Physics 905

Fundamentals of Accelerator Physics

Problem Set #3

March 24, 2016 due March 31, 2016

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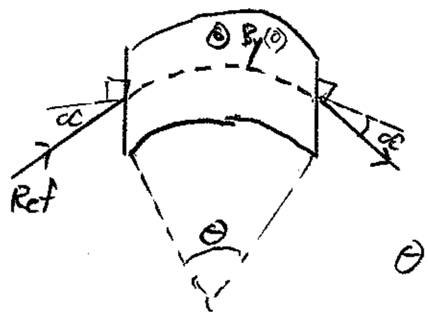
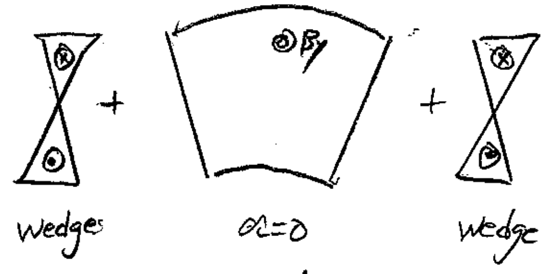
Problem: 1 Dipole Edge Corrections

17 pts a) Horizontal Correction:

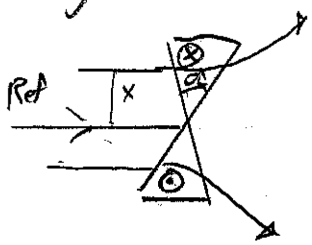


θ = ref bend angle.

Resolve as:



The sector dipole has the transfer matrix derived in class.
For the wedge



For a short sector argue from the Lorentz force equation that the particle experiences the following corrections for orbit displacement x through the wedge.

$$\Delta x' = \frac{B_y(0)}{(B\rho)} \tan\alpha \cdot x$$

$$\Delta x = 0$$

to show that

$$\overline{M}_{wedge} = \begin{bmatrix} 1 & 0 \\ \frac{\tan\alpha}{\rho} & 1 \end{bmatrix} \qquad \frac{1}{\rho} = \frac{B_y(0)}{(B\rho)}$$

This kick will be applied entering and exiting the magnet.

3 pts b) Show that the x-transfer Matrix of the full dipole can be expressed as

$$\overline{M}_x = \overline{M}_{wedge} \cdot \underbrace{\overline{M}_{sector}}_{\text{derived in class}} \cdot \overline{M}_{wedge} = \begin{bmatrix} \frac{\cos(\theta-\alpha)}{\cos\alpha} & \rho\sin\theta \\ -\frac{\sin(\theta-2\alpha)}{\rho\cos^2\alpha} & \frac{\cos(\theta-\alpha)}{\cos\alpha} \end{bmatrix}$$

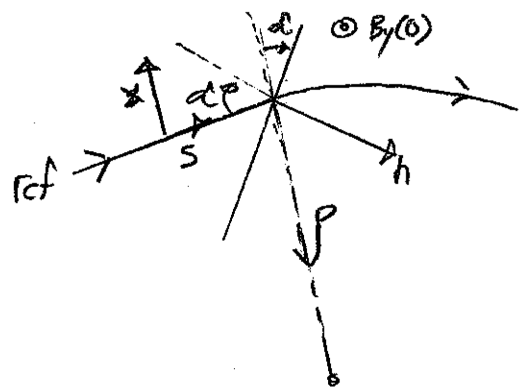
Hint:

$$\cos\theta + \tan\alpha \sin\theta = \frac{\cos(\theta-\alpha)}{\cos\alpha}$$

$$\sin\theta - \tan\alpha \cos\theta = \frac{\sin(\theta-\alpha)}{\cos\alpha}$$

10pts c) Vertical Correction

There is also a vertical correction to the kick due to slanted edge entry which results from fringe fields.



h = horizontal

Using arguments analogous to part a):

$$\Delta y' = q v \frac{\int B_x(y)\, ds}{\gamma m v^2} = \frac{1}{(B\rho)} \int_{edge} B_x(y)\, ds$$

Argue that

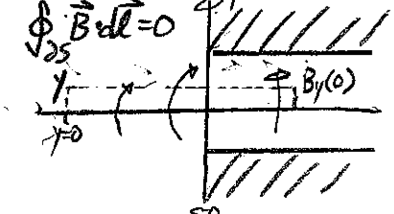
$$\Delta y' = \frac{-\tan\alpha}{(B\rho)} \int_{edge} \vec{B}\cdot d\vec{s}$$

$$\int_{edge} \vec{B}\cdot d\vec{s} = y\, B_y(0)$$

Hint:

$\nabla \times \vec{B} = 0$

$\oint_{\partial S} \vec{B}\cdot d\vec{l} = 0$



$\vec{B}(s>0) = B_y(0)\hat{y}$

to obtain

$$\Delta y' = -\frac{\tan\alpha}{\rho}\, y \,, \quad \Delta y = 0$$

$$\bar{M}_y\Big|_{edge} = \begin{bmatrix} 1 & 0 \\ -\frac{\tan\alpha}{\rho} & 1 \end{bmatrix}$$

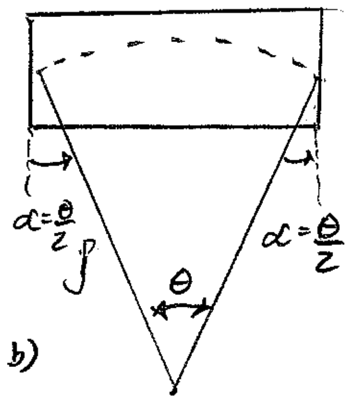
for the edge correction going in. The same correction applies going out.

d) Show that

$$\bar{M}_y = \underbrace{\bar{M}_y\Big|_{edge} \cdot \bar{M}_y\Big|_{sector} \cdot \bar{M}_y\Big|_{edge}}_{\text{Take for granted}} = \begin{bmatrix} 1-\theta\tan\alpha & \rho\theta \\ -\frac{\tan\alpha}{\rho}(2-\theta\tan\alpha) & 1-\theta\tan\alpha \end{bmatrix}$$

show.

e) For a box dipole (most common type)



Use results of b) and d) to show that:

Transfer Matrices Box Dipole

$$\overline{M}_x = \begin{bmatrix} 1 & \rho \sin\theta \\ 0 & 1 \end{bmatrix}$$

$$\overline{M}_y = \begin{bmatrix} 1 - \theta \tan\frac{\theta}{2} & \rho\theta \\ -\frac{1}{\rho}\tan\frac{\theta}{2}\left(2 - \theta\tan\frac{\theta}{2}\right) & 1 - \theta\tan\frac{\theta}{2} \end{bmatrix} \simeq \begin{bmatrix} \cos\theta & \rho\sin\theta \\ -\frac{\sin\theta}{\rho} & \cos\theta \end{bmatrix}$$

for $\theta \ll 1$.

Problem 2 FRIB Solenoids 15 pts

FRIB will use superconducting solenoids for beam focusing in the 3 linac segments. Consider U_{238}^{78+} (post stripper) accelerated to a kinetic energy of 128 MeV/u. Solenoids have axial length ~ 1/2 m and a field strength of 7 Tesla is employed.

5 pts a) Estimate $(B\rho)$ for the ion in Tesla-meters.

5 pts. b) Estimate the focal length f of the solenoid assuming $B_z \simeq 7$ Tesla over the axial length of 1/2 m. and is zero outside (Hard Edge). Is the thin lens approximation reasonable? Why?

5 pts c) Suppose superconducting cable with an average current density of J is wound to a thickness T around the beam pipe. Derive a formula to roughly estimate the central field B_z in terms of J and T. If the coil thickness is $T = 2$ cm what is J in Amps/mm^2 to produce $B_z = 7$ Tesla?

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Problem 3 Magnetic Optics 30 pts

1pts a) From the Lorentz force equation, show that a static magnetic field $\vec{B}(\vec{x})$ cannot change the particle kinetic energy $W = (\gamma - 1)mc^2$. Make no approximations.

$$m \frac{d}{dt} (\gamma \vec{\beta}) = q \vec{\beta} \times \vec{B}$$

$$\gamma = \frac{1}{\sqrt{1 - \vec{\beta}^2}} \quad ; \quad \vec{\beta} = \frac{1}{c} \frac{d\vec{x}}{dt}$$

5pts b) In class, it was shown for a solenoid magnet with azimuthal symmetry ($\partial/\partial\theta = 0$), that the magnetic field can be expanded in terms of the on-axis field as

$$B_r(r, z) = \sum_{\nu=1}^{\infty} \frac{(-1)^\nu}{\nu!(\nu-1)!} \frac{\partial^{2\nu-1}}{\partial z^{2\nu-1}} B_{z0}(z) \left(\frac{r}{2}\right)^{2\nu-1}$$

$$B_z(r, z) = B_{z0}(z) + \sum_{\nu=1}^{\infty} \frac{(-1)^\nu}{(\nu!)^2} \frac{\partial^{2\nu} B_{z0}(z)}{\partial z^{2\nu}} \left(\frac{r}{2}\right)^{2\nu}$$

$$B_{z0}(z) \equiv B_z(r=0, z)$$

Take $W = (\gamma - 1)mc^2 \simeq$ const and employ standard paraxial approximations to show that if nonlinear applied force terms are dropped ($\propto x^2, xy, xy'$ etc.) that the equations of motion are:

$$x'' - \frac{B_{z0}'}{2(B\rho)} y - \frac{B_{z0}}{(B\rho)} y' = 0 \qquad (B\rho) = \frac{\gamma \beta mc}{q}$$

$$y'' + \frac{B_{z0}'}{2(B\rho)} x + \frac{B_{z0}}{(B\rho)} x' = 0 \qquad B_{z0}' = \frac{\partial B_{z0}(z)}{\partial z}$$

3pts c) Qualitative answer only! Are the results of part c) inconsistent with part a)? If so, could they still be ok to use? Why?

2pts d) If we take $\vec{B} = \nabla \times \vec{A}$, show we can generate the linear field components of the solenoid as

$$B_r = -\frac{1}{2} \frac{\partial B_{z0}}{\partial z} r$$

$$B_z = B_{z0}$$

from $\quad \vec{A} = \frac{1}{2} B_{z0} r \hat{\theta}$

10pts e) Use the paraxial approximation and the results from d) to show for a solenoid that

$$\begin{aligned} P_\theta &= [\vec{x} \times (\vec{p} + q\vec{A})] \cdot \hat{z} \\ &\simeq m\gamma\beta c (xy' - yx') + \frac{qB_{z0}}{2}(x^2+y^2) \end{aligned}$$

Show that the equations of motion in b) imply that

$$P_\theta = \text{const.}$$