Physics 905
Fundamentals of Accelerator Physics

Problem Set #5
April 7, 2016 Due April 14, 2016

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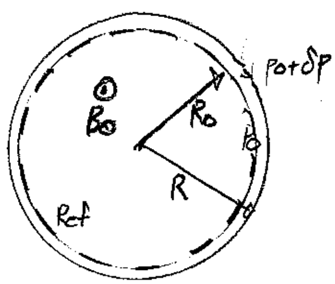
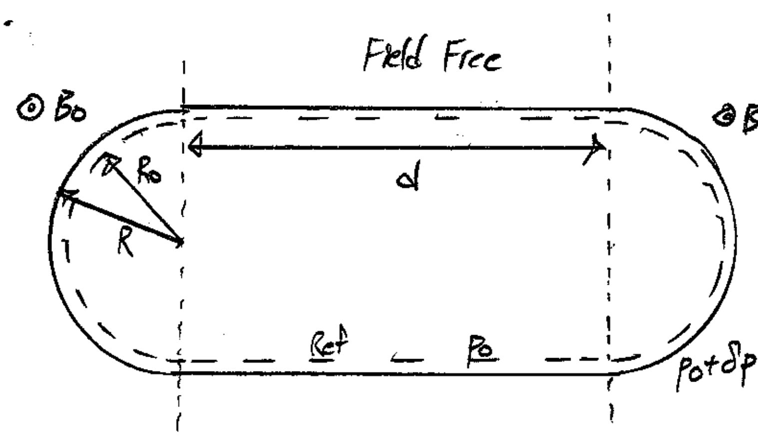
Problem #2: Slip Factor 20 pts

5 pts a) Consider a circular accelerator / storage ring composed of a uniform magnetic field $B_y = B_0 \hat{y} = \text{const}$. The ideal ref path for particle of momentum p_0 has radius R_0 in the plane $\perp$ to B_y. A particle with momentum $p = p_0 + \delta p$ will have a different closed path and radius R.



Calculate the slip factor η in terms of γ for this situation.

5 pts b) Next, repeat a) for a racetrack accelerator with two uniform dipole bends seperated by a field free drift of distance d. Calculate the slip factor in terms of γ and d/R_0.



10 pts c) For $d = 2R_0$ in c), plot η as a function of γ and note where it changes sign. Is this the "transition gamma"? What speed in $\beta = v/c$ does this correspond to?

Problem #1 Chromatic Effects in Solenoids 15 pts. 1/

pts a) Use a Larmor frame equation of motion to derive an equation of motion for chromatic effects in solenoid transport. Follow the procedure in class to show that.

$$\tilde{\eta}'' + \kappa_0 \tilde{\eta} \simeq 2\delta \kappa_0 \tilde{x}_0$$

$\tilde{x}_0$ = Larmor frame unperturbed orbit for $\delta = 0$

$$\kappa_0 \equiv \left(\frac{B_{z0}(s)}{2(B\rho)_0} \right)^2$$

$(B\rho)_0$ = Design rigidity.

$\tilde{x} = \tilde{x}_0 + \tilde{\eta}$ = Larmor frame orbit.

pts b) Are chromatic effects expected to be larger or smaller for solenoid focusing relative to quadrupole focusing when comparing "equivalent" focusing strength? Why?

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Problem #3 Dispersion Function 55 pts

The dispersion function in a periodic ring satisfies:

$$D''(s) + K(s) D(s) = \frac{1}{\rho(s)} \qquad \begin{cases} \rho(s) = \text{Bend Radius} \\ K(s) = \text{Focus Function} \end{cases}$$

$$D(s+L_p) = D(s) \qquad \begin{cases} \rho(s+L_p) = \rho(s) \\ K(s+L_p) = K(s) \end{cases} \qquad L_p = \text{Lattice Period}$$

10 pts a) Argue the solution for D is unique.
Hint: Let D_1 and D_2 be two independent solutions and look for a contradiction.

This implies there is a unique closed orbit $x = \delta \cdot D$ for every value of off-momentum δ. This aids interpretation of results.

5 pts b) Argue that the solution for D can be expressed in an extended 3x3 Transfer Matrix form as:

$$\begin{bmatrix} D \\ D' \\ 1 \end{bmatrix}_s = \begin{bmatrix} \begin{bmatrix} \bar{M}(s|s_i) \end{bmatrix} & \begin{matrix} d(s|s_i) \\ d'(s|s_i) \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix} \begin{bmatrix} D \\ D' \\ 1 \end{bmatrix}_{s=s_i}$$

where $\bar{M}(s|s_i)$ is the usual 2x2 Transfer Matrix for Hill's equation.

How will the periodicity requirement $D(s+L_p) = D(s)$ be specified in this 3x3 formulation? Does this provide adequate constraints to calculate D, D' at any point in the lattice? Explain your answer.

15 pts c) Show for ρ=const and

$$K = \text{const} > 0: \quad \Rightarrow \quad \begin{aligned} d(s|s_i) &= \frac{1}{\rho K}\left[1 - \cos(\sqrt{K}(s-s_i))\right] \\ d'(s|s_i) &= \frac{1}{\rho\sqrt{K}} \sin(\sqrt{K}(s-s_i)) \end{aligned}$$

$$K = \text{const} < 0: \quad \Rightarrow \quad \begin{aligned} d(s|s_i) &= \frac{1}{\rho|K|}\left[-1 + \cosh(\sqrt{|K|}(s-s_i))\right] \\ d'(s|s_i) &= \frac{1}{\rho\sqrt{|K|}} \sinh(\sqrt{|K|}(s-s_i)) \end{aligned}$$

Use Green function results from class and forms derived for $\bar{M}$

5pts d) Use $\kappa = \frac{1}{\rho^2}$ in part c) to show for a sector dipole that the 3x3 transfer matrix through a bend of length l is:

$$\bar{M} = \begin{bmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ \frac{-\sin\theta}{\rho} & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{bmatrix} \qquad l = \rho\theta \qquad \theta = \text{bend angle}$$

$$\begin{bmatrix} D \\ D' \\ 1 \end{bmatrix}_s = \bar{M} \begin{bmatrix} D \\ D' \\ 1 \end{bmatrix}_i$$

Show for a small angle bend, $\theta \ll 1$ that

$$\bar{M} \simeq \begin{bmatrix} 1 & l & \frac{l\theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{bmatrix}$$

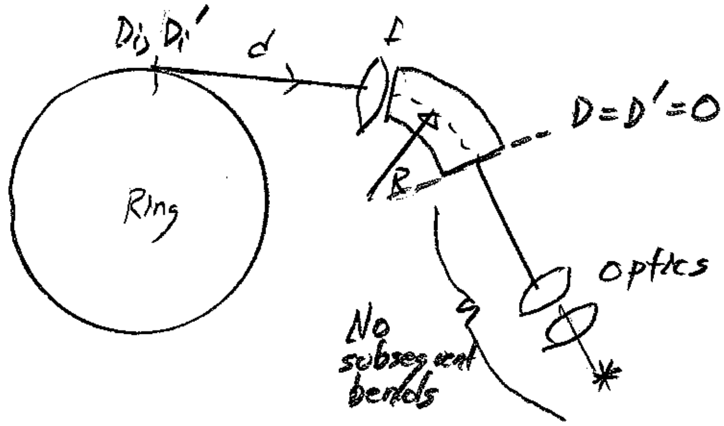
10pts e) Derive the 3x3 transfer matrix, for D for:

1) A drift with $\kappa = 0$ and $\rho \to \infty$
2) A Thin lens at $s = s_i'$ with $\kappa = \frac{1}{f}\delta(s - s_i')$ $f = \text{const}$ and $\rho \to \infty$.
3) Within a uniform sector bend with large radius where we take $\kappa \simeq 0$ and $\rho = \text{const}$.

Use direct methods as opposed to the Green's function.

Show that results agree with the Green's function for 1) and 2) and for the small angle bend result derived from the Green's function in 3).

10pts f) A particle is kicked out of a ring with dispersion $D = D_i$ and $D' = D_i'$ just after the kick and then is transported through an extraction line with a drift length d, a thin lens focusing kick with focal length f, and then a sector bend of radius R and length l; and finally through a series of optics to the target.



Using results from part e), derive constraints on the lattice parameters d, f, R, and l that can be enforced to ensure that $D = 0 = D'$ after the magnet to have zero dispersion in the straight transport and focusing line to the target?

Are these constraints practical to implement in the lab? Why? Qualitative answer only needed.