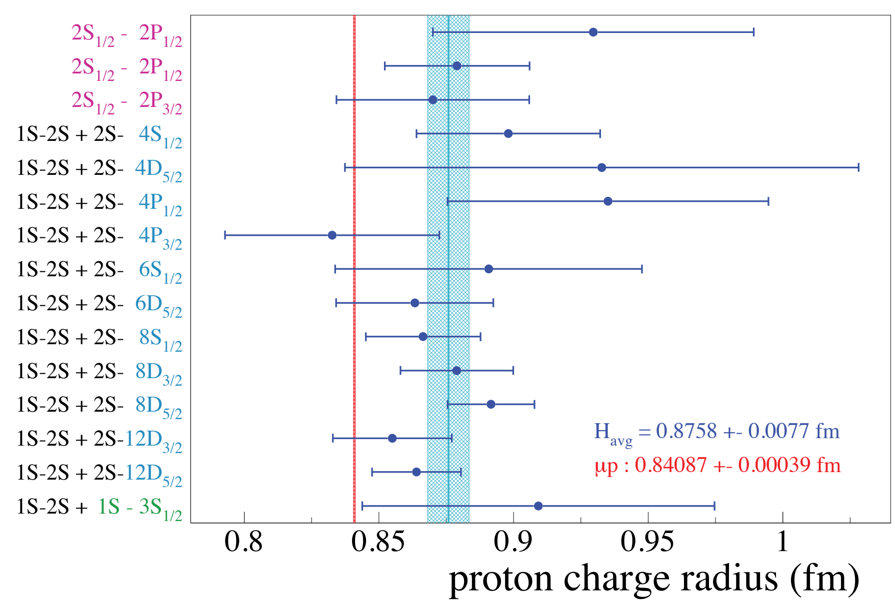
Bohr radius: $$a_0 = \frac{\hbar}{m_e c \alpha}$$

New radius: 0.84087(39) fm

Proton radius determinations over time



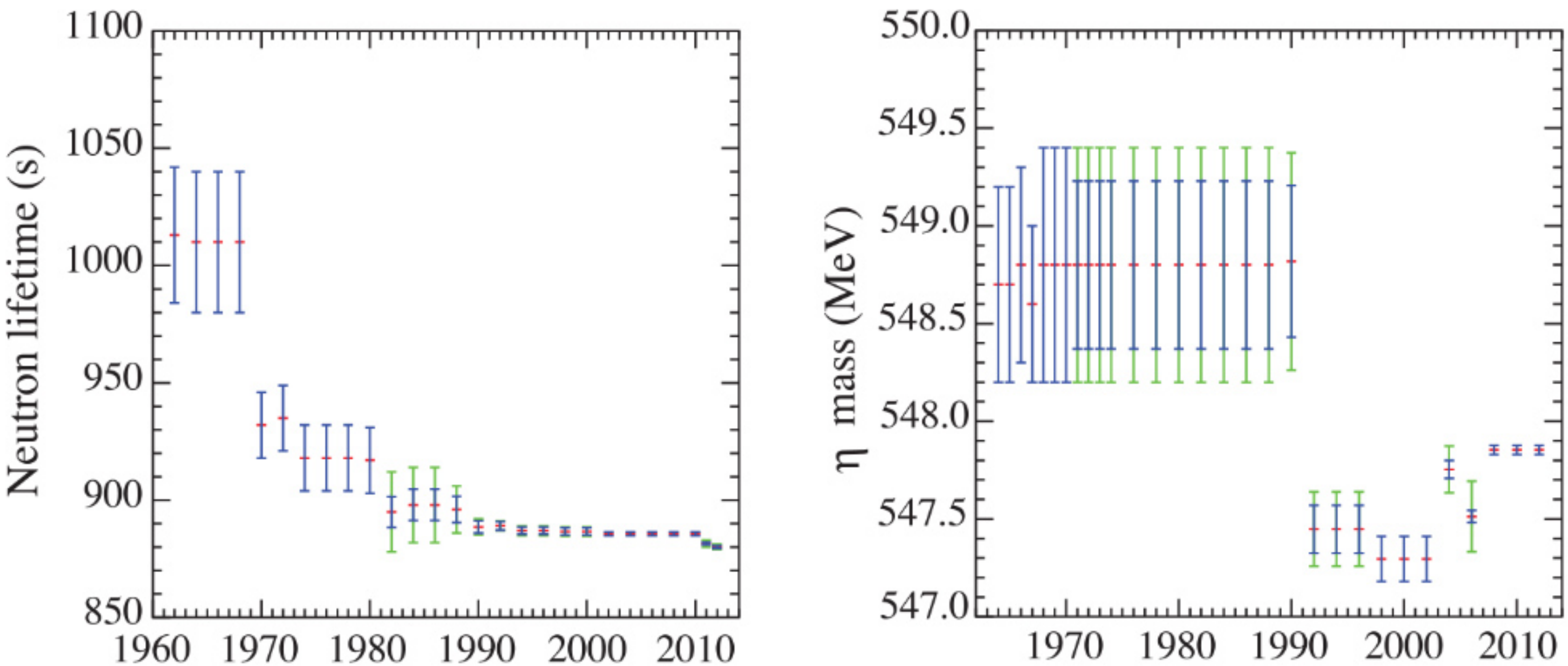
Proton charge radii obtained from hydrogen spectroscopy



Pohl et al. http://www.annualreviews.org/doi/abs/10.1146/annurev-nucl-102212-170627

New Measurement Deepens Proton Puzzle

Aug. 2016: Pohl *et al.* (Science 353) determined the charge radius of the deuteron, a nucleus consisting of a proton and a neutron, from the transition frequencies in muonic deuterium. Mirroring the proton radius puzzle, the radius of the deuteron was several standard deviations smaller than the value inferred from previous spectroscopic measurements of electronic deuterium. This independent discrepancy points to experimental or theoretical error or even to physics beyond the standard model.



Examples of how the measured values of constants can vary dramatically before converging on their correct values (from PDG)

Isotope Shift

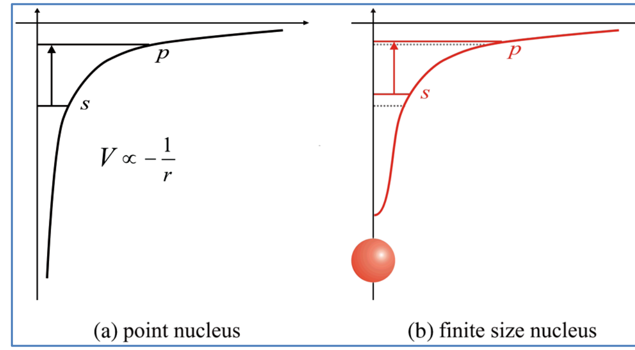
Laser trapping of exotic atoms.
RMP 85, 1383 (2013)

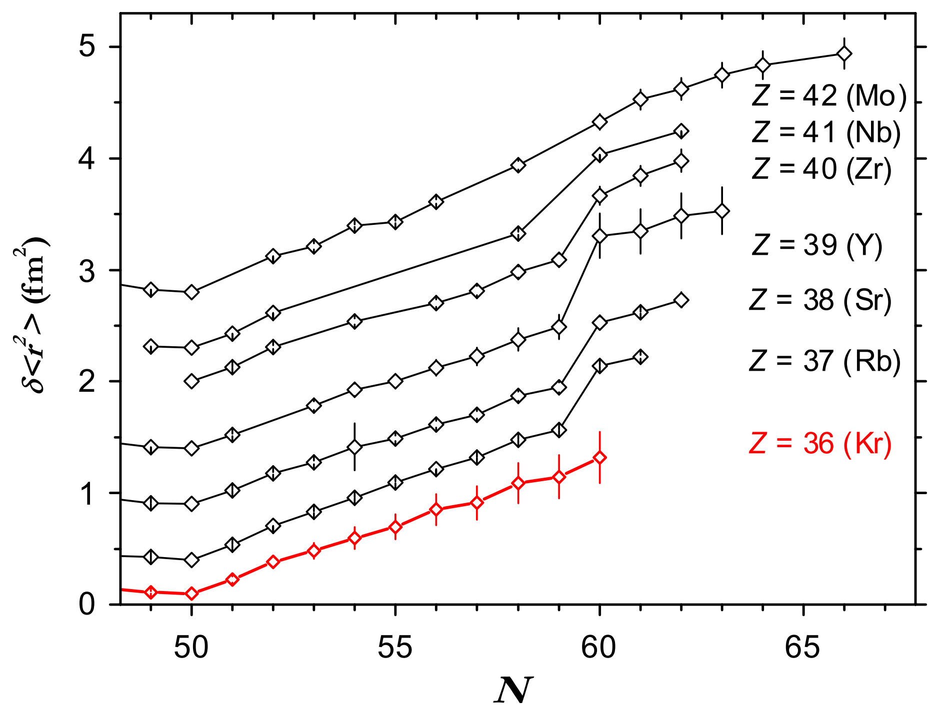
TABLE I. Contributions to the electronic binding energy and their orders of magnitude in atomic units. a_0 is the Bohr radius, $\alpha \approx 1/137$. For helium, the atomic number $Z = 2$, and the mass ratio $\mu/M \sim 1 \times 10^{-4}$. g_I is the nuclear g factor. α_d is the nuclear dipole polarizability.

Contribution	Magnitude
Nonrelativistic energy	Z^2
Mass polarization	$Z^2\mu/M$
Second-order mass polarization	$Z^2(\mu/M)^2$
Relativistic corrections	$Z^4\alpha^2$
Relativistic recoil	$Z^4\alpha^2\mu/M$
Anomalous magnetic moment	$Z^4\alpha^3$
Hyperfine structure	$Z^3 g_I \mu_0^2$
Lamb shift	$Z^4\alpha^3 \ln\alpha + \cdots$
Radiative recoil	$Z^4\alpha^3(\ln\alpha)\mu/M$
Finite nuclear size	$Z^4\langle r_c/a_0\rangle^2$
Nuclear polarization	$Z^3 e^2 \alpha_d/(\alpha a_0^4)$

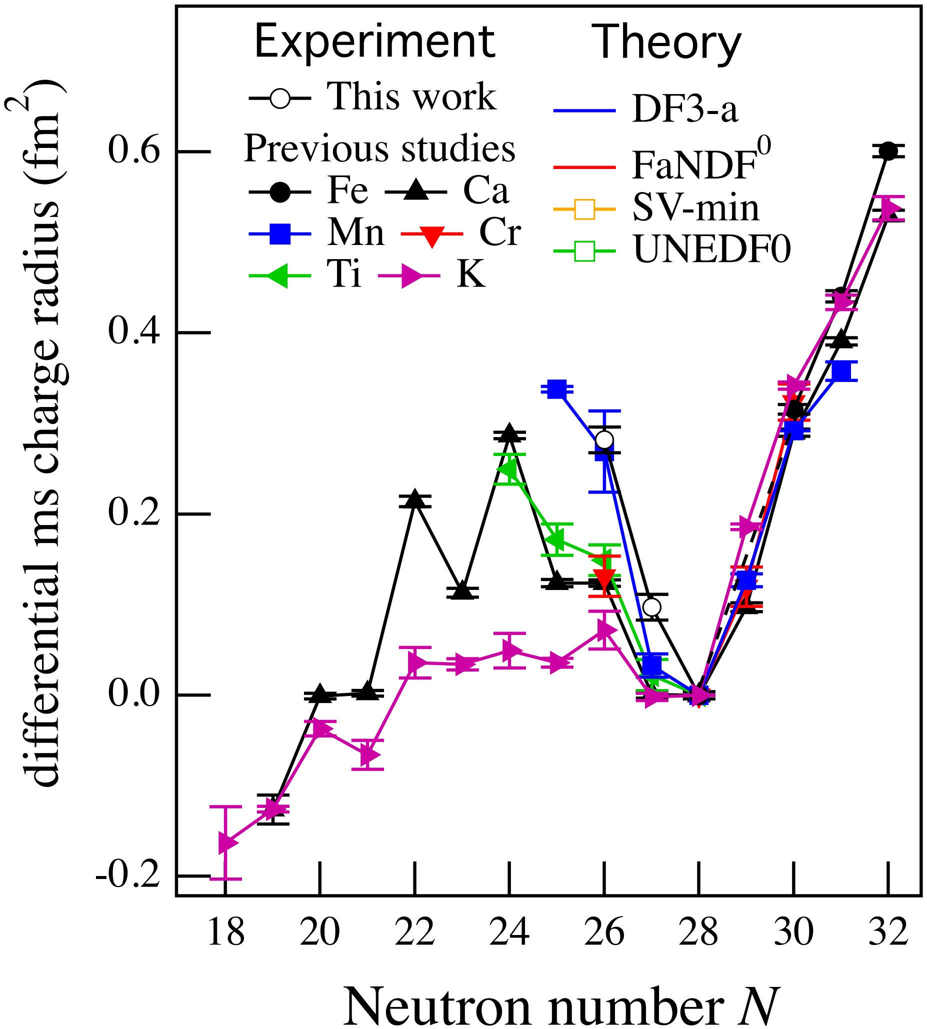
$$\alpha = \frac{1}{137}$$

μ=reduced electron mass

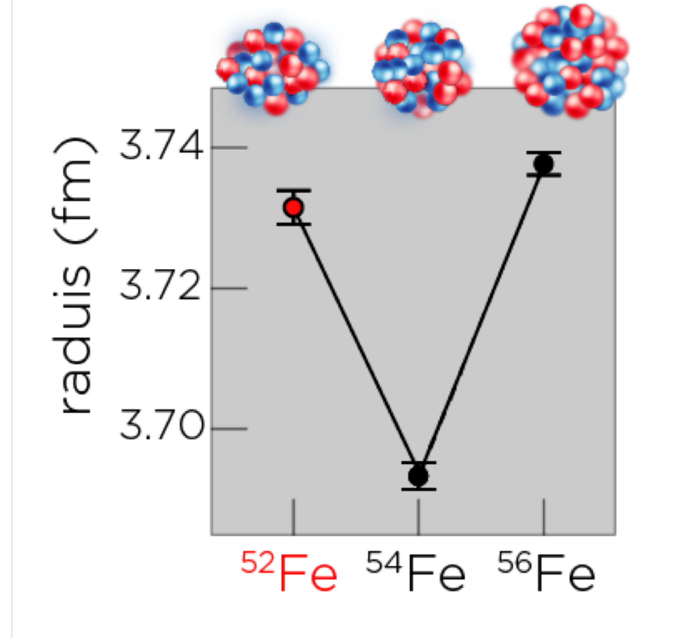




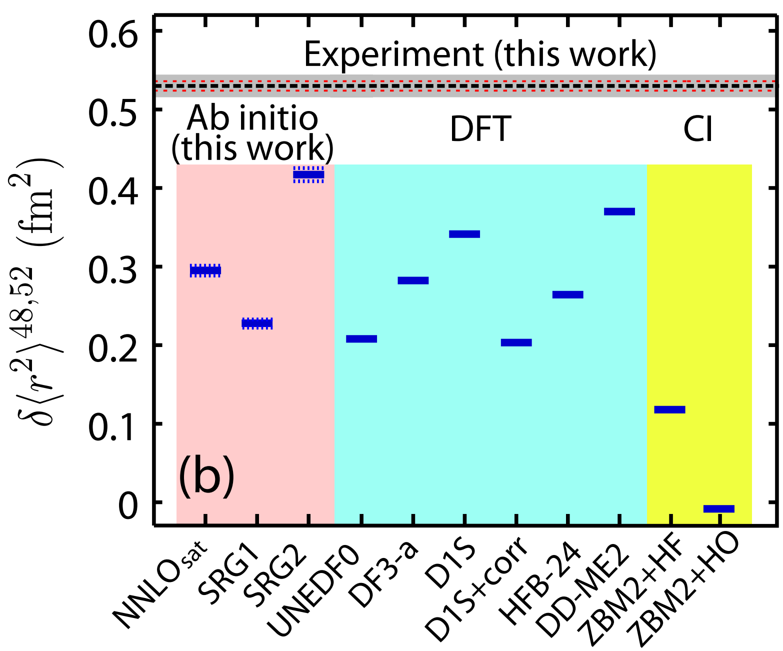
Difference in mean-square charge radii for the N~60 region, PRL 105, 032502 (2010)



Phys. Rev. Lett. 117, 252501 (2016)
BECOLA @ NSCL



Nature Physics 12, 594 (2016)



Neutron radii

- Proton-Nucleus elastic
- Pion, alpha, d scattering
- Pion photoproduction

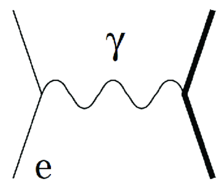
Involve strong probes

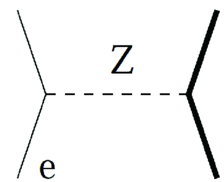
Phys. Rev. Lett. 112, 242502 (2014)

http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.112.242502

Parity-violating electron scattering

Z_0 of Weak Interaction





M_Z=90.19 GeV!

Parity Violating Asymmetry

$$A = \frac{\left(\frac{d\sigma}{d\Omega}\right)_R - \left(\frac{d\sigma}{d\Omega}\right)_L}{\left(\frac{d\sigma}{d\Omega}\right)_R + \left(\frac{d\sigma}{d\Omega}\right)_L} = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}}\left[\underbrace{1 - 4\sin^2\theta_W}_{\approx 0} - \frac{F_n\,(Q^2)}{F_P(Q^2)}\right] \sim 7\cdot 10^{-7}$$

Weinberg angle: $\sin^2\theta_W = 0.23120 \pm 0.00015$

	proton	neutron
Electric charge	1	0
Weak charge	0.08	1

A comment: Yukawa potential

$$V_Y(r) = -g^2 \frac{e^{-\mu r}}{r}$$

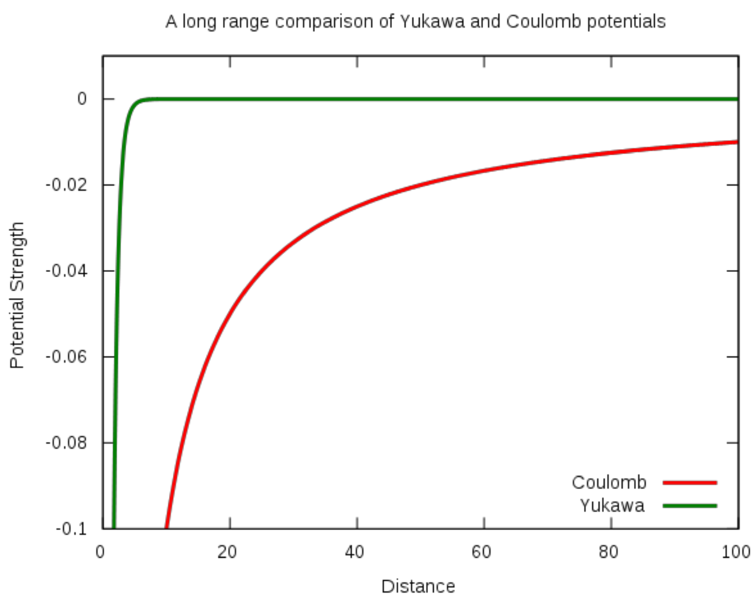
$$\lambda_B = \frac{1}{\mu} = \frac{\hbar}{m_B c}$$

Compton wavelength of the boson (force carrier)

Mass of the boson

$$-\frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2} + \nabla^2 \psi = \mu^2 \Psi$$

Klein-Gordon equation



Lead (^{208}Pb) Radius Experiment : PREX

Analysis is clean, like electromagnetic scattering:

1. Probes the entire nuclear volume
2. Perturbation theory applies

E = 850 MeV, $\theta = 6^0$
electrons on lead

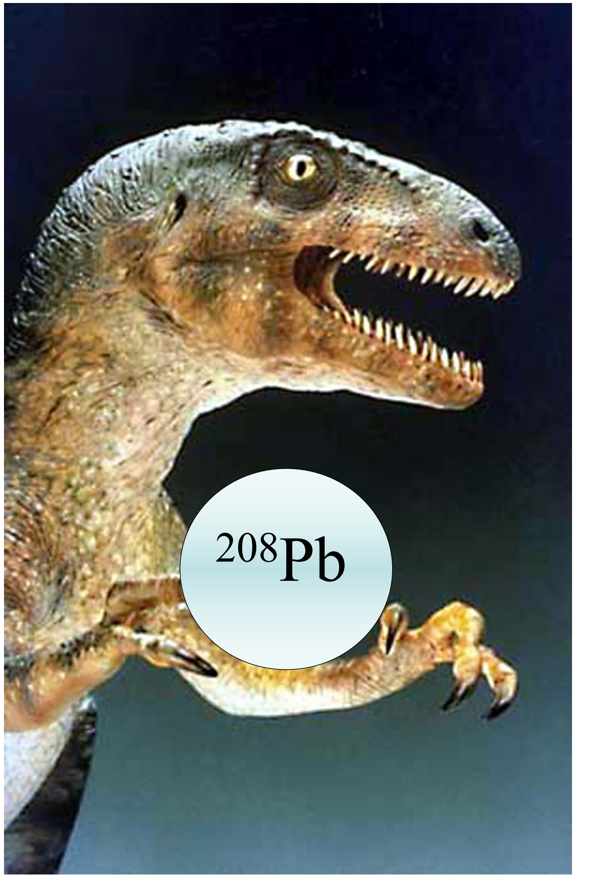
Phys. Rev. Lett. 108, 112502 (2012)

http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.108.112502

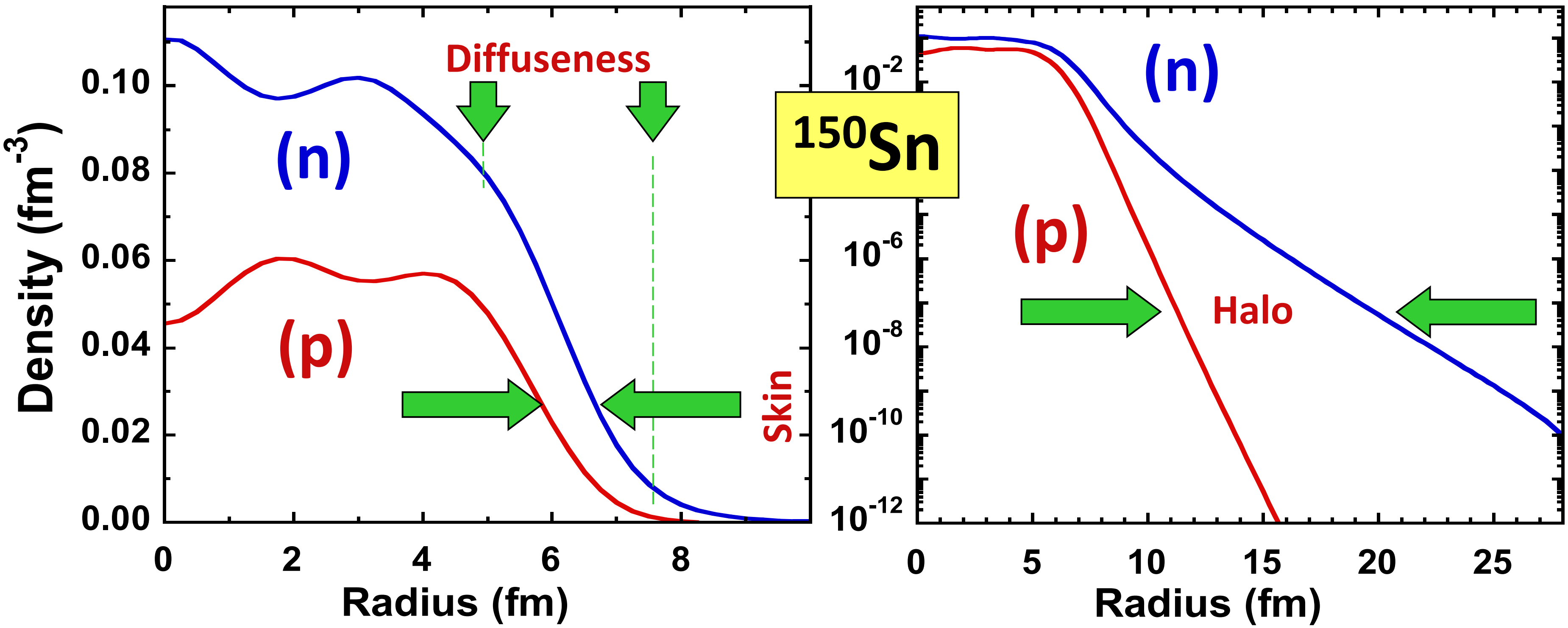
$$R_{skin} = R_n - R_p$$

PREX: $0.34^{+0.15}_{-0.17}\, fm$

Theory: $0.168 \pm 0.022\, fm$



Neutron & proton density distributions



S. Mizutori et al., Phys. Rev. C61, 044326 (2000)