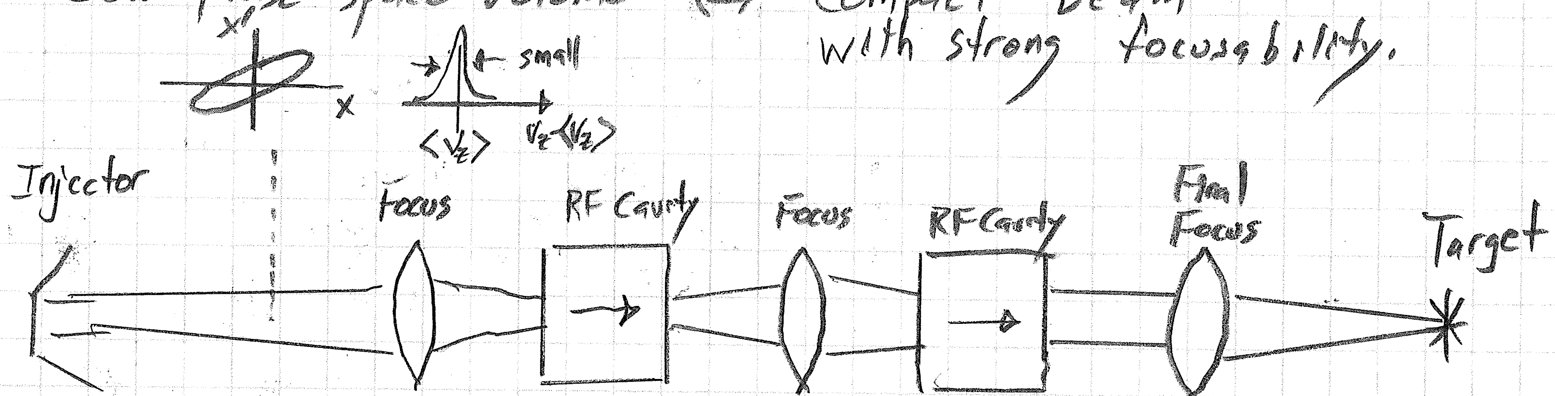
PHY 905
Spring 2020
Lund & Hao
02, injectors. pdf

Injectors

Ideally, want a distribution of monoenergetic particles with high current density and minimal phase-space volume.

- Low phase-space volume $\Leftrightarrow$ compact beam with strong focusability.



- Injectors highly important: start with a bad beam, and likely impossible to "fix".
  - Small spreads $\Rightarrow$ compact, cheap, and robust accelerator

- Injectors often have intricate physics
  - Much potential gain with improvements
  - Smaller systems, typically lower cost

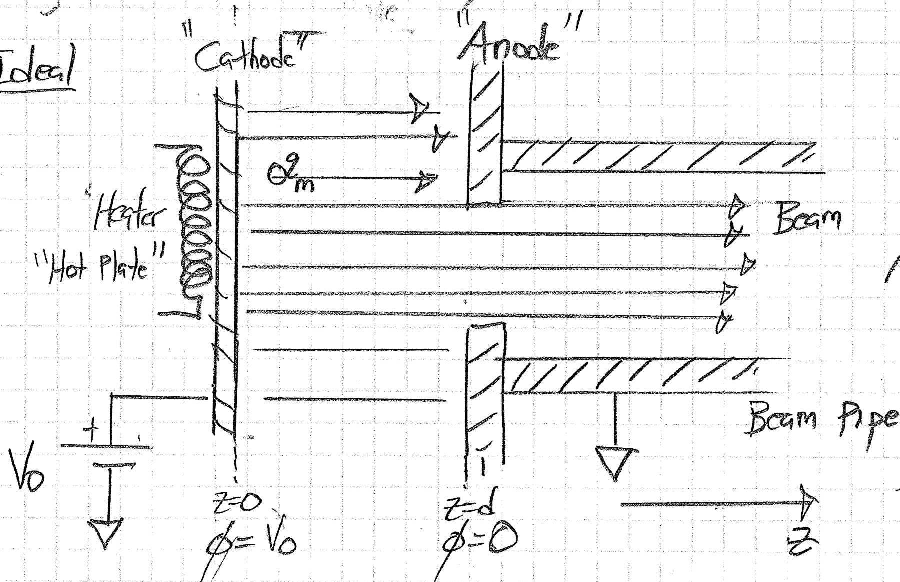
  } Ideal for student projects!

- Not easy topic: but rich and high impact.

Injector Geometry

Ideal Diode

Ideal



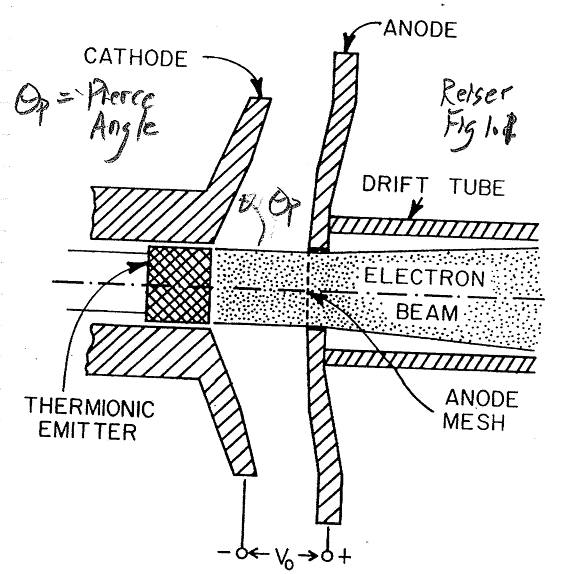
Accelerator
Focusing
Optics

V_0 = Source bias

q = particle charge
m = particle mass

R = Radius Apeture
d = Anode-Cathode Seperation

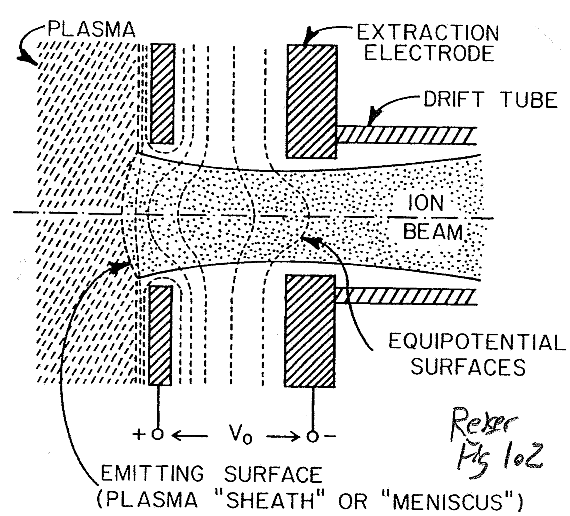
Electrons



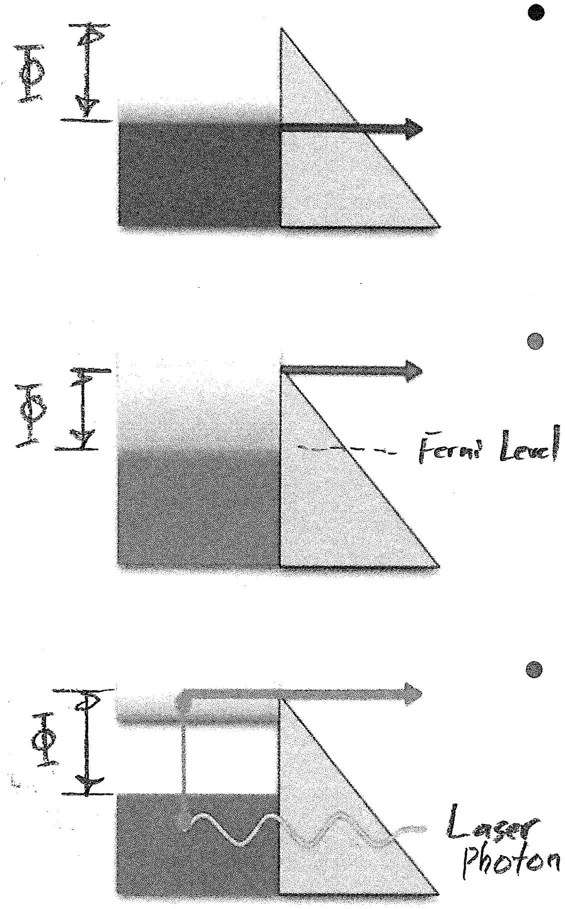
Plasma
- ECR
- Discharge...

Hot Plate
- Doped Tungsten
- Al-Silicate

Ions



The Canonical Emission Equations
(slide courtesy of Dr. K. Jensen, Naval Research Laboratory)



- **Field Emission**
  Fowler Nordheim
  E.L. Murphy, and R.H. Good, Physical Review 102, 1464 (1956).

- **Thermal Emission**
  Richardson-Laue-Dushman
  C. Herring, and M. Nichols, Reviews of Modern Physics 21, 185 (1949).

- **Photoemission**
  Fowler-Dubridge
  L.A. DuBridge
  Physical Review 43, 0727 (1933).

Listed chronologically

$$J_{FN}(F) = A_{FN}F^2 \exp\left(-\frac{B\Phi^{3/2}}{F}\right)$$

A_{FN} = const , B = const
F = Applied Field Measure ∝ E-field

$$J_{RLD}(T) = A_{RLD}T^2 \exp\left(-\frac{\Phi}{k_B T}\right)$$

T = Source Temp (Absolute)
A_{RLD} = const

$$J_{MFD}(\lambda) = \frac{q}{\hbar\omega}(1-R)F_{\lambda}(\omega)\{\hbar\omega-\Phi\}^2 I_{\lambda}$$

$\hbar\omega$ = photon Energy

Others – Factors associated with Laser & Material/Barrier

Electron Emission

Thermal

Emitted from Maxwellian tail of Fermi-Dirac distribution of conduction electrons with current density

$$\boxed{J = A T^2 e^{-W/(k_B T)} \qquad A = \frac{4\pi e m k_B^2}{h^3} = 1.2\times10^6 \frac{\text{Amp}}{\text{meter}^2 \cdot {}^\circ\text{K}^2}}$$

Richardson - Dushman Eqn.
Phil. Mag 28 633 (1914)
Phys Rev. 21 623 (1923)

T = cathode temp. (absolute)
W = work function (few eV)

k_B = Boltzmann's const (°K units)
$= 8.6175\times10^{-5}\ \frac{\text{eV}}{{}^\circ\text{K}}$

m = e^- mass $= 9.11\times10^{-31}$ kg
e = e^- charge $= 1.6\times10^{-19}$ Coulomb
h = Plancks' constant
$= 6.63\times10^{-34}$ J·sec

- In lab A often $\sim 2\times$ lower than formula.
- Thermionic cathode fabrication difficult.
  - Low W
  - Long life at high temp.
  - Smooth surface

  } Tungsten common
  $W = 4.5$ eV
  $T \sim 2500\ {}^\circ\text{K} \Rightarrow k_B T \sim 0.2$ eV
  $J \sim 10\text{-}20\ \frac{\text{Amp}}{\text{cm}^2}$

Photoemitters (not covered)

Often used for short pulse e^- sources for XFEL facilities and e-microscopes.

Field Emitters

Used on electron microscopes etc for compact bright sources.

Very Diverse Topic.

Much in refs. USPAS cources on Photoemission & High Brightness Electron Injectors.

Ion Emission

Generally more complex than electron sources.

Many different technologies for:

- Light Ions
- Heavy Ions
- Negative Ions (H^-)

} Plasma often used or "Hot Plate" emitter.

Often plasma is confined by a magnetic field so ions can be born "magnetized"

- Generates phase-space correlations with more intricate beam dynamics downstream.

At NSCL / FRIB ECR = Electron Cyclotron Resonance ion sources used

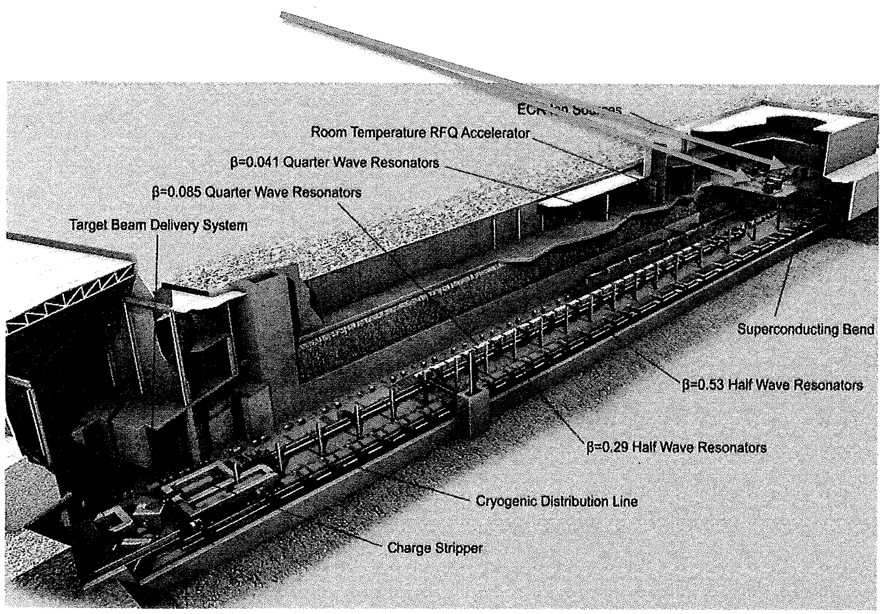
- Produce a spectrum of high charge state ions that must be seperated downstream.

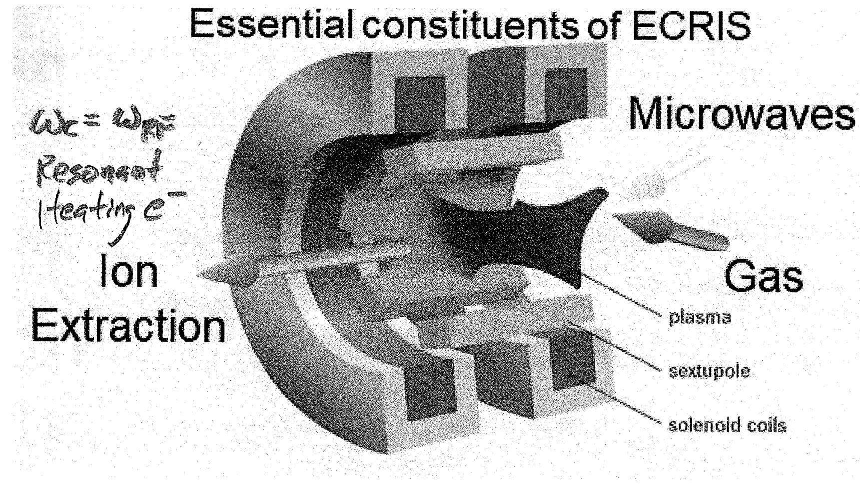
6/

Accelerator Facilities Use Electron Cyclotron Resonance Ion Source (ECRIS) to produce High Intensity, High Charge State Ion Beams

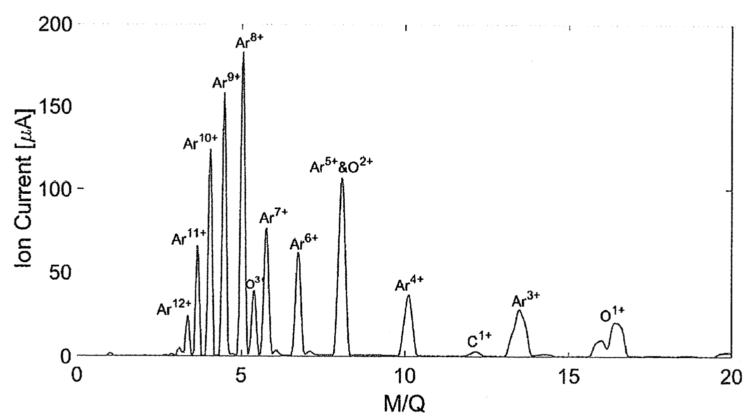
From B. Isherwood

FRIB
Two ECRIS:
1. Room temperature 14 GHz ARTEMIS-B (Commissioning)
2. Superconducting 28 GHz VENUS based ECRIS (2020)





- Magnetic confinement and microwave heating are used to induce ionization
  - GHz range microwaves ignite source plasma
  - Solenoids and hexapole field confine the plasma

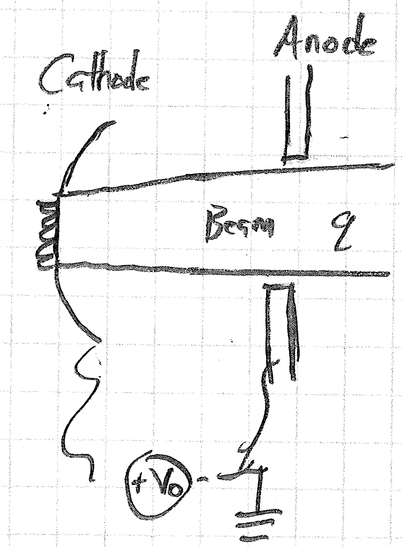


Charge State Distribution from ARTEMIS-B

18th International Conference on Ion Sources, Lanzhou, China: Slide 4

National Science Foundation
Michigan State University
NSF Grant 1632761

For both electrons and ions, cannot extract arbitrary current. Will ultimately be limited by charge already emitted pushing back.



- Particles accelerate in gap potential
- For fixed current density J at emission, charge density drops with distance from emitter

Poissons' Equation

$$\frac{d^2\phi}{dz^2} = \frac{-qn}{\varepsilon_0}$$

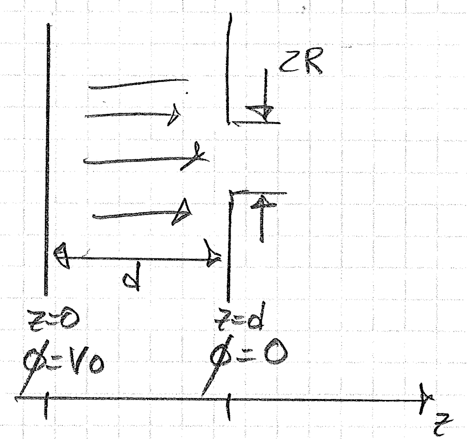
$$E_z = -\frac{\partial\phi}{\partial z}$$

E_z will push back and suppress emission.

Limiting Current: "Child-Langmuir" value.

Analyze for steady flow; $\partial/\partial t = 0$.

Model ions as a cold fluid:



$n(z,t)$ = Density
$V_z(z,t)$ = Flow Velocity

m = ion mass
q = ion charge

Charge Conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

$\rho = qn$
$\vec{J} = qnV_z\hat{z}$

$$\Rightarrow \boxed{\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nV_z) = 0}$$ Continuity Equation

Momentum (NonRel. + Cold Fluid)

$$n\frac{d}{dt}(m\vec{V})_z = \underbrace{qn\left[\vec{E} + \vec{V}\times\vec{B}\right]_z}_{\text{Field Coupling}} - \cancel{\nabla(P)_z}^{\,0}$$

Cold Fluid
No Pressure

$\frac{d}{dt} = \frac{\partial}{\partial t} + V_z\frac{\partial}{\partial z}$ convective Der. z-comp

$\vec{E}$ = Elec. Field
$\vec{B}$ = Mag Field = 0

$$\boxed{mn\frac{\partial V_z}{\partial t} + mnV_z\frac{\partial V_z}{\partial z} = qnE_z}$$ Force Equation

Field (Electrostatic)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{E} = -\nabla\phi \qquad E_z = -\frac{\partial \phi}{\partial z}$$

$$\boxed{\frac{\partial^2 \phi}{\partial z^2} = \frac{-qn}{\varepsilon_0}}$$ Poisson Equation

Analyze equations assuming steady $\partial/\partial t = 0$ flow. $n = n(z)$, $V_z = V_z(z)$, $\phi = \phi(z)$

1) Continuity Equation

$$\cancel{\frac{\partial}{\partial t} n}^{\,0} + \frac{\partial}{\partial z}(n V_z) = 0 \quad \Rightarrow \quad \boxed{q n V_z = \text{const} \equiv J} \quad (1)$$

- Notice this implies $n \to \infty$ as $V_z \to 0$ (near emitter). This is idealized, but still expect very large n near emitter.

2) Force Equation

$$\cancel{m \frac{\partial V_z}{\partial t}}^{\,0} + m V_z \frac{\partial}{\partial z} V_z = q E_z = -q \frac{\partial \phi}{\partial z}$$

$$\frac{1}{2} m \frac{\partial}{\partial z} V_z^2 = -q \frac{\partial \phi}{\partial z} \quad \Rightarrow \quad \frac{1}{2} m V_z^2 \Big|_{z=0}^{z} = -q \phi \Big|_{z=0}^{z}$$

$z=0$: $\phi = V_0$, $\Phi = 0$ $z=d$: $\phi = 0$, $\Phi = V_0$

$V_z(0) \simeq 0 \Rightarrow$ particles emitted near rest. $\phi(0) = V_0$

$$\frac{1}{2} m V_z^2 = q V_0 - q \phi \quad \Rightarrow \quad \boxed{\frac{1}{2} m V_z^2 = q \Phi} \quad (2) \qquad \Phi \equiv V_0 - \phi$$

convenient ref choice

3) Poisson Equation

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{-qn}{\varepsilon_0} \quad \Rightarrow \quad \frac{\partial^2 \Phi}{\partial z^2} = \frac{qn}{\varepsilon_0} \overset{\text{Use(1)}}{=} \frac{J}{\varepsilon_0 V_z} \overset{\text{Use(2)}}{=} \frac{J}{\varepsilon_0 \left(\frac{2q\Phi}{m}\right)^{1/2}}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial z^2} = \frac{J \, \Phi^{-1/2}}{\varepsilon_0 (2q/m)^{1/2}}}$$

$$\underset{\text{solve}}{\Rightarrow} \quad \text{Mult} \ \frac{\partial \Phi}{\partial z} \quad \Rightarrow \quad \frac{\partial \Phi}{\partial z} \frac{\partial^2 \Phi}{\partial z^2} = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} \frac{\partial \Phi}{\partial z} \Phi^{-1/2}$$

$$\Rightarrow \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \Phi}{\partial z}\right)^2\right] = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} \frac{\partial}{\partial z}\left(2 \Phi^{1/2}\right) \quad \Rightarrow \quad \frac{1}{2}\left(\frac{\partial \Phi}{\partial z}\right)^2 \Big|_{z=0}^{z} = \frac{J}{\varepsilon_0 (2q/m)^{1/2}} 2 \Phi^{1/2} \Big|_{z=0}^{z}$$

But $\Phi(z=0) = V_0 - \phi(0) = 0$

$\frac{\partial \Phi}{\partial z}(z=0) = -\frac{\partial \phi}{\partial z}\Big|_{z=0} = E_z(0) = 0$

$$\Rightarrow \boxed{\frac{\partial \Phi}{\partial z} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2} \left(\frac{m}{2q}\right)^{1/4} \Phi^{1/4}}$$

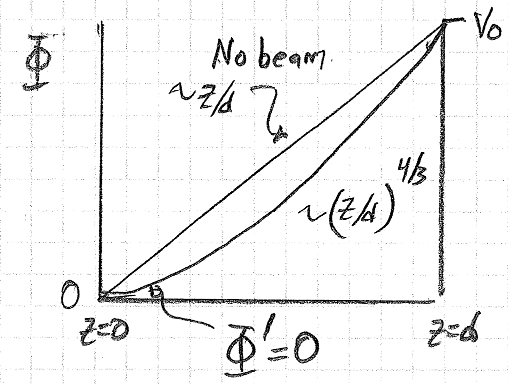
Space-Charge Limited Current
as much charge drawn off as possible for $E_z\big|_{z=0} = 0$ with a steady solution reached at stagnation.

Integrate Poisson's Equation:

$$\frac{d\Phi}{\Phi^{1/4}} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2}\left(\frac{m}{Zq}\right)^{1/4} dz$$

$$\frac{4}{3}\,\Phi^{3/4}\Big|_{z=0}^{z} = \left(\frac{4J}{\varepsilon_0}\right)^{1/2}\left(\frac{m}{Zq}\right)^{1/4} z\Big|_{z=0}^{z} \Rightarrow \boxed{\Phi(z) = \left(\frac{3}{4}\right)^{4/3}\left(\frac{4J}{\varepsilon_0}\right)^{2/3}\left(\frac{m}{Zq}\right)^{1/3} z^{4/3}}$$

Require: (consistency) $\Phi(d) = V_0 \Rightarrow \Phi = V_0\left(\frac{z}{d}\right)^{4/3} \Rightarrow V_0 = \left(\frac{3}{4}\right)^{4/3}\left(\frac{4J}{\varepsilon_0}\right)^{2/3}\left(\frac{m}{Zq}\right)^{1/3} d^{4/3}$



Solve for J:

$$\left(\frac{4J}{\varepsilon_0}\right)^{2/3} = \left(\frac{4}{3}\right)^{4/3}\left(\frac{Zq}{m}\right)^{1/3}\frac{V_0}{d^{4/3}}$$

$$J = \frac{\varepsilon_0}{4}\left(\frac{4}{3}\right)^{4/2}\left(\frac{Zq}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2}$$

$$= \frac{4}{9}\varepsilon_0\left(\frac{Zq}{m}\right)^{1/2}\frac{V_0^{3/2}}{d^2}$$

– or –

Current Density

$$J = \text{const}\,\frac{V_0^{3/2}}{d^2} = \text{Child-Langmuir Current Density}$$

Also called "Child-Langmuir Law"

$$\text{const} = \frac{4}{9}\varepsilon_0\left(\frac{Zq}{m}\right)^{1/2}$$

Current

$$I = \pi R^2 J = \frac{4\sqrt{2}\,\pi}{9}\varepsilon_0\left(\frac{q}{m}\right)^{1/2}\left(\frac{R}{d}\right)^2 V_0^{3/2} = \begin{cases} 0.232\left(\frac{R}{d}\right)^2\left(V_0\,[\text{kV}]\right)^{3/2}\ \text{A} & \text{electrons (sign flip)} \\ 5.43\left(\frac{Q}{M}\right)^{1/2}\left(\frac{R}{d}\right)^2\left(\frac{V_0}{1\,\text{kV}}\right)^{3/2}\ \text{mA} & \text{ions} \end{cases}$$

R = Radius Beam Aperture

$m = A\,m_u$
$q = Qe$

Want for source: $J = \text{const} \cdot V_0^{3/2}/d^2$

J high $\Rightarrow$ V_0 large, d small

But must also suppress:

1) <u>Voltage Breakdown</u>

$$V_0 \lesssim 100\,\text{kV} \begin{cases} \left(\frac{d}{1\text{cm}}\right) & \text{for } d \lesssim 1\,\text{cm} \\ \left(\frac{d}{1\text{cm}}\right)^{1/2} & \text{for } d > 1\,\text{cm} \end{cases}$$

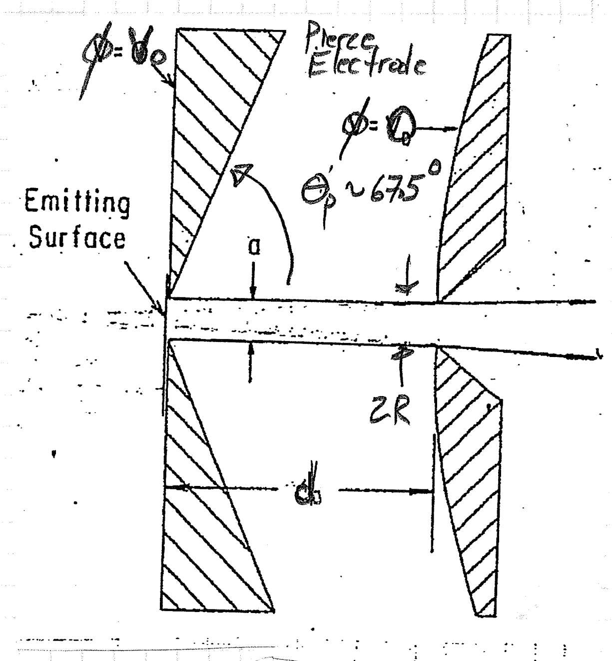
Suppress to avoid distruction of hardware

J. Kwan, LBNL

Practical observation based with real hardware. see A. Chao Accelerator Handbook World Press.

2) <u>Optical Abberations</u>

Nonlinear fields must be suppressed to preserve good beam quality.



Angle helps suppress radial beam expansion due to space charge for finite radial extent beam.

A.T. Forrester Wiley, 1988

Real Beam 3D <u>not</u> 1D.

Find angled "Pierce Electrodes" can be used to keep beam approx 1D. But even with detailed code optimization, And must have

$$d \gtrsim (3 \text{ to } 4)\, R$$

R = beam apeture radius.

Note that:

<u>current density</u>

$$J = \text{const}\ \frac{V_0^{3/2}}{d^2} \sim \begin{cases} V_0^{-1/2} \sim d^{-1/2} & d \lesssim 1\,\text{cm} \quad (V_0 \sim d) \\ V_0^{-5/2} \sim d^{-5/4} & d > 1\,\text{cm} \quad (V_0 \sim d^{1/2}) \end{cases}$$

<u>Current</u>

$$I = \pi R^2 J \sim R^2 \frac{V_0^{3/2}}{d^2} \sim V_0^{3/2}$$ "Child-Langmuir Scaling"

$R \sim d$ to control abberations.

$\therefore$ J <u>decreases</u> as source size increases

But I <u>increases</u> as source size increases

Also, want sources that deliver maximum current in phase-space volume measures of the beam.

Will find later that normalized rms x- and y-emittances measure $\perp$ phase-space areas of the beam:

$$\varepsilon_{nx} = (\gamma\beta)\left[\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2\right]^{1/2}$$

$$\varepsilon_{ny} = (\gamma\beta)\left[\langle y^2\rangle\langle y'^2\rangle - \langle yy'\rangle^2\right]^{1/2}$$

$\perp$ Phase-Space Volume $\sim \varepsilon_{nx}\varepsilon_{ny}$

Beam is not converging or diverging:

$$\langle x x' \rangle = 0$$

For ~uniform density beam in aperture:

$$R = 2\langle x^2 \rangle^{1/2}$$

For NR ions born at temperature T:

$$\gamma \simeq 1$$

$$\beta^2 \langle x'^2 \rangle = \frac{1}{c^2} \langle v_x^2 \rangle \qquad \frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} k_B T \qquad \frac{1}{2} k_B T \text{ per DOF}$$

$$\beta \langle x'^2 \rangle^{1/2} = \frac{1}{c} \left(\frac{k_B T}{m} \right)^{1/2}$$

Thus

$$\varepsilon_{nx} = \gamma\beta \left[\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2 \right]^{1/2} \simeq \beta \langle x^2 \rangle^{1/2} \langle x'^2 \rangle^{1/2} = \frac{R}{2} \left(\frac{k_B T}{m c^2} \right)^{1/2}$$

$$\boxed{\text{Phase Volume} \sim \varepsilon_{nx} \varepsilon_{ny} = \frac{R^2}{4} \frac{k_B T}{m c^2}}$$

Assume x and y plane T equal.

This leads naturally to a measure of source performance called "Brightness" defined as

$$J = \frac{4}{9} \varepsilon_0 \left(\frac{2q}{m} \right)^{1/2} \frac{V_0^{3/2}}{d^2}$$

$$B \equiv \text{Transverse Brightness} = \frac{\text{Current}}{\text{Phase-Space Vol.}} = \frac{I}{\varepsilon_{nx} \varepsilon_{ny}} = \frac{\pi R^2 J}{\frac{R^2}{4} \frac{k_B T}{m c^2}}$$

$$\boxed{B = \frac{16\pi}{9} \varepsilon_0 \left(\frac{2q}{m} \right)^{1/2} \frac{V_0^{3/2}}{d^2 \left(k_B T / m c^2 \right)}} \propto \frac{V_0^{3/2}}{d^2 T}$$

Note:

- B independent of source radius R
- Want ions as low a temp T as possible and high a voltage V_0 as possible $\Big\}$ Hard to do

Summary: Sources

- Well functioning sources crucial for accelerator:
  - Bright: compact phase-space $\Leftrightarrow$ compact beam
    - smaller (more economical) structures
    - lower losses to mitigate damage
- Improving sources allows running higher intensity to get more out of accelerator.
  - Upgrade source $\Leftrightarrow$ Upgrade accelerator
- Need also high reliability for long periods of time
  - Source not operational, then expensive machine will not work
- Sources high leverage, but also difficult:
  - Technology and physics strongly coupled
  - Rely on difficult material science, plasma physics, etc.

  Possible to teach a whole course on just one type of source technology.
- Many types of sources. Only scratched surface in this lecture:

Electrons	Ions	Radioactive	Antiparticle
Thermionic emitters	hot plate (low Q)	Capture/cool	Accel. drives source - capture, cool, inject
Field Emission (point)	plasma (higher Q) discharge ECR .... laser driven		
Photocathodes	e⁻ beam driven		

- Source projects great for PhDs: rich physics, improvements high leverage