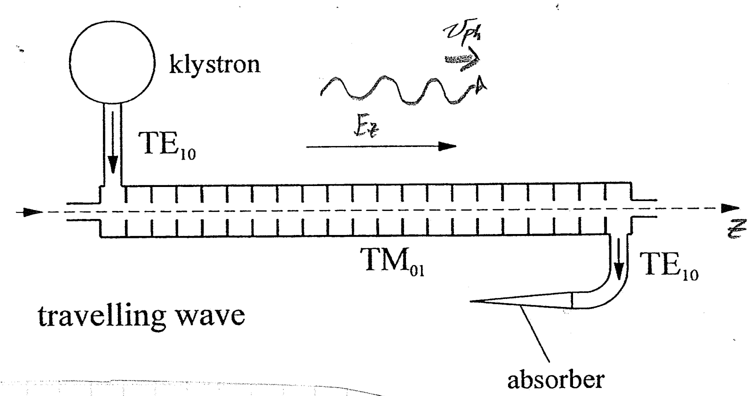
Traveling Wave RF Acceleration

See for more info:
Wangler, RF Linear Accelerators, Chapter 3
Wille, The Physics of Particle Accelerators, Chapter 5.

Rather than exchanging energy with an RF standing wave in a fixed cavity, a traveling RF wave can be setup to resonate with a particle to accelerate it.

Consider an EM wave propogating along $\hat{z}$ in a waveguide structure:



Wille

$$E_z(z,t) = E(z)\cos\left[\omega t - \int_0^z k(z)\,dz + \phi\right]$$

$$k(z) = \frac{\omega}{v_p(z)} \qquad v_p = \text{phase velocity of wave.} = \frac{\omega}{k}$$

For efficient acceleration of a particle with axial velocity v_z, want

$$\boxed{v_z \simeq v_p}$$

* deviations will result in wave "slipping" in phase
particle

Consider a particle of charge q with $v_z = v_{ph}$ at each instant in time. Then the particle arrives at position z at time

$$\boxed{t(z) = \int_0^z \frac{dz}{v_z(z)}}$$

So the E_z at the particle is:

$$E_z = E(z)\cos\left[\omega t - \int_0^z k(z)\,dz + \phi\right] \qquad k(z) = \frac{\omega}{v_p(z)}$$

$$= E(z)\cos\left[\cancel{\omega\int_0^z \frac{dz}{v_z(z)}} - \cancel{\omega\int_0^z \frac{dz}{v_p(z)}} + \phi\right] = E(z)\cos\phi$$

In the traveling wave approach, this particle is called the <u>synchronous particle</u> and $\phi = \phi_s$.
The synchronous particle will gain kinetic energy.

$$\boxed{\Delta W_s = q\int_0^z E(z)\cos\phi_s\,dz}$$

As previously discussed, $v_p > c$ for a simple cylindrical pipe waveguide. But periodic structures can be placed in the waveguide to produce partial reflections to reduce the phase velocity to satisfy $v_p \lesssim c$. The structure can be analyzed as a periodic array of coupled resonant cavities. Various techniques exist to analyze this situation and gain intuition; See Wangler Chapter 3.

<u>Iris Loaded Waveguide for $v_p < c$</u>

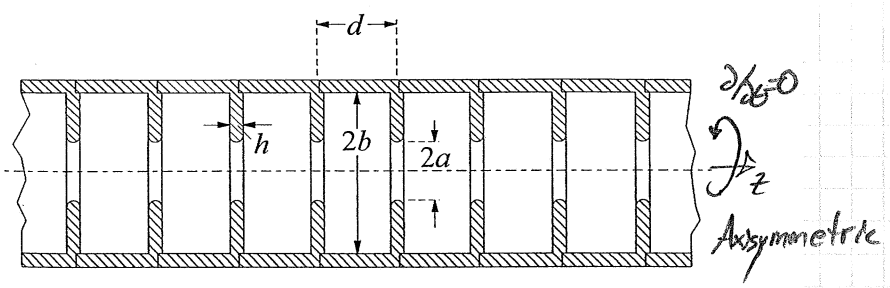
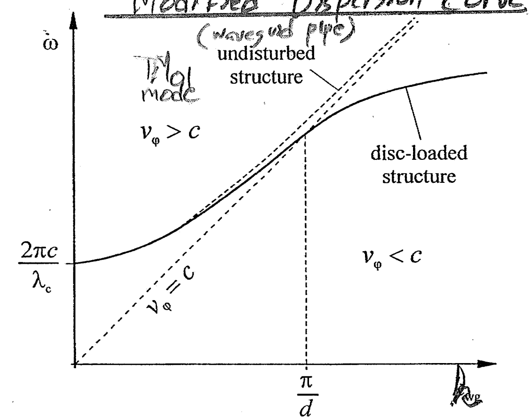


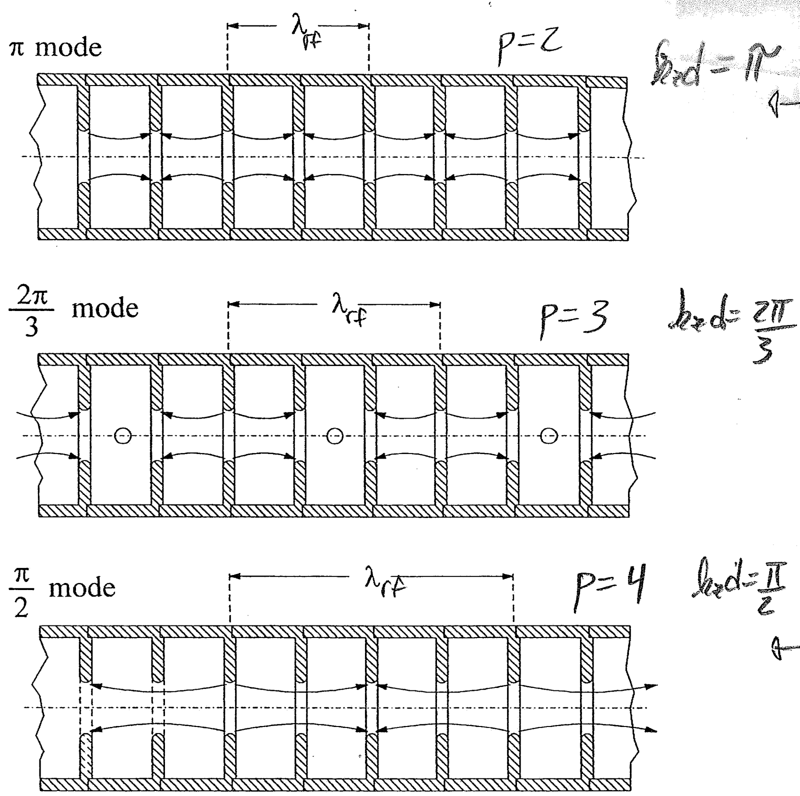
Fig. 5.6 Cross-section through a typical linac structure. The phase velocity of the RF wave is reduced to the particle velocity by the insertion of irises.
Wille

<u>Modified Dispersion Curve</u>



Wille
Fig. 5.7 Dispersion curve for a cylindrical waveguide, with and without irises. The frequency ω is plotted as a function of the wavenumber k_z of the waveguide.

Analogously to cavities in a linac, various wave symmetries are compatible with acceleration!



Requires long settle time so not good for short pulse

Short settle time and high shunt impedance. <u>Most common operation</u>

Low shunt impedance so energy gain modest relative to RF power

Wille

Fig. 5.10 The field configurations of the three most important modes in linac structures.

In smooth waveguides all k above cutoff allowed.

For disc loaded waveguide only

$$\lambda_{rf} = p\,d \qquad p = 1, 2, 3, \ldots$$

d = Iris seperation

allowed.

$$\Rightarrow \quad k_z = \frac{2\pi}{\lambda_{rf}} = \frac{2\pi}{pd}$$

$p = 1, 2, 3, \ldots$

Analysis shows the max energy gain of a particle : see Wille

☆ Max $\iff$ Synchronous condition : $v_z = v_p$

☆ Note: Traveling wave structures often used for relativistic electrons with $\beta \simeq 1$, So there is no need for phase focusing and operation is for max acceleration. The only need is for the bunch to fit into the bucket.

$$\boxed{\Delta W = K \cdot \sqrt{P_{RF}\, l \left(\frac{R_s}{L}\right)}} \quad \text{Max Kinetic Energy Gain}$$

K = correction factor $\simeq 0.8$ typical

P_{RF} = Supplied RF Power

l = Interaction length

$\left(\frac{R_s}{L}\right)$ = Shunt Impedance per meter of structure.

Empherical formula applied that works well.

$$\left(\frac{R_s}{L}\right) = 5.12 \times 10^{8} \cdot \frac{\beta(1-\eta)^2}{p + 2.61\,\beta(1-\eta)} \left(\frac{\sin D/2}{D/2}\right)^2$$

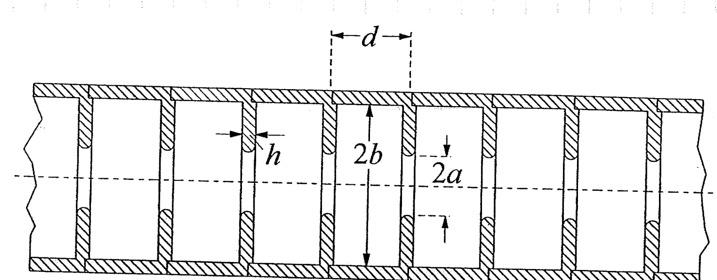
$\beta = v_p/c$

$\eta = \frac{h}{d}$

p = # irises per wavelength

$D = \frac{2\pi}{p}(1-\eta)$

SLAC Structure



$2b = 82.5$ mm

$2a = 22.6$ mm

$h = 5.8$ mm

$d = 35.0$ mm

For the SLAC traveling wave linacs:

$\beta \simeq 1$ Relativistic e^-

$p = 3$ $\frac{2\pi}{3}$ Mode

$\left(\frac{R_s}{L}\right) = .53 \times 10^6 \frac{\Omega}{\text{meter}}$

$P_{RF} = 35\,MW$

$l = 3\,\text{meter}$

Gives

$$\Delta W = K\sqrt{P_{RF}\, l \left(\frac{R_s}{L}\right)} = 59.7\,MV$$

$$\Rightarrow \frac{\Delta W}{l} = \frac{59.7\,MV}{3\,\text{meter}} = 19.9\,\frac{MV}{\text{meter}}$$

Typical Values achieved for traveling wave structures $15\frac{MV}{m} \to 20\frac{MV}{m}$

Values $\sim 100\frac{MV}{m}$ possible for short structures.

Typically only short pulses possible to avoid heat damage to normal conducting structure.

Want very high gradients for any future linear $e^+ e^-$ collider ?

- Otherwise linac too long
- Synchrotron radiation precludes ring.