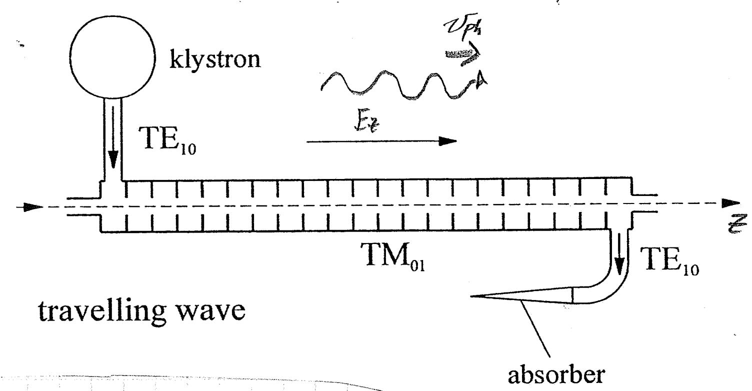
Traveling Wave RF Acceleration

See for more info:
Wangler, RF Linear Accelerators, Chapter 3
Wille, The Physics of Particle Accelerators, Chapter 5.

Rather than exchanging energy with an RF standing wave in a fixed cavity, a traveling RF wave can be setup to resonate with a particle to accelerate it.

Consider an EM wave propogating along $\hat{z}$ in a waveguide structure:



Wille

$$\boxed{E_z(z,t) = E(z)\cos\left[\omega t - \int_0^z k(z)\,dz + \phi\right] \qquad k(z) = \frac{\omega}{v_p(z)} \qquad v_p = \text{phase velocity of wave} = \frac{\omega}{k}}$$

For efficient acceleration of a particle with axial velocity v_z, want

$$\boxed{v_z \simeq v_p}$$

* deviations will result in wave "slipping" in phase particle

Consider a particle of charge q with $v_z = v_{ph}$ at each instant in time. Then the particle arrives at position z at time

$$\boxed{t(z) = \int_0^z \frac{dz}{v_z(z)}}$$

So the E_z at the particle is:

$$E_z = E(z) \cos\left[\omega t - \int_0^z k(z)dz + \phi\right] \qquad k(z) = \frac{\omega}{v_p(z)}$$

$$= E(z) \cos\left[\cancel{\omega \int_0^z \frac{dz}{v_z(z)}} - \cancel{\omega \int_0^z \frac{dz}{v_p(z)}} + \phi\right] = E(z) \cos\phi$$

In the traveling wave approach, this particle is called the synchronous particle and $\phi = \phi_s$. The synchronous particle will gain kinetic energy.

$$\boxed{\Delta W_s = q \int_0^z E(z) \cos\phi_s \, dz}$$

As previously discussed, $v_p > c$ for a simple cylindrical pipe waveguide. But periodic structures can be placed in the waveguide to produce partial reflections to reduce the phase velocity to satisfy $v_p \leq c$. The structure can be analyzed as a periodic array of coupled/resonant cavities. Various techniques exist to analyze this situation and gain intuition; See Wangler Chapter 3.

Iris Loaded Waveguide for $v_p < c$

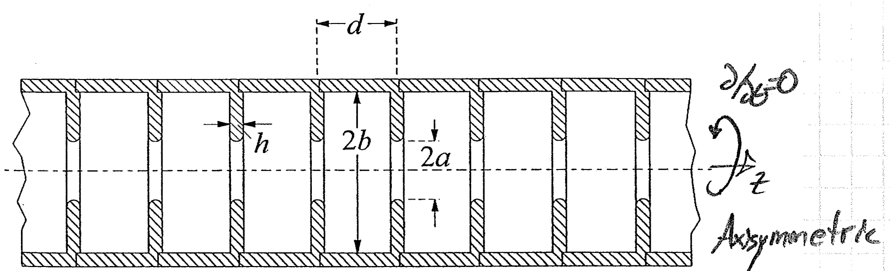
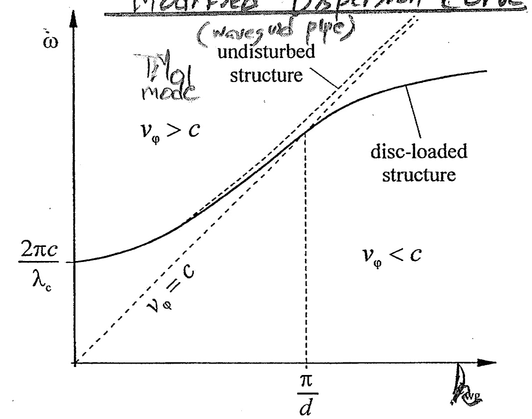


Fig. 5.6 Cross-section through a typical linac structure. The phase velocity of the RF wave is reduced to the particle velocity by the insertion of irises.

Wille

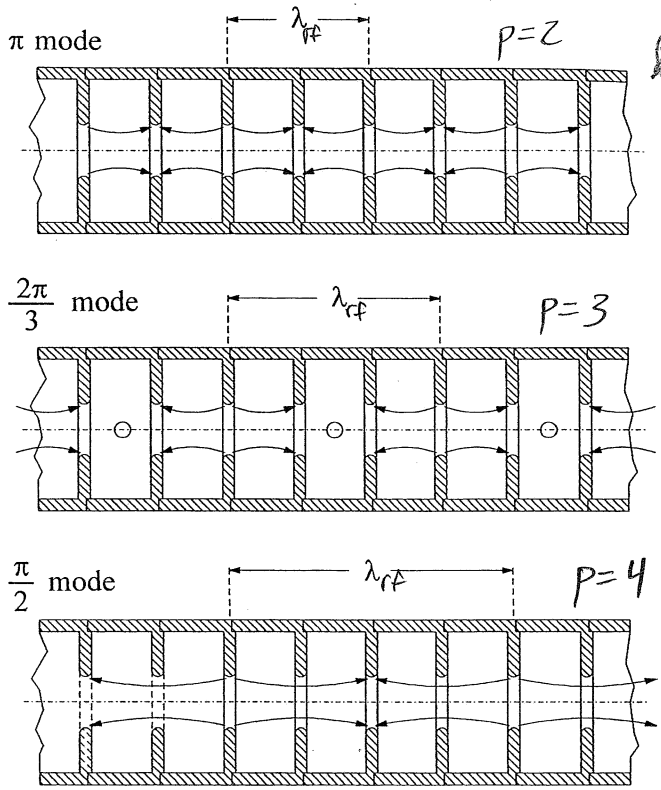
Modified Dispersion Curve



Wille

Fig. 5.7 Dispersion curve for a cylindrical waveguide, with and without irises. The frequency ω is plotted as a function of the wavenumber k_z of the waveguide.

Analogously to cavities in a linac, various wave symmetries are compatible with acceleration!



$p=2$ $k_z d = \pi$ ← Requires long settle time so not good for short pulse

$p=3$ $k_z d = \frac{2\pi}{3}$ ← Short settle time and high shunt impedance. Most common operation

$p=4$ $k_z d = \frac{\pi}{2}$ ← Low shunt impedance so energy gain modest relative to RF power

Wille

Fig. 5.10 The field configurations of the three most important modes in linac structures.

In smooth waveguides all k above cutoff allowed.

For disc loaded waveguide only

$$\lambda_{rf} = p\,d \qquad p = 1,2,3,\dots$$

d = Iris seperation

allowed.

$$\Rightarrow \quad k_z = \frac{2\pi}{\lambda_{rf}} = \frac{2\pi}{pd}$$

$$p = 1,2,3,\dots$$

Analysis shows the <u>max</u> energy gain of a particle : see Wille

☆ Max $\Longleftrightarrow$ Synchronous condition : $v_z = v_p$

☆ Note: Traveling wave structures often used for <u>relativistic electrons</u> with $\beta \simeq 1$, So there is <u>no need for phase focusing and operation is for max acceleration</u>. The only need is for the bunch to fit into the bucket.

$$\boxed{\Delta W = K \cdot \sqrt{P_{RF}\, \ell \left(\frac{R_s}{L}\right)}}$$

Max Kinetic Energy Gain

K = correction factor $\simeq 0.8$ typical

P_{RF} = Supplied RF Power

ℓ = Interaction length

$\left(\frac{R_s}{L}\right)$ = Shunt Impedance per meter of structure.

Empherical formula applied that works well.

$$\left(\frac{R_s}{L}\right) = 5.12 \times 10^8 \cdot \frac{\beta(1-\eta)^2}{p + 2.61\,\beta(1-\eta)} \left(\frac{\sin D/2}{D/2}\right)^2$$

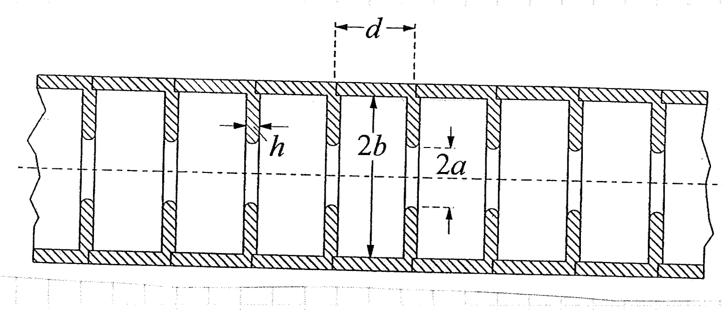
$\beta = v_p/c$

$\eta = \frac{h}{d}$

p = # irises per wavelength

$D = \frac{2\pi}{p}(1-\eta)$

SLAC Structure



$2b = 82.5$ mm

$2a = 22.6$ mm

$h = 5.8$ mm

$d = 35.0$ mm

For the SLAC traveling wave linacs:

$\beta \simeq 1$ Relativistic e^-

$p = 3$ $\frac{2\pi}{3}$ Mode

$\left(\frac{R_s}{L}\right) = .53\times 10^6 \frac{\Omega}{\text{meter}}$

$P_{RF} = 35\,MW$

$l = 3\,\text{meter}$

Gives

$$\Delta W = K\sqrt{P_{RF}\, l \left(\frac{R_s}{L}\right)} = 59.7\,MV$$

$$\Rightarrow \frac{\Delta W}{l} = \frac{59.7\,MV}{3\,\text{meter}} = 19.9\,\frac{MV}{\text{meter}}$$

Typical Values achieved for traveling wave structures $15\frac{MV}{m} \rightarrow 20\frac{MV}{m}$

Values $\sim 100\frac{MV}{m}$ possible for short structures.

Typically only short pulses possible to avoid heat damage to normal conducting structure.

Want very high gradients for any future linear $e^+ e^-$ collider?

- Otherwise linac too long
- Synchrotron radiation precludes ring.