Physics 905
Fundamentals of Accelerator Physics
Problem Set #2
March 17, 2016 due March 24, 2016
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Problem #1 Bending Magnets 15 pts

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Suppose that we want to design a proton sychrotron to accelerate from an injection energy of 50 GeV to 300 GeV. The circumfrance of the ring will be $C = 4000\,m$, composed of 120 sections that typically contain 8 bending magnets and 2 quadrupole magnets. The bending magnets are left out of 20 sections to provide space for injection, extraction, acceleration, vacuum pumping, etc. Bending magnets in the remaining sections each have an effective length of $\ell = 3.25\,m$. (assume sector mags)

2pts a) What is the fraction of C filled by bending magnets?

2pts b) What is the angular deflection per dipole in milliradians?

3pts c) What is the "average radius" $= C/(2\pi)$ of the ring, and what is the actual bending radius of a dipole magnet? Why are they different?

3pts d) Evaluate the needed range of magnetic field in the bending magnets from injection to final energy.

5pts e) Iron dipole bending magnets are employed. Magnets have 10 cm gaps between the iron poles. How many amp-turns (NI) are required to drive the magnet at maximum excitation?

25 pts

Problem #2 Ion Diode : Child-Langmuir Current Density

Consider a hot-plate type ion diode of voltage V_0 and gap length d. Let the current density J be composed of two species

species 1: mass m_1 charge q
species 2: mass m_2 charge q

Set

$$J = J_1 + J_2 \qquad \text{with} \qquad J_1 = \alpha J$$
$$J_2 = (1-\alpha) J$$

What is the "effective mass" M_{eff} in terms of m_1 and m_2 that should be used in the resulting Child-Langmuir Law:

$$J = \frac{4}{9} \varepsilon_0 \left(\frac{2q}{M_{eff}} \right)^{1/2} \frac{V_0^{3/2}}{d^2}$$

Problem #3 2D Iron Quadrupole Magnet 20pts

Consider the axial midplane fields of an axially long magnetic quadrupole with:

$$B_x = B'y \qquad B' = \text{const} \quad \text{(Gradient)}$$
$$B_y = B'x$$

3pts a) Show for a general 3D magnet that $\vec{B}$ within the vacuum apeture of the magnet can be derived from a scalar magnetic potential ϕ as: $\vec{B} = -\nabla\phi$ with ϕ satisfying $\nabla^2\phi = 0$

5pts b) Surfaces of $\phi = \text{const}$ can correspond to magnetic poles. Derive ϕ consistent with the 2D fields and show they are hyperbolic. Sketch poles with closest radial approach (aperture) R. Sketch superimposed field lines and indicate "North" and "South" polarities for $B' > 0$.

2pts c) Show that $|\vec{B}| = B'R$ at $r = \sqrt{x^2+y^2} = R$ to identify the apeture field.

5pts d) Sketch a geometry with truncated radial extent poles, and include an iron return yoke. Place coils with correct polarity / symmetry to "charge" the poles. Assume that the fields are only weakly perturbed by the truncation. Derive a formula for the needed coil amp-turns (NI) to generate gradiant B'. Steps analogous to the ideal dipole analyzed in class.

5pts e) What would happen if the poles are actively rotated by an azimuthal angle of $\pi/4$. (Skew Quad) Derive x and y equations of motion for a particle evolving in this rotated quadrupole. Are these Hills' equations? If not, could coordinates be chosen to make them Hills' equations? Generally, due to errors, magnets will have normal field and small skew field error terms. Does this change the situation for analysis? Why?

Problem #4 Transfer Matrix 25 pts.

Consider a particle orbit evolving consistently with Hill's equation

$$\frac{d^2}{ds^2}x + K(s)\,x = 0$$

for some unspecified lattice focusing function $K(s)$. The solution can be expressed as a transfer map:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_s = \underbrace{\overline{M}(s|s_i)}_{2\times 2 \text{ Matrix}} \cdot \underbrace{\begin{bmatrix} x_0 \\ x'_0 \end{bmatrix}}_{\text{initial cond.}}$$

where

$$\overline{M}(s|s_i) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

10pts a) Show that for all s that the determinant of $\overline{M}$ satisfies

$$W = \det \overline{M} = \underline{1}.$$

This is called the "Wronskian" invariant.

Hint: calculate $\frac{d}{ds}W$ and use equations of motion for how the M_{ij} must evolve to be consistent with Hill's equations.

10 pts. b) Show for any transfer map

$$\overline{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

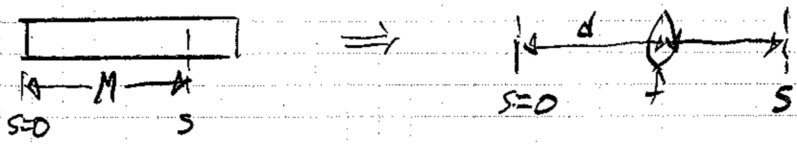
we can always resolve it as

$$M = \underbrace{\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}}_{\text{drift 2}} \underbrace{\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}}_{\text{Thin Lens}} \underbrace{\begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}}_{\text{drift 1}}$$

Identify d_1, d_2, f in terms of M_{11}, M_{22} and M_{21}

This implies that one can always replace thick lenses with thin lenses + drifts. But will the orbit be the same or not within the element?

5pts c) For a thick focusing lens with $K = \text{const} > 0$



show that

$$d(s) = \tan[\sqrt{K}\,s/2]/\sqrt{K}$$

$$\frac{1}{f(s)} = \sqrt{K}\sin[\sqrt{K}\,s]$$

How must we interpret this replacement if $2d \neq s$?